A genetic approach on cross-layer optimization for cognitive radio wireless mesh network under SINR model

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\section*{A B S T R A C T}

Due to the limited spectrum resources and the differences of link loads, how to obtain maximum network throughput through cross-layer design under signal-to-interference-and-noise ratio (SINR) model is recognized as a fundamental but hard problem. For this reason, the throughput maximization problem jointly with power control, channel allocation and routing under SINR model is researched. First, by formulating the optimization model and digging up its special structure, we show that the throughput maximization problem can be decomposed into two sub-problems: a channel allocation and power control sub-problem at the link-physical layer, and a throughput optimization sub-problem at the network layer. As to the link-physical layer sub-problem, since the joint optimization on channel allocation and power control is NP hard, we apply genetic algorithm for searching the optimal solution. As to the network layer sub-problem, we use linear programming technique for throughput optimization. To reflect the interplay property among these three layers, the fitness of each individual in the genetic algorithm is evaluated by solving the network layer sub-problem. Therefore, an effective cross-layer optimization framework based on genetic algorithm is obtained, which can find optimized power control, channel allocation and route selection in polynomial time. In order to enhance the convergence process during evolution, the integer based representation scheme and corresponding genetic operators are well designed with appropriate constraint control mechanisms. Extensive simulation results demonstrate that the proposed scheme obtains higher network throughput compared to the previous works with comparable computational complexity.

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\section*{1. Introduction}

Wireless mesh networks (WMNs) generally consist of wireless mesh routers and mesh clients, where most of mesh routers are rarely mobile and may not have power constraints. Compared with their single-hop counterpart WLANs, WMNs are self-organized with the mesh routers automatically connecting with one another in a multi-hop manner to form ad hoc networks, which provide improved reliability as well as larger service coverage and lower up-front cost. In recent years, WMNs are considered to be an effective solution for “last-mile” broadband wireless internet access [1].

With the increasing number of users and a growing demand for better quality of service (QoS), limited spectrum resources create serious obstacles to obtain high-performance data services in WMNs. Further, fixed spectrum assignment model makes this problem much worse. Fortunately, the emergence and development of
The authors in Ref. [18] aimed at increasing the network selection mechanism based on the robust performance. To improve the network performance, another way to mitigate co-channel interference is to adjust the power level of each radio. For this reason, power control methods or joint power control and channel assignment algorithms have been proposed [9–12]. Since both channel assignment and power control determine the connectivity between mesh nodes, we jointly refer them as topology control mechanisms.

Routing is also considered as a key issue to enhance network performance [13]. As we know, routing unambiguously determines the flow requirements of each link, while the effective capacity of each link is determined by channel allocation and power control. Therefore, routing in upper layers and channel allocation and power control in bottom layers is inseparable in cognitive radio based wireless networks. Separately considering or promoting the network performance on one aspect cannot achieve optimal performance for the entire network.

At present, there has been some research work on the cross-layer optimization to improve network performance. The authors in Ref. [14] investigated how to satisfy the flow demands and propose an optimized mechanism of joint routing and channel allocation. Considering that some channels may have higher capacities, the authors in Ref. [15] proposed a channel assignment that exploits both channel diversity and data rates to improve network performance. The authors in Ref. [16] presented linear mixed integer programming (LMIP) based algorithm to achieve a good trade-off between fairness and throughput. By specially considering the dynamic feature of the spectrum availability, the authors in Ref. [17] presented the robust performance for each link and propose an optimal route selection mechanism based on the robust performance. The authors in Ref. [18] aimed at increasing the network throughput and present a cross-layer optimization approach for multi-hop cognitive network. To satisfy the end-to-end delay constraint and optimize the usage of the scarce radio resources, the authors in Ref. [19] proposed a cross-layer optimization approach for multi-radio multi-channel wireless mesh networks. However, all the above efforts use the unit-disk graph model (or protocol model) for interference characterization. In this model, the impact of interference is solely determined by whether or not a node falls within the interference range of other transmitting nodes, which does not accurately capture physical layer characteristics. As a result, the accuracy (and validity) of results developed under this model remains unclear [20].

