Automatic accurate surface reconstruction of a class of wrap-around models

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Abstract: For a wrap-around surface, many issues, such as difficulty in parameterisation of data points, incapability of automatic modelling, and high-accuracy surface reconstruction, need to be resolved. Therefore, an automatic accurate non-uniform rational B-spline (NURBS) surface reconstruction method is proposed in the present paper. First, after a method that recognises the central axis from data points is given, the data points in physical space are projected to a plane domain. Consequently, the boundary curve is extracted. Second, by local surface fitting of the data points on sector-distributed cross-sectional plane, the unorganised scatter points become ordered array points. A base surface is created using skinning technology. Finally, using robust arithmetic of the multivariate Bernstein-form polynomials, an algorithm for calculating the closest point is proposed. Using base-surface parameterisation, an accurate surface fitting is finally implemented based on least square iterative procedure. Examples verify the feasibility and validity of the proposed surface reconstruction method.

Keywords: point clouds; boundary extraction; surface reconstruction.


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1 Introduction

Rapid development in manufacturing technology of products based on reverse engineering (Varady et al., 1997) is one of the most impressing high-technology progress in current manufacturing, drawing increasing attention from all industries. Research regarding measurement, modelling, and machining intelligent machining system based on reverse engineering enjoys rapid growth, and the system finds substantial applications in die and aviation manufacturing (Ding et al., 2003; Weiss et al., 2002). The system receives high demand on precision and automation for surface measurement and modelling. Although the current proliferation of CAD software can process surface reconstruction, it is a laborious and error-prone process because it involves frequent manual operations through interactivity. Therefore, automatic accurate surface reconstruction is very difficult.

Wrap-around surface is a commonly used surface in die cavities and big shell parts. Reconstructing this surface is difficult due to the absence of clearly defined rectangular boundaries. Weir et al. (2000) constructed a base surface by manually selecting the grid data of point sets, and a non-uniform rational B-spline (NURBS) surface was obtained by iterative procedure. Zhao et al. (2006) proposed a modelling strategy for a wrap-around NURBS surface by deciding the central axis of point clouds in advance. Sun et al. (2006) proposed a modelling method of bilinear surface for a wrap-around model. The drawback of this method was that the four boundary corner points required to be manually selected from sampled points. Thus, reconstructing the wrap-around model automatically was a difficult task. The difficulty in reconstructing a high-precision free-form surface lies in the parameterisation of the point clouds. Various methods of parameterising the data points are available in which base-surface parameterisation is one of the commonly used methods. Piegl and Tiller (2001) attempted to use various base surfaces to project the points onto these surfaces and recover the parameters based on surfaces in rectangular domain. Ma and Kruth (1995) presented a simple technique of constructing B-spline base surfaces for randomly measured points using least square fitting by interactively defining section curves. Parameterisation was realised by projecting the measured points to a base surface. Azariadis (2004) presented a method of dynamic base surfaces adapted to a three-dimensional (3D) shape implied by clouds of points. Therefore, for parameterisation of irregularly spaced and randomly distributed data points, a simple base surface is usually created, which roughly represents the global shape of the cloud of points. This shape can be a plane, a cylinder, or bilinearly blended Coon surfaces. Obviously, these surfaces may fail if the shape of the point clouds is complex.

The present paper proposes a new method of surface reconstruction to achieve automatically wrap-around surface reconstruction and maintain tight tolerances. First, the boundary curves of point clouds are automatically extracted by identifying the central axis from the point clouds. Subsequently, the ordered data points in the cross-sectional plane are obtained via local surface fitting, and the base surface is constructed using skinning technology. Finally, accurate and smooth surface fitting is implemented based on an iterative procedure by considering the closest points of data points on the base surfaces as parametric values.
2 Automatic extraction of boundary curves of point clouds

To extract automatically the boundary curves of the point clouds, the data points in physical space are projected to a plane domain using the mapping method presented in the current paper. Every data point in the plane domain has its corresponding data point in the physical space. First, the rough central axis of the wrap-around model is obtained. Then, the data points in the physical space are transformed into data points in the plane domain according to the mapping relationship decided by the central axis.

