An Elasticity Inspired Method for Multi-AUVs Formation Control Using Range-only Measurements

Hongli Xu
State Key Laboratory of Robotics
Shenyang Institute of Automation, CAS
Shenyang, China
xhl@sia.cn

Guannan Li
State Key Laboratory of Robotics
Shenyang Institute of Automation, CAS
Shenyang, China
liguannan@sia.cn

Multi-AUVs formation control has received a lot of theoretical research achievements during the past ten years. Recent researches pay more attention to the application-oriented problems such as communication constraints, range-only measurements, actural mission restrictions and so on. Particularly the range-only measurements is a general problem for AUV because it is more difficult to measure relative orientation or to deliver location information in real time than to measure relative distance through acoustic sensor. In response to this problem, this paper discusses three AUVs formation in which each AUV can only measure the relative distances of the other AUVs and presents a novel method named by Elasticity Inspired (ELASI) Method. The performance of the proposed formation controller is verified by numerical simulations.

Keywords—formation control; multi-AUVs system; range-only measurements; elasticity inspired method

I. INTRODUCTION

Cooperative control of multi-AUVs systems has become a subject that triggered the interest of the research community in recent years because of its potential applications in fields such as oceanographic research, seafloor survey, underwater archeology, etc. Formation control is one of the fundamental problems in Multi-AUVs cooperation. In formation the AUVs try to keep a specific orientation and distance with respect to each other while moving as a group.

There are two kinds of formations according to the assigned tasks. In one case, the formation is defined in the leader coordinate based on the status of a real or virtual leader in the multi-AUVs system. The followers keep a desired formation configuration in the leader coordinate and these trajectories will parallel to the leader’s trajectory. In the other case, the members in the formation are expected to form certain geometry in the global coordinate. They keep relative global positions constant and therefore their trajectories may be unparallel. Obviously, the main advantage of the latter kind of formation is needless for followers to know the leader’s state in real time. It is possible to control the formation only by the measured relative distances, and is more significant for multiple AUVs in underwater environment because it is more difficult to measure relative orientation or to deliver location information in real time than to measure relative distance through acoustic sensor.

Now there have been some strategies for multi-robot formation control using range only measurements. Ming Cao[5] proposed an algorithm to coordinate a set of mobile agents when the agents can only measure the distances to their respective neighbors. To control the shape of the formation, the solution in this paper involves subsets of non-neighbor agents cyclically localizing the relative positions of their respective neighbor agents while these are held stationary, and then moving to reduce the value of a cost function which assumes the zero value precisely when the formation has correct distances. Jorge M. Soares[2~3] solved the problem of keeping an AUV in a moving triangular formation with respect to two leaders by proposing a control strategy that estimates the formation speed and heading form the ranges obtained to the two leading vehicles, and uses simple feedback laws for speed and heading commands to suitably defined common and differential errors to zero. Sunghwan Kim[4] deal with the similar problem and proposed a nonlinear formation controller using the typical input-out feedback linearization method. Brian D. O. Anderson[5] concerned with the problem of agents inferring the relative positions of its neighbors with range-only measurements and limited communication, with using graph theoretic methods for its solution. Additional efforts applied the known algorithms for formation shape control based on distance preservation. Calafiore[6] considered the problem of autonomous distributed estimation of the position of each agent using noisy measurements of inter-agent distances and proposed a computational scheme based on a distributed gradient algorithm with Barzilai-Borwein step sizes.

In this paper, a formation control method with the name of Elasticity Inspired (ELASI) Method is proposed. The basic idea of this method is that all vehicles in the multi-AUVs system are connected with virtual springs that can create virtual force varying with their relative distances, and then the desired motions of the vehicles can be controlled by the virtual force.

In Section 2, we introduce the ELASI method which is employed for formation control using range-only measurement in global coordinate. Section 3 presents the implementation of this method in formation defined in leader coordinate and gives comparison between the proposed method and an available Line of Sight (LOS) method. Section 4 analyses some of the achieved simulation results. The paper is finished by presenting discussion, conclusions and future work in Section 5.
II. ELASI METHOD FOR FORMATION CONTROL IN GLOBAL COORDINATE

A. ELASI method

ELASI method is a novel method for formation control that can be applied to multi-AUVs formation control using range-only measurements. The basic idea of this method is that each vehicle in the system is connected with several virtual springs, which have a feature that they can create virtual force according to the difference between their actual lengths and their desired lengths. The virtual force will drive the vehicle to its desired position.

