Network Calculus Based Dimensioning for Industrial Wireless Mesh Networks

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Abstract. Wireless Mesh network is widely adopted in both wideband mobile communication network and short distance RF industrial network. As for the latter field, network robustness and real-time are two important issues. Network dimensioning is therefore required for this kind of networks. We models wireless mesh network as four tuples and present a topology transform methodology to change the mesh topology to a cluster tree. Research results from network calculus theory are then applied to obtain key merits such as buffer requirement, bandwidth requirement and end to end delay bound.

Introduction

Short distance RF wireless mesh network (WMN) is a new type of wireless network emerged in recent years. In WMN, node can “talk” with each other by multi-hops, every node can own more than one routing path. Therefore, the WMN obtains increased reliability and robustness. Due to this advantage, today’s short distance RF wireless network standards adopt mesh topology as a robust solution, e.g. wirelessHART specifies a full mesh structure [1]. ISA100 employs “mesh” as one of the topologies it supported [2].

Network calculus in the mathematical tool based on min-plus and max-plus algebra. It is usually used to analyze the deterministic queue system [3]. Koubaa and Schmitt use network calculus theory to analyze the sensor networks firstly [4, 5]. Cluster tree and grid based network are discussed. But no one mentioned mesh. In this paper, we first give an initial approach to analyze mesh network using network calculus theory. Actually, a topology transform method is used to change the mesh to a cluster tree. Applying the results from network calculus theory, we give the input and output stream and estimate the buffer requirement, bandwidth requirement and end to end packet delay.

In this paper, we firstly introduce two application field of WMN. Research works on network dimensioning or worst case analysis in wireless sensor network are also presented. We give some fundamental background theory on network calculus. The system model and topology transform method are also presented. Then, typical metrics such as buffer requirement, bandwidth requirement and delay bound are computed for an illustrated network. We believe that network calculus theory provides an alternative method to dimension WMN.

Related Works

At present, WMN is widely adopted in both wideband mobile communication network and short distance RF industrial network. In the first field, mesh topology is formed based on IP access network. Many access point (AP) can coordinated and cooperate to provide wideband mobile communication service [6, 7]. In the second field, mesh is applied to low power wireless networks. The purpose of this kink of network is to monitor and control the industrial device and process. Because of the high robustness requirement, mesh topology is widely adopted by many standards, e.g. wirelessHART standard specifies a full mesh structure [1]. ISA100 employs “mesh” as one of its supported topologies [2]. Compared with the first field, low bandwidths and scarce resource is the characteristics. We call this kind of WMN industrial wireless mesh network (iWMN). How to dimension and estimate or predictive the performance is a problem for iWMN.
Research on network calculus began at the work of R. L. Cruz and C. S. Chang [8, 9]. At present, network calculus theory is used to analyze the flow control, network scheduler and QoS control [10, 11]. Schmitt applied network calculus to sensor network firstly. A worst case dimensioning framework for sensor network is presented in [5]. Koubaa modeled the cluster tree network into a tri-tuples and derived the recursive expression of the input/output flow [12]. He further analyzed the Zigbee beaconed mode. The relation between network depth and maximum number of child routers is presented in [4]. Koubaa also analyzed how parameters setting in IEEE802.15.4 affect the performance such as latency and throughput [13]. Buffer queue sharing and statistics network calculus can be applied to estimate the end-to-end delay in WMN[16]. Li presented deep analysis on end-to-end delay in case of inter-session coding is used in wireless networks[17].

**Network Calculus Fundamentals**

System model used in network calculus is illustrated in Fig.1. The network node is modeled as a queue and a service function \( \beta(t) \). For a given data flow, the input function in the cumulative arrival function denoted by \( R(t) \), which represents the number of bits that arrive during the interval \([0,t]\). The output function is denoted as \( R'(t) \), which represents the number of bits that leave the node during the interval \([0,t]\). Network calculus theory assumes the arrival function is upper bounded by the arrival curve \( \alpha(t) \) and a minimum service curve guaranteed to \( R(t) \). The output function \( R'(t) \), of the flow \( R(t) \) constrained by the arrival curve that traverses a node offering a service curve \( \beta(t) \),is constrained by an output bound \( \alpha'(t) \), \( \alpha'(t) = (\alpha \ominus \beta)(t) \), where \( \ominus \) in the min-plus deconvolution.

