Identification and experimental assessment of two-input Preisach model for coupling hysteresis in piezoelectric stack actuators

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Abstract

Piezoelectric stack actuators (PEAs) do not always perform as desired because external loads have an effect on the inclination of the hysteresis loop that causes deterioration of the tracking performance. To take account of this loading factor, the two-input Preisach model (TPM) is introduced to estimate the coupling hysteresis in PEAs for loading applications. This paper tackles the identification problem of TPM using a three-dimension interpolation algorithm based on the first-order reversal curves (FORCs) technique. To prove the feasibility of the TPM in describing piezoelectric hysteresis, the coupling hysteresis properties in PEA via experimental data are discussed. To assess the accuracy of the TPM in predicting expansion in a case where the PEA is subject to two inputs, it is compared with the single-input classical Preisach model (CPM) by performing several experiments under various excitation conditions.

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1. Introduction

In piezo-actuated nanopositioning systems, such as motion control [1], scanning probe microscopy [2,3], and manipulators [4], piezoelectric stack actuator (PEA) exhibits inevitably nonlinear hysteresis behavior with respect to two inputs: voltage and external load. Coupling hysteresis limits the accuracy of the characterization of the input-to-output relationship and control design for actuators. Fig. 1(a) and (b) shows the effects on voltage-to-expansion hysteresis curves in a PEA under excitations of a sinusoidal voltage (0.1 Hz) and different loads from 0 N to 100 N. The width of the hysteresis curve is slightly changed as the load increases. However, the inclination of the loop reduces by 13.5% from the case of 0 N to 100 N. Fig. 1(b) shows that the width of the curve increases as the frequency of the sinusoidal excitation voltage increases [no load applied]. It reaches around 10.3% of maximum expansion at 0.1 Hz and around 21.1% at 100 Hz.

In order to eliminate hysteresis behavior or dominate it in a certain range, lots of mathematical characterizations in terms of physics-based forms [5] were proposed. However, several mechanism governing piezoelectric properties are still not completely characterized by physics-based models. This motivates the development of phenomenological hysteresis models to characterize hysteresis nonlinearity in the case when PEAs subject to a certain excitation voltage. In cases of excitation voltage with low-frequency, the most popular models are the classical Preisach model (CPM) [6] and its discrete-time form [7] which have many well defined properties that make it suitable to describe hysteresis and control compensation for PEAs. Two of the most iconic applications were Ge [8] and Song [9]. Besides, the classical Prandtl–Ishlinskii (PI) operator [10] and its modifications [11–13] were also widely used to characterize hysteresis behavior for PEAs combining with available control techniques. Some alternative models to the CPM and PI would be the extended Bouc–Wen model [14] and the Dahl-based hysteresis model [15] due to their concise identification and calculation process. Furthermore, vast dynamic single-input models with rate-dependent property were developed to characterize hysteresis effects in PEAs under excitation condition of reference input signal covering a wide frequency range. Ben Mrad and Hu [16] developed a generalized Preisach model by introducing an average rate of change of the input signal. Based on the similar idea of constructing novel density functions, a modified Preisach model [17] and two generalized PI-type models [18,19] were presented. Besides, Janaideh [20] provided a dynamic PI-type hysteresis model constructed with a time-dependent threshold variable. These dynamic hysteresis models were proved practically possess much better accuracy than the static models to estimate hysteresis nonlinearity for PEAs driven by one dynamic voltage signal. Experimental results reveal that the single-input models discussed above were widely fused with available control...
strategies to characterize and compensate hysteresis. When there is a load of few Newtons acting on PEA, its effect on single-input models is not obvious. So, the models are still acceptable for many applications. However, the accuracy of them will deteriorate considerably as the applied load increased.

To achieve precise positioning, coupling hysteresis models are needed. Various mathematical models to describe coupling hysteresis behavior in magnetostrictive actuators for varying mechanical loads have been devised [6,21–26], and general forms of coupling hysteresis models based on the CPM are given in [6,21]. Suzuki [22] developed a stress-dependent model based on the moving Preisach model [6], while Davino [23] presented a stress-dependent magnetostriction phenomenological model by employing the CPM and a memory less bivariate function. Cavallob [24] proposed a model to characterize magnetostriction hysteresis for a slow-speed applied mechanical load that adopted a similar structure to the CPM. A generalized Preisach operator was developed [25] to estimate the dynamic coupling hysteresis under varying load and excitation frequencies. Finally, Zhang [26] presented a model based on the Prandtl–Ishlinskii (PI) operator by introducing weighted superposition for a so-called one-sided dead-zone operator.