In contrast, signal-to-interference-and-noise ratio (SINR) model is thought to be more realistic than protocol model [21,22]. In this model, concurrent transmissions are allowed and interference (due to transmissions by non-intended transmitter) is treated as noise. A transmission is deemed successful only if the SINR value is larger than a threshold at the receiver. Moreover, achieved transmission capacity is also a function of SINR (via Shannon's formula). However, due to the extreme computation complexity in SINR model, some previous efforts were only done on single-hop networks, e.g., [21,23,24]. For multi-hop networks, various simplifications have been employed. For example, in Refs. [25,26], the transmission power level at each node is identical. In Ref. [27], the cross-layer problems only involve the joint optimization on channel allocation and power control, while the route selection is not considered. It has recently drawn to our attention that the authors in Ref. [22] have proposed a branch-and-bound framework to find the near optimal solution for multi-hop cognitive radio networks. The major differences between their work and ours lie, however in that (i) in our work, the genetic algorithm is applied to find the optimal resource allocation, including finding both the optimal channel allocation and power control level for each mesh router; (ii) we formally analyze the relationship between throughput and resource allocation, and propose linear programming techniques for throughput optimization; (iii) the resource allocation is optimized to maximize throughput, therefore the performance of the entire network is enhanced; and (iv) the multi-radio technology is adopted in our paper, which can improve the achievable throughput even further.

The cross-layer optimization for multi-radio CR based WMNs under SINR model is investigated in our paper. We first formulate this challenging problem mathematically, and dig up its unique structure which allows us to design an effective optimization framework based on genetic algorithm. The optimization process is carried out by utilization of a network layer sub-problem (corresponding to throughput optimization) and a link-physical layer sub-problem (analogous to power control and channel allocation). For the link-physical layer sub-problem, we develop an algorithm based on genetic algorithm (GA). For network layer sub-problem, we use linear programming techniques. To reflect the interaction property of these two sub problems, we define the fitness value of an individual in GA as the value of a linear objective function. In this manner, we successfully solve the cross-layer optimization in polynomial time, balance the demand of link bandwidth at the network layer and increase the link capacity at the link-physical layer, thus to improve the entire network throughput. The optimization framework proposed in this paper also represents a cross-layer strategy for other stochastic optimization techniques.

The rest of this paper is organized as follows. We state our network model and assumptions in Section 2. We theoretically describe the cross-layer optimization problem in Section 3. Section 4 presents the genetic algorithm based nested optimization framework. We evaluate our algorithm in Section 5, and present our conclusions in Section 6.
2. Network model

In this paper, we consider a CR-based WMN consisting of \( V = \{ i | 1 \leq i \leq N \} \) cognitive nodes. Except for one common control channel, we assume there are \( C \) orthogonal channels \( OC = \{ 1, 2, \ldots, C \} \) used for data transmission. Without loss of generality, each node \( i \in V \) in the network is equipped with \( I \) network radios, which can perform spectrum sensing to identify current spectrum "holes" and use them for communications. That is, each node \( i \in V \) perceives a set of available channels \( OC_i \subset OC \) for the given time instance (i.e., those bands are currently not used by primary users). Due to the difference of geographical location, the set of available channels for one node may not be the same as those at other locations. We also denote \( OC_{ij} = OC_i \cap OC_j \) as the set of common channels between node \( i \) and node \( j \). The maximum transmission power is totally quantized into \( T \) levels, and each node can intelligently select its transmission power level, \( t \in T \) for all its radios. For notational convenience, Table 1 lists all the notations in this paper.

2.1. Channel allocation and power control under SINR model

For any node \( i \) and node \( j \) \( (i, j \in V) \), power level \( t \) \( (t \in T) \), and channel \( m \) \( (m \in OC_{ij}) \), we define \( x_{ij}^{m,t} \) as the link resource allocation variable. If \( x_{ij}^{m,t} = 1 \), it means that node \( i \) communicates with node \( j \) using power level \( t \) over channel \( m \). Otherwise, \( x_{ij}^{m,t} = 0 \). Since concurrent transmission may cause the disorder of arriving packages, to guarantee the consistency of routing packets, only one channel and one power level is assigned between node \( i \) and node \( j \) at a time,

\[
\sum_{m \in OC_{ij}} \sum_{t} x_{ij}^{m,t} \leq 1 \quad (i, j \in V, i \neq j) \tag{1}
\]

The channel allocation of node \( i \) is indicated by \( y_{mi}^m \). If \( y_{mi}^m = 1 \), it means \( \exists j \in V, i \neq j, m \in OC_{ij}, t_1, t_2 \in T \). \( x_{ij}^{m,t_1} \) or \( x_{ji}^{m,t_2} = 1 \); otherwise \( y_{mi}^m = 0 \). The number of channels simultaneously used by node \( i \) is limited by the number of network radios,

\[
\sum_{m \in OC_i} y_{mi}^m \leq I(i \in V) \tag{2}
\]