For a vehicle connected with n virtual springs, the lengths of these virtual springs are $l_1, l_2, \ldots, l_n$ respectively with their desired lengths being $d_l_1, d_l_2, \ldots, d_l_n$. The resultant virtual force acting on the vehicle can be represented as:

$$\vec{F} = \sum_{i=1}^{n} \vec{F}_i$$

Then it is rewritten by Hooke’s law of elasticity:

$$\vec{F} = \sum_{i=1}^{n} ((l_i - d_l_i) \cdot \vec{e}_i \cdot k_i)$$

Where $\vec{e}_i$ represents the unit vector along the $i$th virtual spring, and $k_i$ is a parameter that represents the elastic constant of the virtual springs.

The resultant force $\vec{F}$ will drive the vehicle to the desired position where all $l_i - d_l_i = 0$, which means $\vec{F} = 0$ there. The virtual force is exerted on the vehicle directly, so the desired velocity of the vehicle is calculated by:

$$\vec{v} = \vec{v}_0 + k t \cdot \vec{F}$$

In which $k$ is a constant and $\vec{v}_0$ is initial value of $\vec{v}$.

B. ELASI method for formation control in global coordinate

This paper only discusses the formation including three members in which one is defined as leader and the other two as followers. When the three vehicles form a certain triangle, it is said that they have formed the desired formation, even though the geometry may rotate in the horizontal plane. In this case, the formation is defined as $D \| [D_1L|, |D_2L|, |D_1D_2|] \| under global coordinate, as is shown in Figure 1. Then the purpose of the formation control is:

$$|F_1L| \rightarrow |D_1F|, |F_2L| \rightarrow |D_2L| \text{ and } |F_1F_2| \rightarrow |D_1D_2|.$$

In the formation, each vehicle is regarded as a node, and the line connected them are corresponding springs. So for each follower, it is connected with two virtual springs, whose desired lengths are those of the desired triangle sides. Then for follower 1:

$$v_{F_1L} = \frac{v_{F_1} + v_{F_2}}{2} \cdot |F_1L| \cdot \vec{e}_{F_1L} \cdot k$$

$$v_{F_1F_2} = \frac{v_{F_1} + v_{F_2}}{2} \cdot |F_1F_2| \cdot \vec{e}_{F_1F_2} \cdot k$$

And for follower 2:

$$v_{F_2L} = \frac{v_{F_1} + v_{F_2}}{2} \cdot |F_2L| \cdot \vec{e}_{F_2L} \cdot k$$

$$v_{F_1F_2} = \frac{v_{F_1} + v_{F_2}}{2} \cdot |F_2F_1| \cdot \vec{e}_{F_1F_2} \cdot k$$

Where $\vec{e}_{F_1L}, \vec{e}_{F_2L}, \vec{e}_{F_1F_2}$ and $\vec{e}_{F_2F_1}$ are corresponding unit vectors.

![Fig. 1. The formation definition in global coordinate. L is leader; F_1 and F_2 are two followers respectively. Triangle LF_1F_2 is the current position while LD_1D_2 the desired formation.](image)

Algorithms for estimating the distances between vehicles are needed because of the measurement delay caused by acoustic propagation delay in underwater environment. They are presented as follows:

$$|F_1L|_{t+\Delta t} = |F_1L|_t + tag \cdot \vec{v}_{F_1L} \cdot \Delta t$$

Where $tag = sign(l_d - l)$, with $l_d$ being the desired length of the corresponding spring and $l$ being its actual length.

Each follower can measure the distances to its neighborhood vehicles, but cannot get the distance information between other vehicles. Take follower 1 for example, it can get the values of $|F_1L|$ and $|F_1F_2|$, but cannot measure $|F_2L|$. Accordingly, $\vec{e}_{F_1L}$ and $\vec{e}_{F_2L}$ can be estimated by the leader’s state sent to the followers on time, but $\vec{e}_{F_1F_2}$ and $\vec{e}_{F_2F_1}$ cannot be updated because of the assumption of no communication between the followers.

