Experiments on resonant vibration suppression of a piezoelectric flexible clamped–clamped plate using filtered-U least mean square algorithm

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Abstract
This article investigates the adaptive filtered-U least mean square feed-forward algorithm for active resonant vibration control of a clamped–clamped flexible piezoelectric plate structure under persistent harmonic excitation. Different from the widely used filtered-X least mean square algorithm based on the finite impulse response filter, the filtered-U least mean square algorithm uses the infinite impulse response filter. An infinite impulse response filter can be constituted simply by using two adaptive transversal filters. The filtered-U least mean square algorithm can model the system accurately with much fewer coefficients. Moreover, the filtered-U least mean square algorithm has better control performance and stability in the presence of vibration control feedback, owing to the inherent zero-pole structure of the infinite impulse response filter. In this investigation, the filtered-U least mean square algorithm is implemented only in experiments. Two experimental cases are carried out, including the reference signal extracted from the function signal generator and the lead zirconate titanate sensor. A proportional–derivative feedback control algorithm is also applied as a comparison. The experimental results demonstrate the feasibility and performance of the designed proportional–derivative controller and filtered-U least mean square controller.

Keywords
clamped–clamped piezoelectric flexible plate, active vibration control, feed-forward control, filtered-U least mean square

Introduction
Nowadays, flexible structures have been widely used in the field of aerospace due to its advantages of low weight and low energy loss (Hurlebaus and Gaul, 2006). The materials used in aerospace tend to produce lighter and larger structures to improve the efficiency and reliability of the system. However, this kind of materials has the characteristics of low rigidity and small material damping ratios (Gandhi and Thompson, 1992). Therefore, the vibration or flutter can be easily excited by external disturbances (Qiu et al., 2009). If the vibration or flutter is not suppressed effectively, structural fatigue and damage may be caused. Thus, the research of active vibration and flutter control of flexible structures is an issue of great importance.

There has been a large amount of researches on active vibration control of flexible plate or beam structures (Bailey and Hubbard, 1985; Hu and Ma, 2005). Control strategies are usually based on the closed-loop feedback. A lot of excellent results have been achieved (Fanson and Caughey, 1990; Fazelzadeh and Jafari, 2008; Han et al., 1997; Kumar et al., 2007; Qiu et al., 2015; Sharma et al., 2007). The characteristic of the feedback control is that the controller only acts after the error signal appears. Obviously, the response of the controller in feedback control always lags behind the error signal. However, feedback control strategy is not so useful for resonant vibration control under persistent resonant excitation. The feed-forward control acts according to the external disturbances. The controller is supposed to respond in the presence of disturbance rather than the error signal for feed-forward control. This feed-forward control scheme can effectively
eliminate the vibration of the controlled object caused by the external persistent disturbance.

The adaptive feed-forward control methods have been proved to have good performance and high reliability for resonant vibration suppression under persistent harmonic excitation. The principle of the adaptive control algorithm is based on the specified update strategy, and the adaptive controller is adjusted according to the dynamic characteristics of the controlled system. In the process of dynamically adjusting, the controller arrives to the optimal value gradually. Thus, the controlled object reaches its expectation. The adaptive control algorithms can provide an efficient way for complex systems with uncertainties. Therefore, the adaptive control methods have attracted widespread attention in recent years. A multi-input multi-output (MIMO) adaptive feed-forward controller was proposed for the alleviation of turbulence-induced rigid body motions and structural vibrations on aircraft (Wildschek et al., 2014).

Several new algorithms for adaptive feed-forward compensation, taking into account the existence of an inherent internal positive feedback coupling, are applied to an active vibration control system (Landau et al., 2011). Based on the adaptive filtering theory, the adaptive feed-forward filtered-X least mean square algorithm (FXLMS) control algorithm was first used for noise control (Burgess, 1981; Kuo and Morgan, 1995; Widrow et al., 1975) and then extended to vibration control (Kim et al., 1999; Pu et al., 2014; Yan et al., 2012; Yang et al., 2005). A feed-forward adaptive controller was developed for vibration suppression of intelligent structures with built-in piezoelectric sensors and actuators (Yang et al., 2005). The experimental results show good control performance. Kim et al. (1999) designed an adaptive feed-forward with feedback controller to improve the robustness of the algorithm for both the transient and persistent external disturbances. An improved FXLMS algorithm based on reverse model linearization was put forward by Yan et al. (2012) to address the nonlinearity of the actuators in an active vibration isolation system. Simulation and experiments showed that the improved FXLMS algorithm has a better effect on periodic vibration control with nonlinearity, as compared with that of the traditional FXLMS feed-forward control. A new variable step size FXLMS algorithm with an auxiliary noise power scheduling strategy for online secondary path modeling was proposed for active vibration control of a beam (Pu et al., 2014).