Let \( g_{ij} \) denote the channel gain from transmitter \( i \) to receiver \( j \). In general, it is expressed as the negative exponential function of distance,

\[
g_{ij} = d_{ij}^{-\gamma} (i, j \in V, i \neq j) \tag{3}
\]

where \( \gamma \) is the path loss factor, \( d_{ij} \) is the distance between node \( i \) and node \( j \). When transmitter \( i \) selects channel \( m \) and power level \( t \), the receiving power of node \( j \), denoted as \( p_{ij}^{m,t} \), can be calculated as,

\[
p_{ij}^{m,t} = p_i^t \cdot d_{ij}^{-\gamma} \cdot x_{ij}^{m,t} (i, j \in V, i \neq j, m \in OC_{ij}, t \in T) \tag{4}
\]

where \( p_i^t \) is the transmission power in level \( t \) assigned to transmitter \( i \). Define the interference of receiver \( j \) hearing from other connections on band \( m \) in all power levels as \( \bar{P}_{ij}^m \), that is,

\[
\bar{P}_{ij}^m = \sum_{k \in V, k \neq i} \sum_{h \in OC_{ij}, h \neq j} p_i^t \cdot d_{kj}^{-\gamma} \cdot x_{kj}^{m,t} (j \in V) \tag{5}
\]

Let \( P_N \) be the noise power, and \( s_{ij}^{m,t} \) be the SINR from node \( i \) to node \( j \) on channel \( m \) and power level \( t \), that is,

\[
s_{ij}^{m,t} = \frac{P_{ij}^{m,t}}{\bar{P}_{ij}^m + P_N} (i, j \in V, i \neq j, m \in OC_{ij}, t \in T) \tag{6}
\]

Based on this definition, the transmission from node \( i \) to node \( j \) over channel \( m \) and power level \( t \) is successful, if and only if,

\[
x_{ij}^{m,t} = 1 \iff s_{ij}^{m,t} \geq \zeta, (i, j \in V, i \neq j, m \in OC_{ij}, t \in T) \tag{7}
\]

where \( \zeta \) is the SINR threshold for successful transmission. Considering that (7) is not suitable for mathematical programming, we further transform this constraint as,

\[
s_{ij}^{m,t} \geq x_{ij}^{m,t} \cdot \zeta, (i, j \in V, i \neq j, m \in OC_{ij}, t \in T) \tag{8}
\]

Given the SINR threshold \( \zeta \), when there is no interference from other transmitters, it is clear to see that the maximum transmission range \( R_T \) and noise power \( P_N \) can be calculated as,

\[
\begin{align*}
R_T &= \left( \frac{P_N}{\zeta} \right)^{\frac{1}{2}} \\
P_N &= \frac{\left( \frac{P_N}{\zeta} \right)^{\frac{1}{2}}}{2}
\end{align*}
\]
Combine (8) with (6), if the maximum transmission range \( R_p \), the path loss factor \( \gamma \), and the SINR threshold \( \sigma \) are given in priori, variable \( P_h \) can be removed and \( s_{q,t}^{m,t} \) can be calculated in a much simpler way,

\[
\frac{s_{q,t}^{m,t}}{T} = \frac{\alpha \cdot t \cdot d_{ij}^{-\gamma} \cdot \lambda_{q,t}^{m,t}}{R_T^{-\gamma} + \sum_{h \in V \setminus \{k\}} \sum_{i \in E} t \cdot \lambda_{ih} \cdot d_{ij}^{-\gamma}} \tag{10}
\]

Denote \( U_j(m, t) \) as the effective capacity of link \( e(i, j) \) on channel \( m \) and power level \( t \). According to Shannon's capacity formula, it can be calculated as,

\[
U_j(m, t) = H_m \cdot \log_2 \left( 1 + \frac{s_{q,t}^{m,t}}{C_1} \right) (i, j \in V, i \neq j, m \in OC_j, t \in T) \tag{11}
\]

In practice, if the SINR is very small, the effective capacity is too small to carry traffic flow. Therefore, with the given SINR threshold \( \omega \), the minimum capacity is \( H_m \cdot \log_2 (1 + \omega) \). As only one channel and one power level are assigned to each link \( e(i, j) \), the effective capacity of link \( e_q \) is defined as the sum of \( U_j(m, t) \),

\[
U_q = \sum_{m \in OC_q} \sum_{t \in T} U_j(m, t) (i, j \in V, i \neq j) \tag{12}
\]

2.2. Multi-path routing constraints

Assume there are \( Q \) end-to-end routing sessions in the network, and routing session \( q \in Q \) is represented as \( (s_q, d_q, r_q) \), where \( s_q, d_q, r_q \) represent the source, destination and its minimum traffic demand of session \( q \) respectively. Define \( f_{ij}^q \) (and \( f_{ij}^q \)) as the traffic flow travelling from node \( i \) to \( j \) (and from node \( j \) to \( i \)) for session \( q \), where \( i, j \in V, i \neq j \). Define a common scaling fairness factor \( \lambda (\lambda > 0) \) for all sessions with the given minimum rate requirement \( r_q \). Similar to Ref. [29], the multi-path routing constraints can be defined as follows,