By comparison, little research has focused on the coupling hysteresis characterization in PEA for load applications. In this paper, the two-input Preisach model (TPM) is used to estimate piezoelectric coupling hysteresis, which was firstly proposed to characterize hysteresis in magnetostrictive materials with respect to two variables: magnetic field and stress [6]. If the hysteresis nonlinearity in a PEA can be predicted accurately by the TPM, then the compensation controller based on the TPM can be designed to eliminate the hysteresis. Experimental assessments of the properties of coupling hysteresis in a PEA, based on experimental data, are developed firstly to confirm the feasibility of the TPM. Secondly, the numerical implementation of the TPM needs more effort due to the fact that the high accuracy of the TPM largely depends on how well it captures the inputs–to–output nonlinear behavior. Several smart algorithms, such as genetic algorithm [14], least square algorithm [27], and particle swarm optimization algorithm [28] and its modification [29], contribute to identification of hysteresis models. These algorithms are engaged in realizing parameters identification based on simulation results and measured values. However, they do not facilitate identification of the TPM due to the two double-integrating structures. Herein, a three-dimension interpolation algorithm [30] based on the first-order reversal curves (FORCs) technique [6,7,16] is introduced to solve the identification problem of TPM. As this algorithm and its modifications have been used in many fields with satisfactory results and involve a simple implementation procedure, its use is considered appropriate for characterizing hysteresis offline.

Fig. 1. Effects on coupling hysteresis curves in PEA. (a) Voltage with frequency of 0.1 Hz and different loads. (b) Voltage with different frequencies and 0 N load.

The main contribution of this study is that it provides a complete identifiable procedure for the TPM by employing a three-dimension interpolation algorithm based on the two databases of first-order reversal functions. By discussing the properties of coupling hysteresis in a PEA, the feasibility of the TPM using in hysteresis characterization is confirmed, and experimental assessments verify the accuracy of the TPM for estimating hysteresis in a PEA that is subject to two inputs: excitation voltage and external load. The study provides methods for assessing whether or not the TPM is valuable for compensation control of coupling hysteresis in a PEA for loading applications. In Section 2 of the paper the identification procedure and error analysis of the TPM are developed, in Section 3 an experimental set-up is established, Section 4 discusses the coupling hysteresis behavior in PEA, in Section 5 comparative tests are performed and compared with the CPM to assess the accuracy of the TPM under conditions of various excitations, and Section 6 provides the study’s conclusions.

2. Two-input Preisach model

2.1. Identification procedure of two-input Preisach model

Considering the coupling hysteresis nonlinearity in PEA that can be characterized by two-input u(t), p(t) and output y(t), u(t), p(t), and y(t) is defined as excitation voltage, external load, and expansion, respectively. Then, the TPM is described as [6]

\[
y_{TPM}(t) = \int_{\alpha=\beta}^{\alpha=\beta} \mu_1(\alpha, \beta, p(t)) y_{\alpha\beta}[u(t)] \, d\alpha \, d\beta
+ \int_{\lambda=\eta}^{\lambda=\eta} \mu_2(u(t), \lambda, \eta) y_{\alpha\beta}[p(t)] \, d\lambda \, d\eta, \\
\text{for } t \in [0, +\infty) \text{ and } i = 0, 1 \ldots
\]

where the dependence of the distribution functions \( \mu_1 \) and \( \mu_2 \) on \( p(t) \) and \( u(t) \), respectively, reflects the cross-coupled effect between the two inputs. In the following, we only consider the first term in the right-hand side of (1), due to the similar process is true for the second term of (1). This term admits a two-dimensional geometric explanation as shown in Fig. 2 for a fixed value \( p(t) \). The global maximum \( \mu_{\text{max}}(u_{\text{max}}) \), the minimum \( \mu_{\text{min}}(u_{\text{min}}) \) of input \( u(t) \), and the line of \( \alpha \geq \beta \) (\( \alpha \) and \( \beta \) are thresholds of \( u(t) \)) lead to a limiting triangular \( T_0 \). Within \( T_0 \), each pair \( (\beta, \alpha) \) determines a uniquely basic hysteresis
operator $y_{ap}[u](t)$. At any time, $T_0$ is subdivided into two regions $A_{(-)}$ and $A_{(+)}. \alpha$ The operator $y_{ap}[u](t)$, within $A_{(-)}$ and $A_{(+)},$ is equal to $-1$ and $1$ respectively, and outside is zero.