C. One way for estimating the follower’s unit vectors

As has been discussed above, additional conditions are needed for the followers to estimate the two unit vectors $\vec{e}_{F_1F_2}$ and $\vec{e}_{F_2F_1}$. One solution to this problem is to keep them
constant theoretically. Even though they may change slightly during runtime, the insensitivity of the ELASI method to orientation can ensure their validity. It can be proved that these unit sectors are constant theoretically when the formation can satisfy the condition \(|D_1L|/|F_1L|=D_2L/|F_2L|\), and the proofs are as follows.

Let the origin formation be \([F_1L, F_2L, F_1L_2]\), and the desired formation be \([D_1L, D_1L_2, D_1D_2]\). The three vehicles are \(L(0,0)\), \(F_1(x_1, y_1)\) and \(F_2(x_2, y_2)\). When \(L\) stays still, after moving for \(\Delta T\), new positions of \(F_1\) and \(F_2\) are \(F_1'(x_1', y_1')\) and \(F_2'(x_2', y_2')\):

\[
\begin{align*}
F_1' &= F_1 + k(F_1L - |D_1L| + F_1F_2 - |D_1D_2|)\Delta T/D_1L + F_1F_2 - |D_1D_2|/D_1D_2)\Delta T \\
F_2' &= F_2 + k(F_2L - |D_2L| + F_1F_2 - |D_1D_2|)\Delta T/D_2L + F_2F_1 - |D_1D_2|/D_1D_2)\Delta T
\end{align*}
\]

Then

\[
y_2'-y_1' = \frac{(y_2 - y_1)K - (y_2 - y_1) - (y_2 - y_1)}{(x_2 - x_1)K - (x_2 - x_1) - (x_2 - x_1)}K = 1 + 2\frac{|F_1'F_2'| - |D_1D_2|}{|F_1'F_2'|}k\Delta T
\]

As can be conducted from the above that \(y_2'-y_1' = y_2 - y_1\) only when

\[
\frac{F_1'L - |D_2L|}{F_1'L} = \frac{F_2'L - |D_1L|}{F_2'L} = \frac{F_1'L - |D_1L|}{F_1'L}
\]

Then

\[
|D_1L|/|F_1L| = D_2L/|F_2L|
\]

The claim is then proved.

**D. Another way for estimating the follower’s unit vectors**

Another way to get the unit vectors is to design an estimator according to ELASI law. For both of the followers sharing a component motion the same as the motion of the leader, it is reasonable to assume that \(L(0,0)\) stays still, then the corresponding estimated position of each follower is \(F_{1,t}(x_{1,t}, y_{1,t})\) and \(F_{2,t}(x_{2,t}, y_{2,t})\). For each follower, it knows \(v_{\text{form}}\) of itself, and an estimated \(v_{\text{form}}\) of the other follower can be gotten according to the estimated positions \(F_{1,t}\) and \(F_{2,t}\). After moving for a period of \(\Delta t\), new estimated position of each follower can be given as:

\[
F_{1,t+\Delta t} = F_{1,t} + v_{\text{form}} \cdot \Delta t
\]

Then the new unit vectors are:

\[
e_{F_1L,t+\Delta t} = -\frac{y_{F_1,t+\Delta t} - y_{F_1,t}}{x_{F_1,t+\Delta t} - x_{F_1,t}}
\]

As each vehicle knows the position of all vehicles in the beginning, the initial values of the estimator can be gotten. The unit vectors for each period can then be estimated by repeating the process. The advantage of this method over the one given above is that it is suitable for random formations, but its performance is not as good.

**III. ELASI METHOD FOR FORMATION CONTROL IN LEADER COORDINATE**

In some missions, it is expected that trajectories of vehicles are parallel. In this case, formations are usually defined in a coordinate based on the status of leader. Available method for this formation is based on LOS theory in [7], and the novel ELASI method can also be applied to this formation.

A system of two AUVs is analyzed, with one acting as the leader and the other as the follower. The definition of the formation is depicted by Figure 2. The formation \(D\) is presented by vector \([Dx, Dy]\) which is defined on a leader-based coordinated in terms of the leader’s position and attitudes.