\[
f_{ij}^q \geq 0, f_{ji}^q \geq 0 \quad (i, j \in V, i \neq j, q \in Q) \tag{13}
\]

\[
\sum_{k \in E} f_{ki}^q = \sum_{j \in E} f_{ij}^q \quad (i \in V, q \in Q) \tag{14}
\]

\[
\sum_{i \in E} f_{ij}^q \geq \lambda r_q \quad (i = s_q, q \in Q, \lambda \geq 0) \tag{15}
\]

\[
\sum_{i \in E} f_{ij}^q \leq U_j \quad (i, j \in E, q \in Q) \tag{16}
\]

where (13) restricts the amount of flow on each link to be non-negative, (14) states that the amount of incoming flow is equal to the amount of outgoing flow at each node, except the source and destination, (15) represents that the outgoing flow from the source is at least \( \lambda r_q \), which also states that the incoming flow to the destination is at least \( \lambda r_q \), and (16) indicates that the sum of the flows over all sessions traversing a link cannot exceed the effective capacity. Here, we should note that if the traffic demand is very small, and the effective capacity of each link is very large, the achieved percentage \( \lambda \) may be larger than 1.

3. Optimization problem

Given CR-based WMN topology graph \( G(V, E) \) and the routing requirement set \( (s_q, d_q, r_q), \quad (q = 1, 2, \ldots, Q) \). The problem of maximizing network throughput with fairness enhancement is equivalent to searching optimal channel assignment and power control for the entire network, which can maximize the common fairness scaling factor \( \lambda \), such that the maximum rate of \( \lambda r_q \) can be transmitted from \( s_q \) to \( d_q \) for each session \( q \in Q \). Formally, the joint problem can be expressed as,

\[
\begin{align*}
\text{max} & : \lambda \\
\text{s.t.} & : (1, 0), \quad (i, j \in N, t \in T, m \in OC) \\
\sum_{m \in OC} \sum_{t \in T} & x_{ij}^{m,t} \leq 1 \\
\sum_{m \in OC} & y_{m}^p \leq I \\
g_{ij} & = d_{ij}^{-\gamma} (i, j \in V, i \neq j) \\
p_{ij}^{m,t} & = p^t \cdot d_{ij}^{-\gamma} \cdot \lambda_{ij}^{m,t} (i, j \in V, i \neq j, m \in OC, t \in T) \\
p_{ij} & = \sum_{k \in V \setminus \{i\}} \sum_{h \in V \setminus \{j,h\}} \sum_{t \in T} p^t \cdot d_{ij}^{-\gamma} \cdot \lambda_{ih}^{m,t} (j \in V) \\
s_{q,t}^{m,t} & = \frac{p_{ij}^{m,t}}{P_N + p_{ij}^q} (i, j \in V, i \neq j, m \in OC, t \in T) \\
s_{q,t}^{m,t} & \geq \lambda \cdot s_{ij} (i, j \in V, i \neq j, m \in OC, t \in T) \\
U_j & = \sum_{m \in OC} \sum_{t \in T} U_j(m, t) (i, j \in V, i \neq j) \\
f_{ij}^q & \geq 0, f_{ji}^q \geq 0 \\
\sum_{k \in E} f_{ki}^q & = \sum_{j \in E} f_{ij}^q (i \in V, q \in Q) \\
\sum_{i \in E} f_{ij}^q & \geq \lambda r_q (i = s_q, q \in Q, \lambda \geq 0) \\
\sum_{i \in E} f_{ij}^q & \leq U_j (i, j \in E, q \in Q) \\
\end{align*}
\]

where \( L \) is a constant, \( (s_q, d_q, r_q), \quad (q = 1, 2, \ldots, Q) \), and \( P_N \) are input parameters. Note that \( x_{ij}^{m,t}, y_{m}^p, f_{ij}^q, \) and \( \lambda \) are optimization variables to determine network resource allocation and route selection. In particular, the objective function is linear with non-linear constraints \( s_{q,t}^{m,t} \), \( U_j(m, t) \), and \( U_j \). By convention, the optimization problem is in the form of mixed integer non-linear programming (MINLP) problem, which is NP-hard generally and cannot be solved by CPLEX. In the next section, we will develop a nested optimization framework based on GA to solve this problem.

