An Efficient Path Planning Algorithm for Mobile Robot Using Improved Potential Field

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Abstract—Traditional APF-based mobile robot path planning approaches possess an inherent problem which is the formation of local minimum that probably prevent robot from arriving at the target. In view of those considerations, an improved potential field function is proposed to settle this problem. The new method includes an improved attractive potential function and an improved repulsive potential function. The attractive potential function takes into account the minimum separation between robot and obstacles, while the repulsive potential function takes into account the relative position between robot and the target. As a result, the target is ensured as the global minimum in working space. Simulation experiments are performed and the results demonstrate the effectiveness of the improved method.

I. INTRODUCTION

For mobile robot, the navigation technique is of key importance. Among navigation techniques, the path planning is an important problem. In the majority of published literatures, it is also referred to as planning path, finding path problem, collision-free motion, obstacle avoidance, motion planning, etc. The problem of path planning for mobile robot is that robot searches an optimum or approximate optimum non-collision path from the initial state to final state according to a certain performance objective [1].

Path planning problem is divided into two types: the global path planning problem in which environment information is known completely and the local path planning in which environment information is known partially. This paper only addresses the global path planning for mobile robot [2]–[4].

Global planning path for mobile robot is an area of research that has received considerable recognition in the past decade. Several methods have been proposed: the heuristic searching way, the dynamic programming method, the free space method, the neutral networks, the ant colony algorithm, the genetic algorithm, the simulated annealing algorithm, the topological approach and the artificial potential function method and so on [1]–[6].

Artificial potential field function method was first put forward by Khatib [6]. The concept of “potential field” comes from physics. By calculating the magnitude and direction of all these field effects, a complete definition of potential field function in the whole space is constructed. All these forces acting on one object can be calculated according to the function of potential field instead of separately calculating each force [2]. In robotics [23], when talking about how robot avoids obstacles or plans a path in the process of walking, the field effects created by obstacles and goal are imagined so that they seem to have attractive and repulsive forces on robot. As a result, robot seems to avoid obstacles naturally and spontaneously. Such field effects are called APF [7] [8] [10] [11]. The method has been widely applied for its concise mathematical expression.

Inherent limitation still exists though this method is popular. When the mobile robot falls into the local minimum of potential function, the final state will never be reached. This situation frequently happens as obstacle is in the vicinity of the target point [12].

It is assumed that obstacles are relative far from the goal in previous researches. As a result, repulsive forces that obstacles impose on the robot are much smaller and ignored. In fact, the assumption is very different from the practical case, where there always exists a small separation between one of the obstacles and the target, or the target is in the close vicinity of some obstacle. The smaller the separation between the obstacle and the target, the bigger the repulsive force exerting on mobile robot. Consequently, mobile robot will deviate from target [9] [12] [13].

For purpose of overcoming the defect, an improved artificial potential function introduces the relative position between the robot and the target into a repulsive potential function and the least distance between the robot and obstacle into an attractive potential function respectively. The improved artificial potential function can get the global minimum and ensure the target attainable.

This paper is organized as follows: An overview of planning path methods in Section I. Section II introduces the traditional artificial potential function and presents the inherent limitation of the method, namely the unreachable target issue. To overcome this defect, an improved artificial potential function is introduced in Section III. Simulation experiments are made in Section IV. Finally, concluding remark is given in Section V.

II. TRADITIONAL ARTIFICIAL POTENTIAL FUNCTION AND LOCAL MINIMUM PROBLEM

A. The Attractive Field Function

For purpose of simplifying the issue of path planning, robot is often viewed as a mass point. The position of robot is expressed as a vector of \( x = [x, y] \) in two-dimensional

\[
\vec{T} = \begin{bmatrix} 1 & 0 & 0; 0 & 1 & 0 \end{bmatrix}
\]

\[
\mathbf{V} = \begin{bmatrix} 0 & 1; 0 & 0 \end{bmatrix}
\]

\[
\mathbf{W} = \begin{bmatrix} 0; 0 \end{bmatrix}
\]

\[
\mathbf{X} = \begin{bmatrix} x; y \end{bmatrix}
\]

\[
\mathbf{Y} = \begin{bmatrix} x; y \end{bmatrix}
\]

\[
\mathbf{Z} = \begin{bmatrix} x; y \end{bmatrix}
\]

\[
\mathbf{A} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{B} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{C} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{D} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{E} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{F} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{G} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{H} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{I} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{J} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{K} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{L} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{M} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{N} = \begin{bmatrix} a; b \end{bmatrix}
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\mathbf{O} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{P} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{Q} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{R} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{S} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{T} = \begin{bmatrix} a; b \end{bmatrix}
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\[
\mathbf{U} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{V} = \begin{bmatrix} a; b \end{bmatrix}
\]

\[
\mathbf{W} = \begin{bmatrix} a; b \end{bmatrix}
\]
plane [3]. The attractive potential function at the point is related to the relative position of the goal and the robot.