The convergence of the FXLMS algorithm has much to do with the accuracy of the control path and the convergence factor. The convergence analysis of the algorithm has attracted many attentions. Morgan (1980) came to the conclusion that if the phase error of the identified control path is more than 90°, the system will become unstable. He got the result by analyzing the experimental results. Elliott and Nelson (1989) gave a rough scope for the convergence factor of the FXLMS algorithm through theoretical derivation. Although the FXLMS algorithm has the merits of good control performance and simple structure, the reference signal is supposed to be highly relevant to the interference source. What is more, the output of the controller should not affect the reference signal to ensure the stability of the control system. Actually, the interference source is often used as the reference signal. However, this can be extremely difficult and unrealistic in engineering applications. Therefore, the filtered-U least mean square (FULMS) algorithm is developed for situations in which interference source cannot be measured. The structure of the FULMS algorithm is based on the infinite impulse response (IIR) filter. The zero-pole structure of the IIR filter can effectively eliminate the feedback impact. The FULMS algorithm can also achieve a good control performance with a lower order structure due to the characteristics of the IIR filter. The FULMS algorithm was first proposed by Eriksson (1991). He used the algorithm for active noise control. Two new kinds of constraints FXLMS and FULMS control algorithms were developed to increase the convergence region of the system (Kim et al., 1994). Kim et al. (2011) implemented an improved FULMS algorithm to suppress noise of a short acoustic duct. The experimental results validated the feasibility and effectiveness of the proposed algorithm. Crawford and Stewart (1997) derived the full-gradient, simplified-gradient and Feintuch-based versions of the FULMS algorithm. Park and Lee (2012) proposed an FU-VSSLMS (variable step size LMS) algorithm with fast convergence and good stability. The test results demonstrated that the FU-VSSLMS algorithm has a superior convergence performance compared with those of the FXLMS and FULMS algorithms. Since the mean square error (MSE) function of the FULMS is not a quadratic function, it may have possibilities to trap in a local minimum. Wang and Ren (1999) deduced the conditions for the global convergence of the FULMS algorithm by using the ordinary differential equation method. Fraanje et al. (2003) analyzed the convergence of the FULMS algorithm when perfect cancellation is not achievable.

This article investigates active resonant vibration control of a smart flexible clamped–clamped plate under resonant sinusoidal excitation. The proportional–derivative (PD) controller and the adaptive FULMS feed-forward algorithm controller are designed to suppress the resonant vibration. Both theoretical and experimental studies are carried out to verify the advantage of the designed FULMS control method.

The contribution of this article mainly lies in two aspects. First of all, the advantages of the FULMS feed-forward are analyzed and compared with those of the FXLMS algorithm. Second, the FULMS algorithm...
is implemented to suppress the vibration under resonant sinusoidal excitation. Different cases of experiments are carried out to investigate the factors that affect the control performance of the FULMS algorithm. A PD controller is also designed as a comparison. A kind of clamped–clamped piezoelectric plate system is designed and constructed for experimental studies. The experimental results demonstrate the satisfactory control performance and robustness of the FULMS and PD controllers.

The rest of this article is organized as follows. In section “Controller design,” the FULMS algorithm is introduced. The advantages and disadvantages of the FULMS feed-forward algorithm are discussed. In section “Experimental results,” the experimental setup of the clamped–clamped piezoelectric plate system is designed and constructed. The experimental comparison studies are conducted using the PD control method and the FULMS control method. Finally, the conclusion is drawn in section “Conclusion.”

Controller design

FULMS feed-forward control algorithm

The FXLMS feed-forward control algorithm is widely used in active noise and vibration control. It uses a finite impulse response (FIR) filter, characterized by an all-zero transfer function as the controller. The weights of the controller are tuned according to the least MSE criterion.