2.1 Automatic recognition of the central axis

Let us assume that the sampled set of data points is \( P_i \) (\( i = 1, 2, \ldots, m \)) and the axis of the surface is \((c, \vec{v})\), where \( c \) denotes the direction vector and \( \vec{v} = P \times c \); \( P \) is an arbitrary point. Several methods are available to calculate the normal vectors of the data points, which can be obtained from local geometrical characteristics of neighbouring point sets of a point (OuYang and Feng, 2005):

\[
n_i = n_i / \| n_i \|.
\]  

(1)

Based on known conclusion (Potmann and Wallner, 2001), the following is obtained:

\[
\min (\vec{v}, c) = \sum_{i=1}^{n} (\vec{v} \cdot n_{i} + c \cdot (P_i \times n_{i}))^2
\]

with the coefficient matrix

\[
\begin{bmatrix}
P_i \times n_1 & n_1 \\
P_i \times n_2 & n_2 \\
\vdots & \vdots \\
P_i \times n_m & n_m
\end{bmatrix} \begin{bmatrix}
P_i \times n_1 & n_1 \\
P_i \times n_2 & n_2 \\
\vdots & \vdots \\
P_i \times n_m & n_m
\end{bmatrix}^{T}
\]

(3)

The solution of equation (2) is the eigenvector based on the least eigenvalue. Then, the axis is expressed as:

\[
l = c / \| c \|
\]

(4)

Figure 1 Mapping method of transforming 3D into 2D
2.2 Boundary curve extraction based on the mapping method

The central axis is obtained as shown in Figure 1. A perpendicular line to the axis is made from an arbitrary point \( P_i \) of data points, and the foot of the perpendicular line is designated as \( P_i' \). The vector of the perpendicular line is \( P_iP_i' \). Another vector is fixed, such as \((0, 0, -1)\). Vector \( a \) is the cross product of the fixed vector and the vector of the axis, and vector \( P_iP_i' \) is obtained from the cross product of vector \( a \) and the vector of the axis. The angle between vectors \( P_iP_i' \) and \( P_iP_i' \) is computed by

\[
\theta = \arccos\left(\frac{P_iP_i' \times P_iP_i'}{|P_iP_i'| |P_iP_i'|}\right)
\]

By assuming that one endpoint of the central axis is \( V \) and \( VP_i' \) is the projection length of the data point on the central axis, a one-to-one correspondence of the original point set in the physical space with the point set in the plane domain is obtained, with \( \theta \) as the \( X \)-coordinate and \( L \) as the \( Y \)-coordinate.

Figure 2  Extraction of boundary curves based on the mapping method, (a) data points of original physical space (b) planar data points by projection (c) boundary curves in the plane domain (d) boundary curves in physical space

Boundary curves of point clouds are usually extracted by interactivity. Automatically extracting these curves is difficult. The boundary extraction method presented in the current paper involves projecting the point sets in the physical space into the plane domain and extracting the boundary curves in the plane domain. Thereafter, the boundary curves in the plane domain are projected back to the 3D space. As a result, the boundary curves of the spatial data points are obtained. The main idea behind the boundary curve of planar data points is simple, which is to find the starting point from the planar point sets and then searching the entire data points either clockwise or counter-clockwise. The boundary points are recognised from the directional change between points. To improve the efficiency of the search, searching technology based on the radius of neighbourhood points is used, where radius \( R \) is set to be in accordance with the density of the data points.
3 Method of generating ordered data for the set of points

The point clouds of a freeform surface measured from a sample part are disordered scatter data. To obtain ordered data, a virtual reference frame is built with the Y-axis in accordance with the central axis, as shown in Figure 3. To obtain the point sets of the cross-sectional contours, the cross-sectional planes through the Y-axis are evenly distributed in a sector. The number of cross-sectional planes is fixed due to the fitting precision requirement, and the number of data points in the sectional plane is fixed. Consequently, rectangular array data points appear. We assume that the entire point cloud is divided by $n + 1$ cross-sectional contours and that $m + 1$ data points are evenly built in every cross-section. Let $D_1$ and $D_2$ be the boundary points located in a cross-sectional plane. Therefore, the position vector of every data point in the cross-section is expressed as follows:

$$X_i = D_1 + \frac{i}{m+1} (D_2 - D_1), \quad i = 1, \cdots, m + 1$$

Figure 3  Sliced cross-section (see online version for colours)