The identification problem of the TPM is now discussed. The major problem is to determine the distribution functions $\mu_1(\alpha, \beta, p)$ and $\mu_2(u, \lambda, \eta)$ by fitting (1) to some measured data. A series of first-order reversal functions $y_{ap}(t)$ and $y_{ap}(u)$ are required (Fig. 3). As shown in Fig. 3, the $y_{ap}$ is the output resulting from monotonic increases of $u(t)$ and load $p(t)$ to the values $\alpha$ and $p$ on the major loop, respectively. The value $y_{ap}(u)$ is subsequent monotonic decrease of $u(t)$ to the value $\beta$. The values of $y_{ap}$ and $y_{ap}(u)$ can be obtained in the similar way. As far as the numerical implementation of the TPM is concerned, it can be resolved by defining the Everett functions [6]

$$\int \Delta \mu_1(\alpha, \beta, p)d\alpha d\beta = \frac{1}{2}(y_{ap} - y_{ap}(p))$$

(2)

$$\int \Delta \mu_2(u, \lambda, \eta)d\alpha d\beta = \frac{1}{2}(y_{ap} - y_{ap}(u))$$

In the case when the current input voltage $u(t)$ is monotonically increasing, the geometrical explanation of the incremental area $\Delta A$ on the $\alpha-\beta$ plane is given in Fig. 4. By diagram technique [7] and (2), the first integral of (1) can be written as

$$y_{1}^{TPM}(t) = \int \int A_{(-)} \mu_1(\alpha, \beta, p)d\alpha d\beta - \int \int A_{(+)} \mu_1(\alpha, \beta, p)d\alpha d\beta$$

$$= \int \int A_{(-)} \mu_1(\alpha, \beta, p)d\alpha d\beta - \int \int A_{(+)} \mu_1(\alpha, \beta, p)d\alpha d\beta$$

$$+ 2 \int \int \Delta A \mu_1(\alpha, \beta, p)d\alpha d\beta$$

(3)

where $y_{1}^{TPM}(t)$ means the first term in the right-hand side of (1) calculated via the TPM. The input voltage $u(t)$ and load $p(t)$ are assumed to have zero initial state, which start at $t_0 = 0$ and end at the current time $t = nT_0$ with $t \in [0, +\infty]$. $T_0$ is the sampled time. The $\{t_1, t_2, ..., t_n\}$ is the time sequence with $t_0 = 0$ and $t$ is satisfied in the expression $\Delta_1 A = y_{TPM}(t_1) - y_{TPM}(t_0)$, with $n$ as a positive integer. The finite sequence $(y_{TPM}(mT_0))_{m=0, 1, ..., n}$ and $(p(mT_0))_{m=0, 1, ..., n}$ are the sampled sequence and the reduced memory sequence [6] of the input function $u(t)$ and load $p(t)$, respectively. Herein, the reduced memory sequence, which has an expression on the value of $y_{TPM}(t)$, is not wiped out by more dominant extrema. $n_1, n_2$ represents the number of exterma (not wiped-out) of $u(t)$ and $p(t)$, respectively.

$$y_{1}^{TPM}(t) = y_{1}^{TPM}(t_1) - y_{1}^{TPM}(t_{n - 1}) + y_{1}^{TPM}(t_{n - 1})$$

(4)

Similarly, if the load $p(t)$ is monotonically increasing and decreasing at current time, respectively, for the second integral of (1) we obtain

$$y_{2}^{TPM}(t) = y_{2}^{TPM}(t_1) - y_{2}^{TPM}(t_{n - 1}) + y_{2}^{TPM}(t_{n - 1})$$

(5)

and

$$y_{2}^{TPM}(t) = y_{2}^{TPM}(t_1) - y_{2}^{TPM}(t_{n - 1}) + y_{2}^{TPM}(t_{n - 1})$$

(6)

From (3)–(6), the expansion (1) can be expressed as

$$u(nT_0) > 0 \quad \text{and} \quad p(nT_0) > 0 : y_{TPM}(t) = y_{1}^{TPM}(t_1) + y_{1}^{TPM}(t_{n - 1}) - y_{1}^{TPM}(t_{n - 1}) + y_{1}^{TPM}(t_{n - 1})$$

(7)

$$u(nT_0) > 0 \quad \text{and} \quad p(nT_0) < 0 : y_{TPM}(t) = y_{1}^{TPM}(t_1) - y_{1}^{TPM}(t_{n - 1}) + y_{1}^{TPM}(t_{n - 1})$$