After computation, the delay bound \( D_{\text{max}} \), which represents the worst-case response time of a packet, can be expressed as:

\[
D_{\text{max}} = \sup \{ \inf (r \geq 0) | \alpha(s) \leq \beta(s + r) \} \geq d(t), \forall t.
\]

The backlog bound \( Q_{\text{max}} \), which represents the maximum queue length of the flow, can be expressed as:

\[
Q_{\text{max}} = \sup \{ \alpha(s) - \beta(s) \} \geq q(t), \forall t.
\]

We using the commonly adopted linear arrive curve \( \alpha(t) = b + r \cdot t \) and rate-latency curve \( \beta_{R,T} = R \cdot (t-T)^+ \), where \( b \) is the maximum burst size of the flow, \( r \) is its average rate, \( R \geq r \) is the guaranteed bandwidth, \( T \) is the maximum latency of the service and \( x^+ = \max(0, x) \).

**Figure 1.** System Model in Network Calculus Theory

**Figure 2.** Delay and Backlog Bounds
The delay bound and backlog are also illustrated in Fig. 2. The delay bound is the vertical distance between the arrival curve and service curve. Backlog is the horizontal distance between the arrival curve and service curve.

There are three useful corollaries for the linear arrival curve and rate-latency service curve (all the symbols used in the following three equations are defined as above):

1. The delay bound is computed as
   \[ D_{\text{max}} = \frac{b}{R} + T \]  

2. The backlog bound is expressed as
   \[ Q_{\text{max}} = b + r \cdot T \] 

3. The output bound of the input flow is expressed as
   \[ \alpha'(t) = \alpha(t) + r \cdot T \] 

**System Model**

Generally, there are two ways to implement the “mesh”, reactive and proactive. Reactive mesh means the node have to find a better path by itself. An example of reactive is the routing algorithm in [14]. It may involve a discovery process. Proactive mesh means that the all the routing are pre-configured. An example of proactive mesh is the routing mechanism specified in wireless Hart[1].

In reference [12], the cluster tree network is modeled as a tri-tuples as \((\text{maxDepth}, \text{Nchild}, \text{Nrouter})\), where maxDepth represents the maximum path depth, Nchild represents the maximum number of child nodes, Nrouter represents the maximum number of the child routers. In this paper, we extend this model to a four-tuples, i.e. \((\text{maxDepth}, \text{Nchild}, \text{Nrouter}, \text{Nmesh})\), where Nmesh is added. Nmesh represents the maximum number of routing path which can be used by any node in the network. There are many related works on the research of reactive routing for sensor networks. But in the environment of industrial application, proactive style is the best choice. For example, in wirelessHART specification, all routing paths are planned and configured by a central network manager [1, 15].

In this paper, only the upstream data flow is considered. The arrival curve of every node is \(\alpha_{\text{data}}(t) = b_{\text{data}} + r_{\text{data}} \cdot t\) and the service by the parent node is \(\beta_{\text{data}}(t) = R_{\text{data}} \cdot (t - T_{\text{data}})\).

![Figure 3. An Example of Topology Transform](image-url)
Topology Transform

Network calculus requires fixed topology. Therefore, network calculus can not be applied to random deployed sensor network or mobile sensor network directly. Fortunately, the environment of industrial sensor network is controllable. The idea of topology transform is to deem that the reserved resource is used by virtual network segment. The reserved resource means the backup path and related slot if slotted superframe is employed. The rules for topology transform is to add additional child node to a given router node. The added node is specified by Nmesh. Certainly, as for the wirelessHART network, the entire routing table is configured by the central network manager. In this case, a real topology can be used to obtain more precise results.

Procedures of mesh network analysis based on network calculus are as following.

1. Begin from leaf node, add communication resource to a node according to Nmesh parameter in mesh network model and increase its Nchild.
2. Change the resource reserved to the virtual node and its routing path.
3. Repeat the above step, until the mesh topology is transformed to cluster tree topology.
4. Estimate every node’s input and output flow.
5. Estimate the metrics such as reserved bandwidth, buffer size and delay.

Worst-case Dimensioning

An example (maxDepth, Nchild, Nrouter, Nmesh), where maxDepth=3; Nchild=2; Nrouter =2; Nmesh=2 is illustrated in Fig. 3. (The example is extracted from the document of wireless HART specification). The left side is the topology before transform. In this simple mesh network, node D has two path and node B and D can relay each other. The short curve with a narrow is noted as sensors. The right side is the topology after virtual transform.

Data Flow. For simplicity, we assume that the latency in every node is same. It is a reasonable assume when the length of the superframe used in networks is same. Except for the gateway node, we assume that a sensor is attached to every network node. We also assume the sensor has a linear date generation model, i.e. the local sensor arrival curve is

\[ \alpha_s = b_s + r_s \cdot t \] (4)

where \( \alpha_s \) is the arrival curve of the sensor, \( b_s \) is the maximum burst size of the sensor data flow and \( r_s \) is the average rate of sensor data.