Thus, for every fixed position vector $X_i$, the neighbouring point set can be found to be distance away from the vector, which is less than a given certain value. Therefore, after the given neighbourhood set of points is fitted into a local surface, the intersection point between the fitted surface and position vector $X_i$ can be computed. The resulting intersection points are the ordered point set. The proposed method can also make the data points accurate and smooth simultaneously. To avoid singularity of explicit surface fitting, the least square local surface fitting method is used based on parameterisation of the local tangent space. Therefore, before the neighbourhood points are located, the local reference frame of the point must be fixed first. To maintain computation precision, one coordinate axis of the local reference frame must be in accordance with the Z-axis of the global reference frame. The Z-axis of the local reference frame used here is (Figure 4)

$$K = R / ||R||$$

The original point is $X_i$. Then, 20 points are found from the point set that have the least distance from line $G = X_i + Rt$. Letting the point set be $\{v_j \mid j = 1, \cdots, n\}$, the formula for computing the distance between these points and line $G$ is:

$$d_j = (v_j - X_i) \times R$$
Comparing the value of $d_j$, the 20 points of the least distance are the neighbourhood set of points of the point.

**Figure 4** Local reference frame

Thereafter, the 20 points are fitted into the local surface. Let us assume that the original point of the local reference frame is $X_i$, the $Z$-axis is $K$, and the neighbourhood set of point is $\{v_j | s = 1, \cdots, n\}$. Hence, the local reference frame of the point is $\{X_i; e_1, e_2, e_3\}$.

As shown in Figure 4, the $K$ direction (the $h$-axis direction) is regarded as direction axis $e_3$ of the local reference frame. Therefore, the equation of the plane through the origin and perpendicular to $K$ is expressed as:

$$K \cdot (X - X_i) = Ax + By + Cz + D = 0$$

(8)

Let $d_j = Ax_j + By_j + Cz_j + D$, $v_j^p = v_j - d_jk$, the unit vector of the $h$-axis $e_3 = k$, the unit vector of the $u$-axis $e_1 = (v_j^p - X_i)/|v_j^p - X_i|$, and the unit vector of the $v$-axis $e_2 = e_1 \times e_3$. Thus, the coordinate component in the local reference frame of a point in the neighbourhood set of points is:

$$(u_j, v_j, d_j) = \left( (v_j^p - X_i) \cdot e_1, (v_j^p - X_i) \cdot e_2, d_j \right)$$

(9)

The neighbourhood set of points of position vector $X_i$ is transformed into a new set of points of the local reference frame through the above coordinate transformation. Consequently, using the method of quadric surface fitting, the ordered points of the local set of points near point $X_i$ can be obtained. Let

$$z(u, v) = \sum_{i=0}^{j} \sum_{j=0}^{i} a_{i-j}u^{i-j}v^j$$

(10)

the neighbourhood set of points of point $X_i$ is fitted using the least square method. Thus, $J = \min_{a_{i-j}} \left[ \sum_{l=1}^{n} \left( z(u_k, v_k) - d_k \right)^2 \right]$.  

(11)

When computing for the solution of the least squares, the normal equation group is obtained by calculating the partial derivatives for the unknown variables. The solution is obtained by solving a linear equation system. Then, equation (12) can be arranged in the form $Ax = B$, where
Automatic accurate surface reconstruction of a class of wrap-around models

\[ A = \begin{bmatrix} u_1^2 & u_1 v_1 & u_1 & v_1 & v_1^2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_m^2 & u_m v_m & u_m & v_m & v_m^2 & 1 \end{bmatrix}, \quad x = \begin{bmatrix} a_{2,0} \\ \vdots \\ a_{0,2} \end{bmatrix}, \quad B = \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}. \]  \tag{12}

To ensure full rank of the coefficient matrix of the normal equation group, the number of neighbourhood points must not be less than six. As the number was already set to 20, the only solution is obtained by:

\[ x = (A^T A)^{-1} A^T B. \]  \tag{13}

After the equation for the local surface at point \( X_i \) is obtained, the next step is to compute the intersection point between line \( G = X_i + Rt \) through point \( X_i \) and the local surface. Then, the ordered data points from point \( X_i \) in the cross-sectional plane are:

\[ L_e = X_i + z(0,0)R. \]  \tag{14}

The rest can be deduced by analogy; the ordered data points are all obtained in the cross-sections. Then, adjusting the angle of the cross-sections, the ordered data points of the entire cloud of points are implemented.

4 Reconstruction method for base surface

A base surface can be constructed if the boundary curves and the inner cross-sectional data points are given. The question of fitting the surface model or interpolating the curves is simplified because the surface skinning technology is transformed; thus, the base surface is created using the method adopted in the current paper. From this condition, the base-surface reconstruction is seen to be composed of two steps. First is the NURBS curve fitting of the contour data points in the cross-sections and second is the surface skinning of the contour-curve family.