(8)

$$u(nT_0) < 0 \quad \text{and} \quad p(nT_0) > 0 : y_{TPM}(t) = y_{1}^{TPM}(t_1) - y_{1}^{TPM}(t_{n - 1})$$

(9)
\[ \dot{u}(nT_0) < 0 \quad \text{and} \quad p(nT_0) < 0 : y_{\text{TPM}}(t) = y_{\text{TPM}}(t_{k-1}) - y(u(t_{k-1}), p(t_{k-1})) + y(u(t_{k-1}), p(nT_0)) + y(u(t_{k-1}), p(t_{k-1})) + y(u(nT_0), p(t_{k-1})) + y(u(nT_0), p(nT_0)) \tag{10} \]

Eqs. (7)–(10) are the discrete expressions of the TPM in terms of the experimentally measured functions (2). This numerical form can be used to calculate the output response of PEA that is subject to any two known inputs. We know that the estimated output \( y_{\text{TPM}}(t) \) is not only depends on the current input voltage \( u(t) \) and the load \( p(t) \), but also the nearest historical extrema of the two inputs and their output values.

The three-dimensional interpolation algorithm is introduced to implement (7)–(10). As shown in Fig. 5, the \( u_{\text{max}}, u_{\text{min}} \) of input voltage \( u(t) \), the condition \( \alpha \geq \beta \), and the load variable (z-axis) lead to a limiting triangular prism \( P_0 \). A two-dimension \( (\alpha, \beta) \) plane for each fixed load exists just as shown in Fig. 2. A cubic mesh covering the \( P_0 \) is created and a discrete set of the functions \( y_{\alpha, \beta, p} \) for each fixed load \( p(t) \) is entered that consists of the mesh values of \( y_{\alpha, \beta} \). The functions \( y_{\alpha, \beta, p} \) and \( y_{\alpha, \beta} \) for all pairs \( (\alpha, \beta, p) \) within \( P_0 \) that correspond to corners of any of the cuboids and triangular prisms need to be determined experimentally. In similar situations, the functions \( y_{\lambda, \alpha, \beta} \) and \( y_{\lambda, \alpha, \beta} \), for all pairs \( (\lambda, \alpha, \beta, p) \) within a limiting triangular prism should also be obtained. This limiting triangular prism is composed of the lines of \( p_{\text{max}}, p_{\text{min}} \) of input load \( p(t) \), the condition \( \lambda \geq \eta \) and the voltage variable (z-axis) (Fig. 5).

For points of the \( (\alpha, \beta, p) \) prism lying within any of the cuboids, based on the eight vertex coordinates (Fig. 6) and the corresponding measured values, the following interpolation equation is adopted

\[ y_{\alpha, \beta, p} = (1 - \alpha)(1 - \beta)(1 - p)y(0, 0, 0) + (1 - \alpha)(1 - \beta)py(0, 0, 1) + (1 - \alpha)y(1, 0, 0) + (1 - \beta)py(1, 1, 1) + \alpha(1 - \beta)(1 - p)y(1, 0, 1) + \alpha(1 - \beta)py(1, 1, 1) + \beta(1 - p)y(1, 1, 0) + \alpha \beta py(0, 1, 1) \]

Simplifying the above equation to get

\[ y_{\alpha, \beta, p} = C_{\alpha, \beta}^{\alpha} + C_{\alpha, \beta}^{\beta} \alpha + C_{\alpha, \beta}^{\alpha} \beta + C_{\alpha, \beta}^{\alpha} \beta p + C_{\alpha, \beta}^{\alpha} \alpha \beta + C_{\alpha, \beta}^{\alpha} \beta p \]

\[ + C_{\alpha, \beta}^{\alpha} \beta p + C_{\alpha, \beta}^{\alpha} \beta p \]

For points of the \( (\alpha, \beta, p) \) prism lying within any of the cuboids, based on the six vertex coordinates (Fig. 6) and the measured output data, we use the interpolation Eq. (12) as follows

\[ y_{\alpha, \beta} = t_{\alpha, \beta}^{\alpha} + t_{\alpha, \beta}^{\alpha} \alpha + t_{\alpha, \beta}^{\beta} \beta + t_{\alpha, \beta}^{p} p + t_{\alpha, \beta}^{\alpha} \alpha \beta + C_{\alpha, \beta}^{\alpha} \beta \]

\[ + t_{\alpha, \beta}^{\alpha} \beta p + C_{\alpha, \beta}^{\alpha} \beta p \]

\[ \tag{11} \]