An expression for the conventional attractive field function \( U_{\text{attract}}(x) \) is described by

\[
U_{\text{attract}} = \begin{cases} 
\frac{1}{2} k (X - X_{\text{goal}})^2, & X > X_{\text{goal}} \\
0, & X \leq X_{\text{goal}}
\end{cases}
\]

(1)

where \( k \) is gain coefficient, \( X - X_{\text{goal}} \) is relative position between robot and goal. From (1), the corresponding attractive force formula is obtained by the negative gradient of this attractive potential:

\[
F_{\text{attach}} = -\nabla U_{\text{attract}} = k (X - X_{\text{goal}})
\]

(2)

The force will drive mobile robot to reach the target.

**B. The Repulsive Field Function**

The conventional repulsive force field function can be represented as

\[
U_{\text{repulse}} = \begin{cases} 
\frac{1}{2} \beta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)^2, & \rho \leq \rho_0 \\
0, & \rho > \rho_0
\end{cases}
\]

(3)

where \( \beta \) is gain; \( \rho \) is the minimum separation between the robot and the obstacle; \( \rho_0 \) is a constant and represents the influence scope of the obstacle.

From (3), the corresponding repulsive force formula is driven as

\[
F_{\text{repulse}} = -\nabla U_{\text{repulse}} = \beta \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)^2 \cdot \frac{1}{\rho^2} \frac{\partial \rho}{\partial X}
\]

(4)

where \( \partial \rho / \partial X \) can be represented as

\[
\frac{\partial \rho}{\partial X} = \left[ \frac{\partial \rho}{\partial x} \quad \frac{\partial \rho}{\partial y} \right]
\]

(5)

Therefore, the total force acting on robot is expressed as

\[
F_{\text{total}} = F_{\text{attach}} + F_{\text{repulse}}
\]

(6)

In fact, if robot is driven by \( F_{\text{total}} \), the target will never be accessible. From Fig. 1, when approaching the goal, robot is close to the obstacle at the same time, whether or not the mobile robot, the obstacle, and the target are collinear. As the attractive force is getting smaller, the repulsive force is becoming bigger. Finally, robot would deviate from the object [2] [16] [18]–[20] [22].

In short, target is not the global minimum of the whole potential field. The conventional potential field-based function has to be improved.

**III. THE IMPROVED ARTIFICIAL POTENTIAL FUNCTION**

As mentioned above, the target point is not global minimum. However, the improved potential field function can overcome the limitation. With robot closing to the target, the attractive force is much larger and the repulsive one is much smaller. A case study in Fig. 2 can illustrate this.

**A. The Improved Attractive Field Function**

The improved attractive field function is expressed as

\[
U_{\text{attract}} = \begin{cases} 
\frac{1}{2} k (X - X_{\text{goal}})^2, & \rho < \rho_0 \leq \left\| X - X_{\text{goal}} \right\| \\
0, & \rho > \rho_0
\end{cases}
\]

\[
\left(1 \right)
\]

(7)

where \( \rho_0 < \left\| X - X_{\text{goal}} \right\| \) is included in (7). From (3), \( (1/\rho - 1/\rho_0) \) is included in (7). Correspondingly, the attractive force function is driven as

\[
F_{\text{attract}} = \frac{1}{2} k (X - X_{\text{goal}})^2 \cdot \left(\frac{1}{\rho} - \frac{1}{\rho_0}\right)^2, \quad \rho_0 \leq \left\| X - X_{\text{goal}} \right\| \leq \rho_0,
\]

(8)

where \( m > 0 \). When \( \left\| X - X_{\text{goal}} \right\| \leq \rho_0 \), we have \( F_{\text{attract}} > 0 \), which satisfies the object attainable.