4.1 NURBS curve fitting of the contour data

B-spline can be defined as:

\[ C(v) = \sum_{i=0}^{n} B_i N_{i,k}(v) \]  \tag{15}

where \( B_i \) are the control points, \( n + 1 \) is the number of control points, and \( N_{i,d}(v) \) is the base function of the B-spline, recursively defined by:

\[ N_{i,k}(v) = \frac{v - t_i}{t_{i+k} - t_i} N_{i,k-1}(v) + \frac{t_{i+k+1} - v}{t_{i+k+1} - t_{i+1}} N_{i+1,k-1}(v) \]  \tag{16}

and

\[ N_{i,0}(v) = \begin{cases} 1 & t_i \leq v < t_{i+1} \\ 0 & \text{Otherwise} \end{cases} \]
where $v = [t_0, \cdots, t_{n+k+1}]$ is the knot vector and $k$ is the degree of the base function.

Given a set of discrete contour points $Q_i = [x_i, y_i, z_i]^T$, $i = 0, \cdots, m$, to find a fitting B-spline curve, location parameter $\{v_i\}_0^n$, degree $k$, the number of control points, and the set of knot $U$ must first be decided. The unknowns are the control points. To compute the location parameters from the data points, the following parameterisation formula are adopted:

\[
v_0 = 0,
\]

\[
v_i = v_{i-1} + \frac{\|Q_i - Q_{i-1}\|}{\sum_{j=1}^m \|Q_j - Q_{i-1}\|}, i = 1, \cdots, m
\]

(17)

where parameter $a$ is equal to 0, 0.5, or 1, which is the generalisation of the three methods, including the uniform, centripetal, and chord-length models. In addition, an appropriate knot vector $U$ must be selected according to $\{v_i\}_0^n$ and the number of control points.

When no constraints are given, the solution vector for the control points can be determined by minimising the following least square error:

\[
\min_{X} E(X) = \sum_{j=0}^m \|C(v_j) - Q_j\|^2
\]

(18)

In theory, the problem of minimisation of the least squares is equivalent to a linear system problem. The solutions can be expressed in matrix form as:

\[
P = \left[ N^T N \right]^{-1} N^T Q
\]

(19)

where

$P \in \mathbb{R}^{(m+1) \times 3}$

$Q \in \mathbb{R}^{(m+1) \times 3}$

$D = \text{diag} (w_1, \cdots, w_m)$

4.2 Skinning technology of the base surface

After the B-spline fitting of the boundary curves and the cross-sectional contours are obtained, the surface can be reconstructed through the curves. When the boundary and cross-sectional contours are fitted, the same number of control points and uniform knot vectors must be used due to the issue of curve compatibility. The base-surface skinning is assumed as:

\[
S(u, v) = \sum_{i=0}^{r} \sum_{j=0}^{s} B_{i,j} N_{i,k}(u) N_{j,k}(v)
\]

(20)

where the interval of knots of the $N_{i,k}(u)$ vector is defined by:
Automatic accurate surface reconstruction of a class of wrap-around models

\[
U = \left\{ 0, \ldots, 0, u_{k+1}, \ldots, u_r, \ldots, 1 \right\}_{k+1}.
\]

The interval of knots of the \(N_{i,j}(v)\) vector is defined by:

\[
V = \left\{ 0, \ldots, 0, u_{k+1}, \ldots, u_r, \ldots, 1 \right\}_{k+1}.
\]

Now, \((r + 1) \times (n + 1)\) control points of the surface must be located. As the cross-sectional contours and boundary curves are all defined by uniform knot vectors, the curves are defined as:

\[
C_j(v) = \sum_{j=0}^{n} d_{i,j}N_{i,j}(v), j = 1, \ldots, h
\]  

where \(h\) is the number of fitting curves.

Therefore, the control point \(B_{i,j} = d_{i,j} N_{i,j}(v)\) of the skinning surface can be obtained by B-spline fitting again of the \(i^{th}\) row control points \(d_{i,j}\) in equation (21).

5 Accurate surface reconstruction of point clouds

The purpose of creating the base surface is to obtain the optimum parameters of the data points. The fitting surface can be maintained with tight tolerance and smoothness simultaneously. Therefore, accurate reconstruction of the cloud of points includes parameterisation of the data points and iterative fitting of the reconstructed surface.