where the constants \( c_{\alpha, \beta}^{\alpha} \) \((i=1,2,\ldots,8)\) and \( t_{\alpha, \beta}^{\alpha} \) \((i=1,2,\ldots,6)\) in the interpolation Eqs. (11) and (12), are estimated based on the measured functions corresponding to the corners of the cuboids and triangular prisms within \( P_0 \). When \( \alpha = \beta, y_{\alpha, \beta} \) is used instead of \( y_{\alpha, \beta, p} \) in (12). For points of the \( (\lambda, \eta, u) \) prism, the constants \( c_{\lambda, \eta}^{\alpha} \) \((i=1,2,\ldots,8)\) and \( t_{\lambda, \eta}^{\alpha} \) \((i=1,2,\ldots,6)\) can be calculated similarly by (11) and (12), respectively.

2.2. Error assessment

The approximations of the first-order reversal functions calculated by (11) and (12) inevitably lead to error due to the different divisions of the meshed limiting triangular prism \( P_0 \). In general, we consider the approximation error incurred in a cuboids element of the \( P_0 \). The eight vertex coordinates of the cuboids \( (\alpha_i, \beta_i, p_{i}) \), \( (\alpha_{i+1}, \beta_{i}, p_{i}) \), \( (\alpha_{i+1}, \beta_{i+1}, p_{i}) \), \( (\alpha_{i}, \beta_{i+1}, p_{i}) \), \( (\alpha_{i}, \beta_{i}, p_{i+1}) \), \( (\alpha_{i+1}, \beta_{i}, p_{i+1}) \), \( (\alpha_{i+1}, \beta_{i+1}, p_{i+1}) \), \( (\alpha_{i+1}, \beta_{i+1}, p_{i+1}) \), \( (\alpha_{i+1}, \beta_{i+1}, p_{i+1}) \). When \( \alpha > \alpha_i \) and \( \beta > \beta_i \) are designed as the sampled points of the three-dimensional implementation algorithm.

Thus, the residual term of the algorithm is given as follows

\[ R = y(\alpha, \beta, p) - y_{\alpha, \beta, p} = \frac{(\alpha - \alpha_i)(\alpha - \alpha_{i+1})}{2} \frac{\partial^2}{\partial \alpha^2} y(\xi_0, \beta, p) \]

\[ + \frac{(\beta - \beta_i)(\beta - \beta_{i+1})}{2} \frac{\partial^2}{\partial \beta^2} y(\alpha, \nu_0, p) \]

\[ + \frac{(\lambda - \lambda_i)(\lambda - \lambda_{i+1})}{2} \frac{\partial^2}{\partial \lambda^2} y(\alpha, \beta, \phi_0) \]

\[ + \frac{(\alpha - \alpha_i)(\alpha - \alpha_{i+1})(\beta - \beta_i)(\beta - \beta_{i+1})}{4} \frac{\partial^4}{\partial \alpha^2 \partial \beta^2} y(\xi_1, \nu_1, p) \]

\[ + \frac{(\alpha - \alpha_i)(\alpha - \alpha_{i+1})(\beta - \beta_i)(\beta - \beta_{i+1})}{4} \frac{\partial^4}{\partial \alpha^2 \partial \beta^2} y(\alpha, \nu_1, \phi_1) \]

\[ + \frac{(\beta - \beta_i)(\beta - \beta_{i+1})(p - p_k)(p - p_{k+1})}{4} \frac{\partial^4}{\partial \beta^2 \partial p^2} y(\xi_1, \beta, \phi_1) \]

\[ + \frac{(\beta - \beta_i)(\beta - \beta_{i+1})(p - p_k)(p - p_{k+1})}{4} \frac{\partial^4}{\partial \beta^2 \partial p^2} y(\alpha, \nu_1, \phi_1) \]

\[ \times \frac{\partial^6}{\partial \alpha^2 \partial \beta^2 \partial p^2} y(\xi_2, \nu_2, \phi_2) \]

\[ \tag{13} \]

where \( (\alpha, \beta, p) \in [(\alpha_i, \beta_i, p_{i}), (\alpha_{i+1}, \beta_i, p_{i}), (\alpha_{i+1}, \beta_{i+1}, p_{i}), (\alpha_i, \beta_{i+1}, p_{i}), (\alpha_i, \beta_i, p_{i+1}), (\alpha_{i+1}, \beta_i, p_{i+1}), (\alpha_{i+1}, \beta_{i+1}, p_{i+1})] \).