**B. The Improved Repulsive Force Field Function**

The improved repulsive force field function is represented as

\[
\text{Fig. 1. The target is in the vicinity of the obstacle. The left shows that robot, obstacle, and target are collinear. The right indicates that robot, obstacle, and target are not collinear.}
\]

\[
\text{Fig. 2. Robot, obstacle, and target are not collinear.}
\]
where $\beta$ is gain coefficient, $\rho$ is the minimum separation between the robot and the obstacle, $\rho_0$ is a constant indicating the influence scope of the obstacle.

From (9), $(X - X_{\text{goal}})$ is included in (9), which ensures that target is the global minimum. The corresponding repulsive force function is driven as

$$F_{\text{repulse}} = -\nabla(U_{\text{repulse}}) = \begin{cases} F_{r1} + F_{r2}, & \rho \leq \rho_0, \\ 0, & \rho > \rho_0, \end{cases}$$

where

$$F_{r1} = \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} (X - X_{\text{goal}})^n,$$

$$F_{r2} = \frac{n}{2} \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 \frac{1}{(X - X_{\text{goal}})^{n-1}},$$

$$n > 0.$$  

There are three forms of repulsive force function as $n$ varies:

1) $1 > n > 0$:

$$F_{r1} = \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} (X - X_{\text{goal}})^n,$$

$$F_{r2} = \frac{n}{2} \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 \frac{1}{(X - X_{\text{goal}})^{n-1}},$$

where $\rho < \rho_0$.

In (12), when $F_{r1} \to 0$, we have $F_{r2} \to \infty$, the final state is reachable.

2) $n = 1$:

$$F_{r1} = \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} (X - X_{\text{goal}})$$

$$F_{r2} = \frac{n}{2} \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 \frac{1}{(X - X_{\text{goal}})^{n-1}},$$

where $\rho \neq 0$, $\rho < \rho_0$.

In (14), when $F_{r1} \to 0$, we have $F_{r2} \to \text{constant}$, the object is accessible.

3) $n > 1$:

$$F_{r1} = \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right) \frac{1}{\rho^2} (X - X_{\text{goal}})^n,$$

$$F_{r2} = \frac{n}{2} \beta \left( \frac{1}{\rho} - \frac{1}{\rho_0} \right)^2 \frac{1}{(X - X_{\text{goal}})^{n-1}},$$

where $\rho \neq 0$, $\rho < \rho_0$.

In (16), when $F_{r1} \to 0$, $F_{r2} \to \text{constant}$, the goal is accessible.

IV. SIMULATION RESULTS

To verify the effectness of the improved method described in the previous section, some simulations have been performed. It is assumed that the mobile robot would run at a constant speed. The direction of mobile robot will be dependent on the direction of the total force exerting on the robot. The movement area is $5m \times 5m$, where there exist some circular obstacles. The distance is $0.6m$ between the target and the obstacle. Two cases described in Fig.1 (a) and
(b) are simulated. The starting point of movement is located in the lower-left corner.

The function parameters involve $n$, $k$, $\eta$, $\rho_0$. Let $n = 2, k = 2, \rho_0 = 60 \text{ mm}$. $\eta$ is determined according to the particular circumstances of the case. Simulation results are illustrated in Fig. 4(a) and (b), Fig. 5(a) and (b), Fig. 6(a) and (b).

Fig. 4(a) and (b) give the snapshots of the mobile robot motion using the conventional potential function when the target is far from the obstacles. These two charts show the robot can reach the goal smoothly.

Fig. 5(a) and (b) indicate the snapshots of the mobile robot motion respectively when the robot, the goal and the obstacle are not collinear. Fig. 5(a) shows the robot can not reach the goal using the traditional potential function. However Fig. 5(b) shows the robot can reach the goal smoothly using the improved potential function.

Fig. 6(a) and (b) suggest the snapshots of the mobile robot motion respectively when the robot, the goal and the obstacle are collinear. Fig. 6(a) shows the robot can not reach the goal using the traditional potential function, while Fig. 6(b) shows the robot can reach the goal smoothly using the improved potential function.
V. CONCLUSION
This paper proposes an improved potential field algorithm applied to path planning for mobile robot. The improved potential field includes into the attractive potential function the least distance between the robot and the obstacle and into the repulsive potential function the relative distance between the robot and the goal. The improved way can guarantee the target point is the global minimum point and robot can reach the target successfully. The validity of the improved approach was verified by simulation results.

REFERENCES