![Fig. 2. The definition of the formation in the leader coordinates. L is leader and F is follower. D is the desired position of a follower.](image-url)
A. ELASI method with one virtual spring

The simplest way to set the virtual spring is to set the line that connects the follower and its destination in the leader coordinate as the virtual spring, as is shown in Figure 2, the virtual spring is line FD. In this case, when \(|FD|=0\), \(D_x=0\) and \(D_y=0\), the formation control is accomplished. Based on the virtual spring law, we can get

\[ v_{\text{form}} = k \cdot \overrightarrow{FD} \cdot \overrightarrow{e_{FD}} \]

(7)

Where

\[ \overrightarrow{e_{FD}} = \overrightarrow{FD} / | \overrightarrow{FD} | \]

Attention should be paid to the adjustment of \(k\), for it is expected that the follower will turn as small an angle as possible to accomplish the formation. A tag in relate to the velocity of the follower \(\overrightarrow{v}\) and vector \(\overrightarrow{FD}\) in global coordinate is set to help deciding \(k\).

\[ tag = (\overrightarrow{FD} \cdot \overrightarrow{v})(\overrightarrow{v_{\text{leader}}} \cdot \overrightarrow{v}) \]

(8)

Parameter \(k\) should be set bigger when \(tag \geq 0\) while smaller when \(tag < 0\).

B. ELASI method with three virtual springs

As is shown in Figure 2, we can also set three virtual springs FL, FM, and FN, with point M and point N being auxiliary points and point D being in line MN.

In this case, the purpose of the formation control is:

\[ |FL| \rightarrow |DL|, \ |FM| \rightarrow |DM| \text{ and } |FN| \rightarrow |DN|. \]

Then the motion that drives the follower to accomplish the formation is:

\[ v_{\text{form}} = \overrightarrow{v_l} + \overrightarrow{v_m} + \overrightarrow{v_n} \]

(9)

Where

\[ \overrightarrow{v_l} = k_l \cdot (|FL| - |DL|) \cdot \overrightarrow{FL} / |FL| \]

\[ \overrightarrow{v_m} = k_m \cdot (|FM| - |DM|) \cdot \overrightarrow{FM} / |FM| \]

\[ \overrightarrow{v_n} = k_n \cdot (|FN| - |DN|) \cdot \overrightarrow{FN} / |FN| \]

It should be noticed that proper choice of \(k_l, k_m\) and \(k_n\) can ensure \(v_{\text{form}} = 0\) only when the follower reaches the desired position. Usually \(k_l\) is set larger than the other two parameters.

C. LOS method

LOS method is a method widely used in formation control, which performs well in formation control in leader coordinate. This method is introduced to contrast with the ELASI method in performance. The basic idea of this method is to set a leader coordinate based the status of the leader, and the follower is required to keep a constant position \(D\ [D_x, D_y]\) in the leader coordinate. So the purpose of the formation control can be described as \(F_x \rightarrow D_x, F_y \rightarrow D_y\).

Then the desired yaw angle for the follower is:

\[ \psi_y^d = \theta + \arctan \frac{F_y - D_y}{\Delta} \]

And its desired surge speed can be given as:

\[ u_y^d = u_l^d - \chi(e_x) \]

Where parameter \(\Delta\) is an empirical value, and \(u_l^d\) represents the desired speed of the leader. Function \(\chi\) is an adjustment on error along the desired path, which should satisfy certain requirements.

IV. SIMULATION RESULTS

This section presents our simulation results of the formation control derived in MATLAB. We first show the trajectory of a triangle formation using range-only measurements in global coordinate, and then that of the formation in a leader coordinate, with a comparison made between the ELASI method and the LOS method in performance.

A. Simulations in global coordinate

A three AUVs system is simulated with one of them acting as leader and the other two as followers. In the first simulation the formation is defined as \(D= [10 10 10]\), which represents the distances of \(LD_1\), \(LD_2\) and \(LD_3\) respectively. And the original positions of vehicles are \(L=[10 10]\), \(F_1=[7.3 20]\) and \(F_2=[3 3]\), thus meeting the condition \(|LF_1| / |LD_1| = |LF_2| / |LD_2|\). The parameter is defined as \(k=0.1\), which is an empirical value. According to previous research in [7], the measurement delay is set as 20 seconds.