\( y(\alpha, \beta, p) \) is the real value of the first-order reversal curves. \( \xi_n, \nu_n \), and \( \phi_n \), \( n = 0, 1, 2 \) are points of the intervals \( (\alpha_i, \alpha_{i+1}), (\beta_i, \beta_{i+1}), \) and \( (p_k, p_{k+1}) \), respectively. Assuming the meshed intervals of the \( \alpha, \beta \) plane and the vertical axis (load variable) are equal to \( h_1 \) and \( h_2 \), respectively. Then we obtain \(|\alpha - \alpha_i| \leq h_1, |\alpha - \alpha_{i+1}| \leq h_1, \)
\[ \| \beta - \beta_j \| \leq h_1, \| \beta - \beta_{j+1} \| \leq h_1, \| p - p_k \| \leq h_2, \| p - p_{k+1} \| \leq h_2, \text{ and the following inequality holds} \]

\[
R \leq \left( \frac{\| D_x^2 y \|}{2} + \frac{\| D_y^2 y \|}{2} \right) h_1^2 + \frac{\| D_x^2 D_y^2 y \|}{4} h_1^4 + \left( \frac{\| D_x^2 D_y^2 y \|}{4} + \frac{\| D_y^2 D_x^2 y \|}{4} \right) h_2^2 + \frac{\| D_x^2 D_y^2 y \|}{8} h_1^4 h_2^2
\]

where \( D_x^2 \) represents the partial differentiation with respect to \( x \).

It is clear that the errors of (11) and (12) are determined by the intervals \( h_1 \) and \( h_2 \) of the limiting triangular prism \( P_0 \). The larger the intervals \( h_1 \) and \( h_2 \) are, the less precise the TPM is. However, decreasing the intervals \( h_1 \) and \( h_2 \) results in a more tedious task for computing. So for practical applications attention should be paid to the choice of the mesh of the limiting triangular prism \( P_0 \).

3. Experimental set-up

In this section, a test platform is developed as indicated in Fig. 7 to assess the properties of coupling hysteresis in PEA and to verify the performance of the TPM. Fig. 8 gives the detailed schematic of the data acquisition system, which consists of a National Instrument DAQ board and a computer control system. The National Instrument DAQ board has a multi-channel Sample/ Hold board that can record three analog signals simultaneously: the expansion of the test actuator \( y(t) \), the excitation voltage \( u(t) \), and the applied load \( p(t) \). A non-contact capacitive sensor with 5 nm resolution and 8 kHz bandwidth measures the expansion \( y(t) \) of the test actuator, which has 100 layers with each being \((1 \times 1) \text{ cm}^2\) wide and 0.1 mm thick. An arbitrary load \( p(t) \) varying from 0 N to 200 N with frequencies in the range of 0–100 Hz in the z-direction, is produced by the deformation of the upper piezo-actuator (load generator) that can be measured by a load cell with 0.6 N (0.03%) accuracy (measurement range 0–2000 N) and high rigidity (1 kN/\text{m}m). Together with a window based user interface, the data acquisition software (Labview) is adopted to acquire signals and convert 12 analog inputs to digital signals for processing.

In order to experimentally assess the performance of the TPM, the two databases of \( y_{\alpha \beta} \) and \( y_{\alpha \eta}, y_{\beta \eta} \) functions first need to be measured. The limiting triangle prism \( P_0 \) (\( \alpha \beta \eta \) coordinate) of Fig. 5 is set that \( 0 \leq \alpha \leq 120 \text{ V}, 0 \leq \beta \leq 120 \text{ V} \) and \( 0 \leq p \leq 100 \text{ N} \). \( P_0 \) is divided into 120 cuboids and triangular prisms with a width equal to 10 V for the \( \alpha - \beta \) plane and 20 N for the axis \( \eta \). Herein, the database of \( y_{\alpha \beta} \) and \( y_{\eta} \) is established under conditions of a sinusoidal excitation voltage with frequency of 0.1 Hz, during which the external load remains constant (from 0 N to 100 N). Then, 120 group parameters of the interpolation equations (11) and (12) can be calculated for estimating the first term in (1). Similarly, the limiting triangle prism in \( \lambda \eta u \) coordinate is set that \( 0 \leq \lambda \leq 100 \text{ N}, 0 \leq \eta \leq 100 \text{ N} \) and \( 0 \leq u \leq 120 \text{ V} \), which has the same mesh division as the case of \( \alpha \beta \eta \) coordinate. The database of \( y_{\lambda \eta}, y_{\lambda \eta} \) functions is developed under conditions of a 0.1 Hz sinusoidal load excitation and a constant voltage (from 0 V to 120 V). It is easy to find that 320 group parameters will be determined for calculating the second term in (1) based on the database of \( y_{\lambda \eta} \) and \( y_{\lambda \eta} \) functions. Once
the two databases are measured, the (7)–(10) are used to estimate the expansion of PEA subject to any two known inputs.