4. Cross-layer optimization approach

This section describes the proposed unified framework for solving the crosser-layer optimization problem using stochastic optimization methods. First, the optimization framework is presented, and then the optimization method based on genetic algorithm is proposed.
4.1. Optimization framework

From what has been discussed above, we could notice that the difficulty of the optimization problem comes from the integer variables \( x_{ij}^m \) and \( y^m \), which define the channel allocation and power control for each link and node. Specifically, if these variables can be obtained by some technique, the non-linear constraints can be eliminated. Hence, the optimization problem can be simplified to an LP optimization problem, that is,

\[
\begin{align*}
\max & : \lambda \\
\text{s.t.} & \quad f_{ij}^q \geq 0, f_{ji}^q \geq 0 \\
& \quad \sum_{(k)\in E} f_{ki}^q = \sum_{(l)\in E} f_{lj}^q \quad (i\neq s, j\neq q, i \in V, q \in Q)^
(18) \\
& \quad \sum_{(l)\in E} f_{lj}^q \geq \lambda r(q) \quad (i = s, q \in Q, \lambda \geq 0) \\
& \quad \sum_{q\in Q} f_{ij}^q \leq U_{ij} \quad ((i, j) \in E, q \in Q)
\end{align*}
\]

where \( f_{ij}^q, \lambda \) are optimization variables, \( (s_q, d_q, r_q) \) \((q = 1, 2, \ldots, Q)\), and \( U_{ij} \) are input parameters. The objective function is linear with only linear constraints, which can be viewed as a linear programming problem. Linear programming problem is much simpler than MINLP, and can be solved by LP solver quickly. From the description of Eq. (18), we can also get the relationship of network throughput and resource allocation. That is, first, throughput is realized by solving the LP problem defined in Eq. (18). Second, for the above LP problem, the aggregated flow rate cannot exceed the link capacity. Third, link capacity is a function of SINR, which in turn is determined by the power level and channel assignment.

Here, we should note that the determination of integer variables \( x_{ij}^m \) and \( y^m \) includes the optimization of channel allocation and power control. Each optimization problem itself is NP hard and difficult to solve. The integration for joint optimization further increases the process complexity. Hence, an alternative is to use some stochastic optimization techniques to find the optimal solution [5]. As one of the most popular stochastic optimization techniques, genetic algorithm (GA) has been successfully applied to many NP hard problems, such as travelling salesman problem, and multi-processor task scheduling. Thus, in our paper, the genetic algorithm is used to find the optimal solution.

Fig. 1 shows our optimization framework based on genetic algorithm. In this framework, we first encode the channel allocation and power control variables to one sequence code, namely individual. By this way, an initial population including a number of individuals is generated. Thenceforth, this population is evolved according to the designed genetic algorithm, where the fitness function is applied to guarantee the evolution direction. In our paper, the individual fitness is defined as the maximum throughput with current resource allocation in the individual representation. Based on the individual representation, we first calculate the effective capacity for each link, and delete those links whose capacity is less than \( H_{m}[\log_2(1 + \alpha)] \). Then, with such topology graph as well as the capacity of each link is given, finding the maximum throughput for the routing sessions \( (s_q, d_q, r_q) \) \((q = 1, 2, \ldots, Q)\) is equivalent to solving Eq. (18). Finally, after a finite number of iterations, the best individual can be generated and decoded to an optimal channel allocation and power control solution. In addition, the best route for each routing session can also be generated from the evaluation of the best individual. We should note that our framework can be applied to any other stochastic optimization techniques, such as particle swarm optimization, and ant colony optimization. The only difference is the design of the evolution procedures, as shown in the rectangle with a bold line. Here the detail of other optimization techniques is omitted due to the space limit.

4.2. GA based channel allocation and power control

GAs are stochastic search algorithms inspired by the genetic mechanisms of biological species evolution [28]. In order to tailor GA for a particular problem, the following issues are inevitable:

- How to represent solutions?
- How to select individuals for mating?
- How to produce offspring?
- How to insert the offspring into the population?

All these issues will be illustrated in detail in the following section.
4.2.1. Solution representation

GAs cannot deal with the variables of the optimization problem directly. They work with chromosome-like codes representing the solutions. Thus, the principal issue in a GA application is how to represent solutions of the problem, i.e. how to use a data structure that is appropriate for the problem. To the problem of joint power control and channel allocation in cognitive radio WMNs, since the problem space corresponds to wireless routers selecting channels and communication power levels for its radios, an integer based chromosome coding mechanism is designed in our paper.

In this representation scheme, each individual contains a set of $N$ substrings $[\text{str}v]_{a,v} (t_v), (v \in N)$, where substring $\text{str}_v$ is constituted by a channel vector $a_v$ and a power vector $t_v$. Channel vector $a_v = (a_{v1}, \ldots, a_{vi})$ contains $l$ digits of non-negative integer values in the range $[0, C]$ except those channels currently used by primary users. Power vector $t_v = (t_{v1}, \ldots, t_{vi})$ contains $l$ digits of non-negative integer values in the range $[0, Q]$. Take into account this representation scheme, in order to achieve feasible channel and power assignment for any node $v$, all non-zero digits in the channel vector should take on different values. This condition implies that for node $v$, any two radios equipped at $v$ will not select the same channel. In addition, since the order of arranging radios is irrelevant, any two channel vectors with the same value but different order are viewed as the same representation. To avoid duplicating options, we only include channel vector with numerical values and arranged in incremental order.