5.1 Parameterisation of data points

When the base-surface parameterisation method is used, the general method regards the corresponding parametric value of the projection points of the data points on the base surface as the parametric value of the data points. The projection point is the closest point of the data point on the surface. In computing the closest points of the surface, iterative optimisation methods are often used to find the solutions. When a good initial value is available, the result of the iterative approach is perfect. However, obtaining the initial value is difficult due to the complexity of the surface. To overcome this shortcoming, the present study employed a mathematical model to find the closest points on the parameter surface using arithmetic operations of the multivariate Bernstein-form polynomials (Berchtold and Bowyer, 2004). Based on this model, the closest points are calculated using the searching strategy of recursive quadtree decomposition.

Let us assume that point \(s(u_0, v_0)\) is the closest point on the parameter surface of measured point \(p_0\). Then, the line between them and the normal line of the surface at point \((u_0, v_0)\) coincide, i.e., point \(s(u_0, v_0)\) is the projection point on the surface of the measured point \(p_0\). Therefore, a vector equation can be constructed, as shown in equation (22), to compute the projection point from a point to the parameter surface.

\[
r(u, v) = (s(u, v) - p) \times n(u, v) = 0.
\]  

(22)
Using the arithmetic operations of the Bernstein-form polynomials, equation (22) can be restated in a Bezier surface form by the following expression:

\[ r(u, v) = \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}(u) B_{j,3n-1}(v) \]  

(23)

where \( e_{i,j} = [e_{i,j}^x, e_{i,j}^y, e_{i,j}^z]^T \) is the control point of Bezier surface \( r(u, v) \). Equation \( r(u, v) = 0 \) is evidently equivalent to the following system of equations:

\[
\begin{align*}
\mathbf{r}(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}(u) B_{j,3n-1}(v) = 0 \\
\mathbf{r}'(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}(u) B_{j,3n-1}'(v) = 0 \\
\mathbf{r}''(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}'(u) B_{j,3n-1}(v) = 0
\end{align*}
\]  

(24)

To obtain a more intuitive mathematical model for finding the closest point, the system graph of equation (24) can be represented as three Bezier surfaces over the \( u-v \) parameter domain using the linear precision property of the Bernstein polynomial. These Bezier surface patches are modelled by the following parametric equations:

\[
\begin{align*}
\mathbf{r}(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}(u) B_{j,3n-1}(v) \\
\mathbf{r}'(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}(u) B_{j,3n-1}'(v) \\
\mathbf{r}''(u, v) &= \sum_{i=0}^{3m-1} \sum_{j=0}^{3n-1} e_{i,j} B_{i,3m-1}'(u) B_{j,3n-1}(v)
\end{align*}
\]  

(25)

where

\[
\begin{align*}
\mathbf{e}_{i,j}^x &= \left[ i/(3m-1), j/(3n-1), e_{i,j}^x \right]^T \\
\mathbf{e}_{i,j}^y &= \left[ i/(3m-1), j/(3n-1), e_{i,j}^y \right]^T \\
\mathbf{e}_{i,j}^z &= \left[ i/(3m-1), j/(3n-1), e_{i,j}^z \right]^T
\end{align*}
\]

are the control points of the Bezier surface in equation (25).

From the above derivation, the vector equation indicates that the Bezier surface in equation (25) intersects with the parametric plane simultaneously. Now, the problem of searching the closest point on the parameter surface is transformed into a geometric problem of finding the intersection of the three Bezier surfaces described by equation (24) with the \( u-v \) parametric plane.
Recursive quadtree decomposition of the $u$-$v$ domain is used to search the regions possibly containing the proper parametric values of the closest point on the surface. The parametric rectangular domain is subdivided into four rectangular domains at the midpoint of $u$ and $v$ using the deCasteljau algorithm. For each rectangular domain, the subdivision is continued until computation accuracy is satisfied.

5.2 Accurate surface fitting of point clouds

After the data point parameters are obtained, accurate NURBS surface can be fitted. To meet the demand for accuracy, the number of control points used should be as small as possible. The lattice of the control points is assumed as $(m + 1) \times (n + 1)$. The base functions of the surface is defined by the DeBoor-Cox formula. Our B-spline fitting surface model is:

$$ R(u,v) = \sum_{i=0}^{m} \sum_{j=0}^{n} S_{i,j} N_{i,k}(u)N_{j,l}(v) $$

where $k$ is the degree of the base functions and the two knot vectors are $U = [u_0, u_1, \cdots, u_{m+k+1}]$ and $V = [v_0, v_1, \cdots, v_{n+k+1}]$.