4. Properties of coupling hysteresis in PEA

4.1. Equal vertical chords property

In order to use the TPM (7)–(10) to characterize hysteresis behavior in PEA, equal vertical chords property and path independence property [6] have to both be satisfied. These properties constitute the necessary and sufficient conditions for the description of coupling hysteresis by the TPM on a set of piece-wise monotonic inputs: excitation voltage and external load. The equal vertical chords property [1] is satisfied, regardless of the past history of variations of $u(t)$ and $p(t)$, if all minor hysteretic loops corresponding to the same consecutive extrema values of excitation voltage $u(t)$ for the same fixed load $p$ have equal vertical chords. We know that the equal vertical chords property is a release of the conguency property [6] with no load applied. Then, we only discuss the equal vertical chords property under the conditions of PEA subject to low-speed excitation voltage and high loads. Fig. 9(a) gives the input voltage $u(t)$ applied on a PEA that leads to two minor loops between 30 V and 78 V. Fig. 9(b) and (c) presents the two minor loops with a PEA subject to external loads of 25 N and 100 N, respectively. $y_0^u$ and $y_0^p$ correspond to the same value of the input voltage $u(t)$ at 55 V and are output values on ascending and descending branches of the minor loops, respectively. The increment between $y_0^u$ and $y_0^p$ of the minor loops 1 and 2 are both insignificant in two cases, though the shape of the voltage-to-expansion curves is not completely the same as the load varies over a large band. For any $u \in (30, 78)$, the corresponding vertical chord is checked and found to be satisfied that does not depend on a particular past history preceding the formation of minor loops. This proves that all comparable minor loops, which mean the loops with the same back-and-forth input voltage for any fix load, have equal vertical chords. The same is true for minor hysteresis loops formed as a result of back-and-forth variations of $p(t)$ for any fixed value of $u$.

4.2. Path independence property

We next proceed to discussion of the path independence property [6]. Consider two points $(p_1, u_1)$ and $(p_2, u_2)$ on the $p$–$u$ plane, a set of paths connect the two points corresponding to a monotonically increasing excitation voltage $u(t)$ and external load $p(t)$. A path independent property is satisfied if the output increment does not depend on a particular monotonic path between the two points $(p_1, u_1)$ and $(p_2, u_2)$. As shown in Fig. 10, for instance, three different monotonic paths 1, 2, and 3 connect the point $(p_1, u_1) = (45, 30)$ with $(p_2, u_2) = (48, 36)$ that represent a excitation voltage monotonically increase from 30 V to 36 V, and a dynamic load from 45 N to 48 N. Let $\Delta y$ denote the output increments from point $(p_1, u_1)$ to $(p_2, u_2)$. Three sets of excitation signals applied on a PEA and the measured values of output increments $\Delta y$ are given in Table 1. The differences are insignificant in all three paths. From here it is easy to conclude the validity of the path independent property.

<table>
<thead>
<tr>
<th>Expansion ($\mu$m)</th>
<th>Path 1</th>
<th>Path 2</th>
<th>Path 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y(p_1, u_1)$</td>
<td>8.184</td>
<td>8.174</td>
<td>8.133</td>
</tr>
<tr>
<td>$y(p_2, u_2)$</td>
<td>10.415</td>
<td>10.400</td>
<td>10.350</td>
</tr>
<tr>
<td>$\Delta y$</td>
<td>2.231</td>
<td>2.226</td>
<td>2.217</td>
</tr>
</tbody>
</table>
This section presents the means to assess the feasibility of the TPM for characterization of piezoelectric hysteresis or hysteresis compensation based on experimental data. Experiments show that the TPM can be potentially used in cases where a PEA is subject to any two known inputs: excitation voltage and external load, which require both the equal vertical chords property and the path independence property to be satisfied.

5. Experimental assessments

Case 1. Sinusoidal excitation voltage with different frequencies and load of 0 N.