Fig. 2 illustrates an integer based feasible individual including 5 nodes where $C = 4$ and $I = 2$. Here, $a_1 = (2 3)$ and $t_1 = (2 0)$ mean the first radio of node 1 takes channel 2 and power level 2, and the second radio of node 1 takes channel 3 and power level 0. Since there has one common channel between node 1 and node 2, one directional connection from node 1 to node 2 can be obtained, and its effective capacity can also be calculated by SINR mode. Since the transmission power allocated on node 3 and 4 is larger than 0, we can get two bidirectional connections between node 3 and 4 over channel 4. As the numerical values in the channel vector are arranged in incremental order, $a_1 = (3 2)$ is an infeasible representation for node 1. Here, we should note that some loops may exist in the topology graph after resource allocation, such as $e_{34}$ and $e_{43}$ in Fig. 2. However, similar to Ref. [29], we use LP-based technique to find the optimal route above the topology graph, and can always avoid routing loops during route selection.

Based on this representation scheme discussed above, each individual can directly map to a potential channel allocation and power control solution. The initial population is composed of a certain number, denoted as $M$, of individuals. A general method to initialize the population is to explore the genetic diversity, that is, the chromosomes of each individual are randomly generated. In addition, we should ensure the feasibility of each individual thus to accelerate the convergence process. Therefore, in our paper, we first randomly generate two feasible vectors for each node according to the representation scheme. Once all feasible vectors are available, they will be combined to form a feasible individual. This process is repeated until $M$ individuals are generated. The formed initial population then acts as the very first generation that kicks off subsequent evolving steps.

4.2.2. Selection

Selection is an operation used for choosing individuals to participate in reproduction, which has a significant influence on driving the search towards a promising area and finding optimal solutions in a short time. In this paper, the famous roulette wheel selection method is used, where the chosen probability is proportional to the individual fitness-evaluation function, and its selected probability is defined as,

$$ p_i = \frac{f(i)}{\sum_{j=1}^{M} f(j)} \quad (19) $$

where $f(i)$ is the fitness of individual $i$, which is the maximum $\lambda$ with current resource allocation, and can be calculated by Eq. (18) with an LP solver.
4.2.3. Crossover and mutation

Genetic algorithm relies on two basic genetic operators: crossover and mutation. Crossover processes the contemporary solutions so as to find better ones. Mutation helps GA to keep away from local optima. Performance of GA to a large extent depends on them.

In our algorithm, due to the local optimization property in channel assignment problem, we adopt the single-point crossover operation, as illustrated on the right side of Fig. 3. To avoid scrambling the node-radio constraint, we limit the crossover points to occur only at the joints of any two adjacent substrings in a chromosome. The single-point crossover operator randomly generates one cutoff point $p$, where $1 \leq p \leq |V| - 1$ and switches the digits after $p$ (here, $p = 4$) of the parent chromosomes to form children chromosomes. The created children chromosomes then replace their parents, and chromosomes that were not selected for the crossover operation are the same as their parents. The specific crossover process is illustrated in Fig. 3.

The mutation operator works by randomly making some minor changes in the chromosomes after the crossover operation is performed. In our algorithm, we view each vector in a chromosome as a single gene. Similar to the biological gene alteration process, the GA mutation operator examines each vector (gene) for all nodes in each individual. Define a trivial probability $p_m$ as the likelihood of a gene to mutate. If a gene is determined to mutate, one digit of the vector will be randomly selected and replaced with a different value. Since the new generated substring may be infeasible, an additional repair execution will be executed, which can help to speed up the convergence process. However, if the vector is a power vector, no repair execution is needed. As shown in Fig. 4, since channel vector in $\text{str}_2$ changes to $[4 \ 2]$ is infeasible, it will be repaired with $[2 \ 4]$, while the power vector in $\text{str}_3$ changes to $[3 \ 2]$, no repairment is needed.

4.2.4. Replacement

After generating a new population through the genetic operators, an elitist model based replacement is employed to modify the old population with a certain number of new individuals. And the worst individuals in the parental population are replaced by their children in the next generation.

Now, we have designed all the key components of the GA operation, such as the genetic representation, population initialization, selection, recombination, mutation, and replacement. Then we can obtain the main procedure of GA-based channel allocation and power control, which are described as follows.