We show the simulation results of how well the proposed algorithms work using range-only measurements in Figure 3 and Figure 4. The resulting trajectory can be seen in Figure 3, which shows that the three vehicles form the desired triangle formation. Figure 4 shows some metrics related to the performance of the algorithm. The errors along line \(F_1L\), \(F_2L\) and \(F_1F_2\), which is denoted as \(\Delta [F_1L] = \Delta [F_2L] = \Delta [F_1F_2]\) respectively, remain fairly stable and close to the reference values after a brief initial transient. The formation can be formed rapidly, with a convergence time less than 70s.

In the second simulation ELASI controller with the given unit vector estimator is also simulated. The desired formation is \(D=[30 30 40]\) while the initial formation is a random one. Other parameters are set the same as above. The trajectory of can be seen in Figure 5, which indicates a random formation finally converging to the desired one. The performance can be seen in Figure 6, where \(\Delta [F_1L] = \Delta [F_2L]\) and \(\Delta [F_1F_2]\) stay close to zero with slight vibrations after 200s. Comparison between the two cases above shows that the unit vector estimator expends the
use of the ELASI method to random formations at the cost of performance.

Fig. 3. Trajectory of the system. L denotes the trajectory of the leader AUV, F₁ denotes the trajectory of follower 1 while F₂ denotes the trajectory of follower 2.

Fig. 4. The formation variables of the system. Δ|F₁L|, Δ|F₂L| and Δ|F₁F₂| respectively denotes the error along line F₁L, F₂L and F₁F₂.

Fig. 5. Trajectory of the system. L denotes the trajectory of the leader AUV, F₁ denotes the trajectory of follower 1 while F₂ denotes the trajectory of follower 2.

Fig. 6. The formation variables of the system. Δ|F₁L|, Δ|F₂L| and Δ|F₁F₂| respectively denotes the error along line F₁L, F₂L and F₁F₂.

Fig. 7. Trajectory of the system. Leader denotes the trajectory of the leader AUV, Follower 1 denotes the trajectory in the case where the follower is controlled with ELASI method using one virtual spring, Follower 2 denotes the trajectory in the case where the follower is controlled with ELASI method using three virtual springs and Follower 3 denotes the case where the follower is controlled with the LOS method.

Fig. 8. The formation variables of the system. x error and y error respectively denote the error along x axis and y axis. Follower 1 denotes the error of the follower controlled with one virtual spring, Follower 2 denotes the error of the follower controlled with three virtual springs and Follower 3 denotes the follower controlled using the Line of Sight method.
B. Simulations in leader coordinate

The proposed formation control method is also applied to formation in a leader based coordinate. A two-AUV system is simulated, with one being the leader that follows a scheduled path, while the other acting as the follower that keeps a constant position in the coordinate based on the leader. The formation is defined as [-20 30]. As an assumption is made that both vehicles can communicate with each other, the follower has the knowledge of the status of the leader.

As has been discussed above, there are two ways to apply the ELASI method to this formation. The follower can be controlled based on one virtual spring, or three virtual springs. Both ways are simulated in this section. In order to show how well this method works, the LOS method, an available formation control method is also simulated.

The resulting path can be seen in Figure 5, where Leader denotes the path of the leader, and Follower1, Follower2 and Follower3 denotes the paths of the same follower controlled with the above three methods in different simulations respectively, and they are put in the same figure for easy contrast. It can be concluded that the ELASI method is comparable with the LOS method in performance for the paths of Follower1, Follower2 and Follower3 are almost the same. Metrics in relate to the performances of the controllers can be seen in Figure 6, where the errors along x axis (x error) and the errors along y axis (y error) of the three cases are contrast with each other. It is obvious that in all the three cases, x error and y error can convergent to zero rapidly, which takes approximately 60s, with the errors by the ELASI method being smaller than that by the available LOS method in some cases.

V. CONCLUSION AND FUTURE RESEARCH

This paper presents an alternative solution to three AUVs formation control problem in which two followers move after a leader with a triangular formation by using inter-AUV range measurements only. The proposed algorithm is inspired by elasticity law, which controls the formation by elastic deformation of virtual springs. The algorithm can be used in global coordinate and leader coordinate, in which the settings of virtual springs may be different. The effectiveness of the method is shown by simulation, including comparison with the available LOS method. Future work will focus on improving the performance of the ELASI method by working out a law for setting proper virtual springs and parameters.

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