For, the output flows of node D1~D4 are identical. According to (3), the output flow is

\[ \alpha_{o_j} = \alpha_s + r_s \cdot T \] (5)

For node C1, its input flow comprises the local sensor data flow and node D2 output flow, i.e. \( \alpha_{c1} = \alpha_s + \alpha_{o2} \). Applying (5) we get

\[ \alpha_{c1} = 2 \cdot \alpha_s + r_{c1} \cdot T \] (6)

The output flow of node C1 is

\[ \alpha_{c1}(t) = \alpha_{c1}(i) \odot \beta_{c1}(t) \], applying (3) we get

\[ \alpha_{c1}^* = \alpha_{c1} + r_{c1} \cdot T \] (7)

According to (6) we note \( r_{c2} = 2r_s \), thus
\[ \alpha_{c_1} = 2 \cdot \alpha_s + 3 \cdot r_s \cdot T \]  \hfill (8)

For node B, the input flow is the sum of the output flow of node D1 and node C1, that is

\[ \alpha_b = \alpha_{d_1} + \alpha_{c_1} \]  \hfill (9)

Applying (5) and (8) to (9) we get

\[ \alpha_b = 3 \cdot \alpha_s + 4 \cdot r_s \cdot T \]  \hfill (10)

Thus, the output flow of node B is

\[ \alpha_b = \alpha_b + r_b \cdot T \]  \hfill (11)

According to (10), we note \( r_b = 3 \cdot r_s \), thus

\[ \alpha_b = 3 \cdot \alpha_s + 7 \cdot r_s \cdot T \]  \hfill (12)

The input/output flow of node C is identical as node B.

Finally, for node A, its input flow is:

\[ \alpha_a = \alpha_a + \alpha_c \]  \hfill (13)

Applying (12) and (8) to (13), we get

\[ \alpha_a = 6 \cdot \alpha_s + 14 \cdot r_s \cdot T \]  \hfill (14)

The output flow of node A:

\[ \alpha_a = \alpha_a + r_s \cdot T \]  \hfill (15)

Applying (14) and (15), we get

\[ \alpha_a = 6 \cdot \alpha_s + 20 \cdot r_s \cdot T \]

**Bandwidth Requirement.** To guarantee the delay requirement, we can compute the maximum bandwidth required by each node. Eq. \( R_x \geq r_y \) have to be satisfied where the subscript x and y can be replaced by any node in the network, but node x should be the parent of node y.

For the above network, we can obtain that \( R_D \geq r_s, R_B \geq 3r_s, R_C \geq 3r_s, R_A \geq 6r_s. \)

**Buffer Requirement.** The buffer requirement comprises two parts, buffer required by burst data and buffer required by backlog data, i.e. \( Q_{\text{max}} = Q_{\text{burst}} + Q_{\text{_back}} \). As for the node, we derive:

\[ Q_{\text{max}}^x = b_x + r_x \cdot T \]  \hfill (16)

where the subscript x can be replaced by any node in the network. Applying \( b_x \) and \( r_x \) to (16), we get:

\[ Q_{\text{max}}^D = b_s + r_s \cdot T, \]

\[ Q_{\text{max}}^B = (3b_s + 7r_s) \cdot T, \]

\[ Q_{\text{max}}^C = Q_{\text{max}}^B, \]

\[ Q_{\text{max}}^A = Q_{\text{max}}^B, \]

\[ Q_{\text{max}}^A = (6b_s + 20r_s) \cdot T. \]
Delay Bound. According to network calculus theory, we can obtain the end to end delay bound by summing the per hop delay along the longest path. In the above topology, the worst case delay is

$$D_{\text{sum}} = D_{B2} + D_{A1} + D_{a} + D_{i}$$

(17)

According to (1), (6), (9) and (14), we get

$$D_{B2} = \frac{b_{2}}{R} + T,$$
$$D_{A1} = \frac{2b_{1} + r_{i} \cdot T}{R} + T,$$
$$D_{a} = \frac{3b_{1} + 4r_{i} \cdot T}{R} + T,$$
$$D_{i} = \frac{6b_{1} + 20r_{i} \cdot T}{R} + T.$$

Thus, applying (17), we get

$$D_{\text{sum}} = 4T + \frac{12b_{2} + 25r_{i} \cdot T}{R}.$$ Note this result is pessimistic because we use the worst-case (minimum) service as every node’s service.

Conclusions

Mesh network is a type of robust network which is widely used in both wideband mobile communication and low power industrial network. A quantitative dimensioning is needed to plan and estimate the performance of a deploying WMN. Dimensioning methodology of cluster tree sensor network based on Network calculus can not apply directly to WMN. The virtual topology transform proposed in this paper is an alternative method to change the mesh topology to a cluster tree. Thus, this paper provides a possible method to dimension WMN applying the network calculus theory.

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References