For each data point $P_r$, a pair of parametric values $(u_r, v_r)$ and a weight $w_r$ are associated. The fitting surface model that approximates the data points can be expressed as:

$$ \min E(S_{i,j}) = \sum_{r=1}^{b} w_r^2 \| R(u_r,v_r) - P_r \| = \| D(S - P) \|^2 $$

With the weight matrix of data points $D = \text{diag}(w_1, \cdots, w_b) \in \mathbb{R}^{b \times b}$, the base function value matrix is $N = (N_{i,k}(u_r))_{r,j} \in \mathbb{R}^{(n+1)(m+1) \times b}$, the data point coordinate matrix is $P = (P_1, P_2, \cdots, P_b) \in \mathbb{R}^{b \times 3}$, and the control point coordinate matrix is $S = (S_{i,j}) \in \mathbb{R}^{(m+1)(n+1) \times 3}$.

The problem of weighted non-linear least squares can be transformed into a linear system problem (Weiss et al., 2002):

$$ (DN)^T (DN) S = (DN)^T (DP) $$

Equation (28) is an overdetermined linear equation; thus, it must be assured a full-rank column. To ensure linear uncorrelation of the elements of all rows in the parameter matrices, the overlap data points must be eliminated when creating the overdetermined linear equations. Therefore, the equations can be solved using the householder transformation method.

6 Example

Wrap-around surface is a commonly used surface in large section of projects. The proposed automatic accurate surface reconstruction of wrap-around models has been
successfully used in the industry. The algorithm was written fully in C++, and the programme for the display was developed in OpenGL. The programme functions include automatic recognition of the central axis, extraction of the boundary curve, and accurate reconstruction of the NURBS surface. The whole process is automatically performed without manual operations.

As shown in Figure 5, the measured point clouds of the wrap-around surface for a shell part is generated by a 3D vision scanner based on line-structured light. The number of data points exceeds 100,000, and the dimension of the figure is 1,900 × 900 × 800 mm. The demand for accuracy of the reconstructed surface is strict. Figure 6 shows the reconstructed NURBS base surface with a control point array of 10 × 8. The surface is quite smooth without any sharp distortion. Obviously, the surface can approximately reflect a geometric shape. After parameterisation of the point clouds using the base surface, the accurate NURBS surface is reconstructed (Figure 7). The control point arrays are 30 × 20 and 50 × 36, the maximum errors of the reconstructed surface are 0.136 and 0.0860 mm, and the average errors are 0.0358 and 0.0129 mm (Table 1). Thus, high-accuracy NURBS surface can be reconstructed using the parameterisation method of base surface, and the accuracy of the reconstructed surface can be improved by increasing the size of the control point array.

Table 1  The experimental result of different control points with different size mesh

<table>
<thead>
<tr>
<th>Control points array</th>
<th>Maximum error $\varepsilon_{\text{max}}$ (mm)</th>
<th>Average error $\varepsilon_{\text{ave}}$ (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30 × 20</td>
<td>0.136</td>
<td>0.0358</td>
</tr>
<tr>
<td>50 × 36</td>
<td>0.0860</td>
<td>0.0129</td>
</tr>
</tbody>
</table>

Figure 5  Measured point clouds (see online version for colours)

Figure 6  Reconstructed base surface (see online version for colours)
Figure 7  Accurately reconstructed surface (see online version for colours)

7 Conclusions

1 For the geometric characteristic of a wrap-around surface, a method for automatic accurate NURBS surface reconstruction is proposed. The new approach can automatically recognise the central projecting axis and extract the boundary curves of the cloud of points. High-precision parameterisation of the model is realised by means of base-surface creation using skinning technology. The whole process of modelling is entirely automatic.

2 A cross-sectional contour slicing method in sector distribution is proposed. The ordered data in the cross-sectional plane are obtained by local surface fitting so that the disordered scatter data points are converted into ordered data. The efficiency and stability of the surface reconstruction are improved.

3 The proposed method is an effective parameterisation method that considers the closest points of data points on base surfaces as a parametric value of data points using the computation method of closest points based on multivariate Bernstein-form polynomials, where accurate iterative fitting of the wrap-around surface is implemented.

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References