Algorithm 1: GA-based channel allocation and power control

\begin{itemize}
  \item \textbf{Input:} Max\_gen – the given number of generations
  \item $M$ – population size
  \item $M'$ – offspring size
  \item $p_c$ – crossover probability
  \item $p_m$ – mutation probability
  \item \textbf{Output:} The best resource allocation and route selection
\end{itemize}

\textbf{Step 1 (Initialization)}:

1. Set $\text{gen} = 0$;
2. Generate initiation population $P$ with randomly generated $M$ individuals using integer based coding and initialization strategy;
3. Delete those links whose capacity is less than $H_m \log_2(1 + \alpha)$;
4. Calculate $f$ value for each individual in $P$ using LP solver;

\textbf{Step 2 (Reproduction)}:

- Set $P' = \emptyset$
  - for $i = 0, \ldots, |M|/2 - 1$
    1. Select $p_{i+1}$ and $p_{i+2}$ from $P$ using selection strategy;
    2. Cross $p_{i+1}$ and $p_{i+2}$ using crossover strategy with probability $p_c$, and resulting two children $o_{i+1}$ and $o_{i+2}$;
    3. Mutate $o_{i+1}$ and $o_{i+2}$ using mutation strategy with probability $p_m$;
    4. Repair the infeasible genes if needed;
    5. $P' = P \cup \{o_{i+1}, o_{i+2}\}$;
    6. Delete those links whose capacity is less than $H_m \log_2(1 + \alpha)$;
    7. Calculate $f$ value for each individual in $P'$ using LP solver;

\textbf{Step 3 (Replacement)}:

Replace the less fit individuals in population $P$ with the children in offspring $P'$.

\textbf{Step 4 (Termination)}:

- $\text{gen} = \text{gen} + 1$
  - if $\text{gen} > \text{Max\_gen}$
    1. \textbf{Return} the best resource allocation scheme in $P$;
    2. \textbf{Return} the best route selection from the evaluation best individual in $P$;
  - else go to Step 2.

4.3. Algorithm analysis

\textbf{Theorem 1.} The optimization framework can converge to an optimal solution if there is no limitation on the number of generations.
The nested framework includes two optimization components: the GA based power control and channel allocation, the optimal route selection based on LP solver. The genetic algorithm can converge to the global optimal solution, because it is executed with crossover probability \( p_c \) close to 1, mutation probability \( p_m \) close to 1/length of the individual length, and the elitist model can preserve better individuals proportionally. According to the Markov chain based convergence analysis in Ref. [30], GA algorithms meet these three conditions can guarantee converge to the optimal solution if there is no limitation on the number of generations. In addition, the route selection based on LP solver can manage to find an optimal route. Therefore, the designed nested optimization framework can converge to the optimal solution.

Proof. The algorithm complexity of nested optimization framework mainly includes two components: GA based power control and channel allocation, and LP based route selection. For the GA based power control and channel allocation, the algorithm complexity depends on its genetic operator parameters, we set population size \( M \) and \( n \) as the number of running generations and the population size of the GA respectively.

**Theorem 2.** The time complexity of nested optimization framework is \( \text{Max}_\text{gen} \cdot M \cdot (O(n) + O(LP)) \).

Proof. The time complexity of nested optimization framework includes two optimization components: GA based power control and channel allocation, and LP based route selection. Therefore, the complexity of the entire nested algorithm is \( \text{Max}_\text{gen} \cdot M \cdot (O(n) + O(LP)) \). Here, \( \text{Max}_\text{gen} \) and \( M \) are the number of running generations and the population size of the GA respectively.

### 5. Performance evaluation

In this section, we present numerical results for the proposed solution. Without loss of generality, we assume the path loss index is \( \gamma = 4 \), the bandwidth \( H_m \) of each channel is set to 54 Mbps, the SINR threshold \( \alpha \) is set as 10 dB.

First, we test our algorithm in a small wireless mesh networks with 16 homogeneous nodes deployed in a \( 4 \times 4 \) grid shape. Each node has 2 radio interfaces, and the maximum transmission range is set to 100 m. For simplicity, we set \( T = 1 \), which means there is no power control. Assume there are 9 available orthogonal channels and only a subset of these bands is available for each node. There are 2 routing sessions in the network, as shown in Table 2. The global optimum solution is obtained by solving the LP problem among all feasible resource allocations, i.e., selecting the optimal route among \( (24)^9 \) (24 links, each link can select at most 9 channels) profiles. The solution is further viewed as the performance benchmark. As for genetic operator parameters, we set population size \( M = 40 \), maximum number of generations as 100, crossover probability \( p_c = 0.9 \), and mutation probability \( p_m = 0.01 \).

Fig. 5 shows the final channel allocation and route selection for these 2 sessions, where the number on each link represents the assigned channel. Fig. 6 shows the performance comparison with the global optimum, which is obtained by enumeration approach and functions as the upper bound of the overall throughput. For the above 16-node grid network with 2 routing sessions, each session has two routing paths from the source to the destination, and the achieved scaling factor \( \lambda \) is 18.0869, which is equal to the upper bound. And based on the minimum rate requirement in Table 2, the flow rates \( \lambda \cdot r_q \) for both sessions are 361.728. From Figs. 5 and 6, we also observe that our algorithm can gradually catch up the global optimum when the number of generation increases, and the final achieved route is almost of no interference.