We firstly discuss the effect of voltage frequency on the performance of the TPM (7)-(10). A set of voltage signals with different frequencies from 0.1 Hz to 100 Hz are applied on the PEA discussed in Section 3 with no load applied. The expansion is measured at 21 points along one cycle of the voltage signal as shown by the example in Fig. 11(a). Under the same excitation conditions, the CPM is also used to predict the output response of the 21 points at frequencies selected via the bilinear spline interpolation and linear interpolation methods [7]. Fig. 11(b) shows that the TPM has no obvious advantage over the CPM. Moreover, the TPM and the CPM both provide high precision (20 nm) when the frequency of excitation voltage is around 0.1 Hz. However, both the average error estimated by the TPM and the CPM increase considerably as the voltage frequency is far away from 0.1 Hz that is used to generate two FORCs databases. It is clear that these two average errors reach up to 118 nm and 116 nm at 100 Hz, respectively. Experiments further verify that the TPM is a modification of the CPM. The difference between the two models may result from the accuracy of the interpolation algorithm and the measured error.

Case 2. Low-speed arbitrary excitation voltage and high constant load.

We now discuss the effect of external load on the accuracy of the TPM in a case where the PEA is subject to an arbitrary voltage with a frequency of 0–0.3 Hz and high constant loads. The excitation voltage has a similar frequency with the pseudo-static voltage signal (0.1 Hz) that is used to generate the database of curves $y_{exp}$ and $y_{sim}$. As shown in Fig. 12(a) and (d), it is clear that the TPM provides good estimates at the external load of 25 N and 100 N respectively, and its accuracy changes slightly as the load increases. However, the performance of CPM deteriorates as the coupling hysteresis effects become more dominant in the PEA for high load applications. From Fig. 12(a), the average error of the CPM is 104 nm at load of 25 N that is more than twice of the TPM (49 nm) case. Furthermore, it reaches up to 234 nm at 100 N, more than four times greater than the TPM shown in Fig. 12(c). As demonstrated in Fig. 12(a) and (c), the maximum errors between measured data and expansions calculated by the TPM are much smaller than the CPM ones. Fig. 12(b) and (d) further demonstrate that the TPM could realize well characterization for both the major and minor hysteresis curves of the PEA as the external load increased. Therefore, experimental results indicate that the TPM can provide a much more satisfied accuracy to characterize coupling hysteresis nonlinearity in PEA under high constant load applied.

Case 3. High-speed arbitrary excitation voltage and dynamic load.

Furthermore, the performance of the TPM is investigated under conditions of an arbitrary excitation voltage with a multi-frequency of 50–100 Hz and a sinusoidal load signal with frequency of 10 Hz and amplitude of 100 N. As shown in Fig. 13(a) and (b), experiments on a PEA show that, with the TPM, predicting can be achieved with 142 nm mean error. However, the accuracy of the CPM is shown to deteriorate as load acting on the PEA varies with time and the range of frequencies contained in the voltage signal gets wider. The mean error of the CPM is 303 nm under same excitation conditions with the TPM. Even though the average error of the TPM compared with Fig. 12 in the case of low-speed excitation voltage and high constant load, increases considerably, by contrast, the TPM remains a much better precision than the CPM. Furthermore, in practical applications of PEA where the voltage excitation is limited to a

Fig. 10. Three different monotonic paths.

Fig. 11. Errors obtained using TPM and CPM with different frequencies of sinusoidal voltage. (a) Particular voltage with the frequency of 0.1 Hz and 21 points selected. (b) Average errors of the TPM and CPM.
narrow band of frequencies, the TPM is still acceptable for modeling hysteresis with dynamic load applied.

6. Conclusions

This paper solves the identification problem of the TPM by adopting a three-dimension interpolation algorithm based on the FORCs technique. Experiments are conducted to test the feasibility of using the TPM to characterize piezoelectric coupling hysteresis, which requires both the equal vertical chords property and path independent property to be satisfied. The performance of the TPM in predicting expansion of PEA in the case of any two known inputs (voltage and external load) is assessed herein. Experimental assessments show that compared with the CPM, the TPM could provide the desired modeling accuracy in a case where the actuator is subject to a high constant load. However, the offline identification procedure limits the accuracy of the TPM as the frequency of excitation voltage increases, and it has no obvious advantage over the CPM. Experimental results further verify that the TPM is a Preisach-type hysteresis model. Even so, the TPM still provides much better accuracy than the CPM under conditions of a high-speed excitation voltage and a dynamic load. Several coupling hysteresis models, based on the Preisch model [21–24] and PI model [26], have the same limitations as the TPM. So the alternative to the TPM would be a dynamic model [25], assuming that it is feasible to describe the piezoelectric coupling hysteresis. Furthermore, the use of an online identification technique, such as the neural network method, should be adopted if the accuracy of the TPM is not satisfactory.
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References


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