The convergence speed in the genetic algorithm is closely related to the crossover probability and mutation probability. We now consider a larger wireless mesh networks with 50 nodes randomly deployed in a \( 1200 \times 1200 \) area, as shown in Fig. 7. Each node has 4 radio interfaces. The maximum transmission range is set to 300 m and is divided into \( T = 16 \) levels. Assume there are 9 available orthogonal channels and only a subset of these bands is available for each node. There are 5 routing sessions in the network, as shown in Table 3. As for genetic operator parameters, we set population size \( M = 40 \), maximum number of generations as 2000, crossover probability \( p_c = [0.5, 0.6, 0.7, 0.8, 0.9, 0.95] \), and mutation probability \( p_m = [0.01, 0.03, 0.05, 0.07, 0.09] \). Here, we should note that

![Fig. 5. Channel assignment and route selection.](image-url)
the enumeration approach is no longer feasible in this scenario due to the enormous strategic space.

Fig. 8 shows the performance of network throughput with different genetic parameters. Fig. 8(a) shows the convergences at crossover probabilities which are 0.5, 0.6, 0.7, 0.8, and 0.9, when the mutation probability $p_m$ is fixed at 0.01. Fig. 8(b) shows the convergences at mutation probabilities which are 0.01, 0.03, 0.05, 0.07, and 0.09, when the crossover probability $p_c$ is fixed at 0.9. From Fig. 8, we can observe that the best performance is achieved when the crossover probability $p_c$ is set to 0.9 and the mutation probability $p_m$ is set to 0.01.

We further compare our algorithm with the binary based representation method as proposed by Ref. [5]. In this representation method, a feasible channel vector contains $C$ bit binary variables, and at most $I (I < C)$ bits of vector $a_v$ should be set to 1. In addition, when $a_v$ is set to 1, the corresponding $t_{vj}$ in power vector is set as a random value from 0 to $T$. As for genetic operator parameters, we set population size $M$ = 40, maximum number of generations as 2000, crossover probability $p_c = [0.5, 0.6, 0.7, 0.8, 0.9, 0.95]$, and mutation probability $p_m = [0.01, 0.03, 0.05, 0.07, 0.09]$. After a series of pilot experiments, the algorithm with binary based representation methods was found to perform better with parameters $p_c = 0.7$ and $p_m = 0.05$. Comparisons of network throughput ratio with different representation methods are shown in Fig. 9.

From Fig. 9, we can get the conclusion that when a different representation scheme is applied, different genetic parameters should be selected. And we also can find that the obtained network throughput ratio with integer based representation is much larger than that obtained with binary based representation. Since in literature [5], the mutation is conducted in an inversion variation based method. This method can make minor changes on the channel allocation. However, the corresponding power allocation is the same as that of the initial population, which means the power control for the entire network is not optimized.
during evolution. Compared with their work, the power control in our algorithm is jointly optimized with channel allocation, and much more network throughput is achieved than that of binary based representation. What’s more, the integer based representation scheme has a more compact and simple data structure, the resulting convergence speed is much faster than that of binary based representation.

Finally, we test our algorithm in different number of channels and radios with different number of routing sessions where the integer based representation is applied. Fig. 10 shows results of the network throughput ratio with different number of channels when the number of radios is set to 4. Fig. 11 shows results of the network throughput ratio with different number of radios when the number of channels is set to 12. From Figs. 10 and 11, we can observe that more routing flows result lower network throughput ratio, and more channels and more radios lead to much higher network throughput ratio as expected. Although the objective value is a non-increasing function of number of channels and radios, when the number of channels and radios become sufficiently large (e.g., 12 channels and 4 radios in this network setting), further increasing the number of channels and radios will not enhance the objective value greatly. This suggests that it does not need such a large number of channels and radios for practical purpose.

6. Conclusion and future work

In this paper, we have investigated the joint optimization model of channel assignment, power control and routing for throughput maximization for cognitive radio based wireless mesh networks. Our model takes into account the number of radio radios of cognitive routers, the number of available orthogonal frequency channels, expected traffic load between different source and destination pairs, and the effective capacity of the logical links under SINR model. Since the above optimization problem is NP-hard, to this end, we have proposed a solution based on GA. In order to ensure the individual validity and fast convergence, the integer based representation scheme as well as the corresponding crossover and mutation rules is also designed to satisfy the constraints in the problem. Through extensive simulations, we have demonstrated that our optimization framework based on GA is quite promising in solving complex cross-layer design models involved in cognitive radio based wireless mesh networks.

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