Figure 1 shows the block diagram of the FXLMS algorithm, where \( x(n) \), \( d(n) \), \( e(n) \), \( y(n) \), and \( z(n) \) are the reference signal, desired response, error signal, control output of the adaptive filter, and control response of the controlled structure at the \( n \)th sampling instant, respectively. \( H_1 \) is the primary path from the disturbance to the error sensor, \( H_2 \) is the secondary path or the control path from the controlling part to the error sensor, and \( H_3 \) is the feedback path from the controlling part to the reference sensor. The path of \( H_3 \) only exists when the reference signal is influenced by the control. \( H_2 \) is the estimation of \( H_2 \). \( W \) is the adaptive FIR filter.

The discrete formation of error signal as shown in Figure 1 can be expressed as

\[
E(z) = \left( H_1(z) - \frac{H_2(z)W(z)}{1 - H_2(z)W(z)} \right)X(z)
\]  

(1)

Assuming that the reference signal is strictly kept constant, the ideal situation of the error signal shown in equation (1) will eventually decrease to 0. After transformation, one can get the adaptive filter transfer function

\[
W(z) = \frac{H_1(z)}{H_2(z) + H_1(z)H_3(z)}
\]  

(2)

The open-loop transfer function of the feedback loop is

\[
H_o(z) = H_3(z)W(z) = \frac{H_3(z)H_1(z)}{H_2(z) + H_1(z)H_3(z)}
\]  

(3)

Equation (3) is mainly used to judge the stability of the system. When the gain of equation (3) is greater than 1, or the phase lag reaches \(-180^\circ\), the system will become unstable.

The closed-loop transfer function of the feedback loop can be expressed as

\[
H_c(z) = \frac{W(z)}{1 - H_3(z)W(z)}
\]  

(4)

From equation (4), it can be seen that when the open-loop gain \( H_3(z)W(z) = 1 \), the system will no longer stay stable. Assuming that the feedback control path is constant, the system can still stay stable at a certain frequency range by constraining the weight range of the controller. However, it is not easy to ensure good stability of the system when the frequency range is wide because it is difficult for the adaptive filter to track all the frequencies.

The FULMS adaptive feed-forward filtering controller with IIR filter structure is introduced to solve the feedback influence on the reference signal (Eriksson, 1991). The main difference between the FXLMS algorithm and the FULMS algorithm is the type of the adopted filter. The FXLMS controller uses an FIR filter with an all-zero transfer function structure. When the reference signal is affected by the feedback control part, it mainly has influence on the poles of the transfer function of the controlled object. Since the FIR filter has no poles in its transfer function, the FXLMS controller cannot be tuned effectively to suppress vibration of the controlled object at this time. Now that the IIR filter has zeros and poles in its transfer function, it can be used to model the system while the reference signal is affected by the feedback control part. Generally speaking, two transverse FIR filters are usually adopted to constitute an IIR filter for convenience. One is used as
the feed-forward filter to tune the zeros, and the other is used as the feedback filter to compensate for the poles in the transfer function. The transfer function of an IIR filter can be expressed as follows

\[
H(z) = \frac{\sum_{i=0}^{N_1} a_i z^{-i}}{1 - \sum_{j=1}^{N_2} b_j z^{-j}} \tag{5}
\]

where \(a_i\) and \(b_j\) are the coefficients of the transfer function.

The structure diagram of the FULMS adaptive filtering algorithm is shown in Figure 2, where \(H_1, H_2, H_3, H_4, x(n), d(n), e(n), y(n),\) and \(z(n)\) have the same meanings as those depicted in Figure 1.

When the reference signal is influenced by the control part, the discrete formation of the error signal in Figure 2 can be written as

\[
E(z) = \left( H_1(z) - \frac{A(z)H_2(z)}{1 - B(z) + A(z)H_3(z)} \right) X(z) \tag{6}
\]

Therefore, the FULMS algorithm can attenuate the error signal to 0 by tuning the control weights of \(A(z)\) and \(B(z)\) in equation (6) even if the reference signal is influenced by the control feedback.

It can be seen from Figure 2 that there are two FIR filters \(A\) and \(B. \) \(A\) is the adaptive feed-forward filter whose order is \(N_1, \) and \(B\) is the adaptive feedback filter whose order is \(N_2. \) \(A\) and \(B\) can be expressed as

\[
\begin{align*}
A(n) & = [a_1(n) \ a_2(n) \ \cdots \ a_{N_1}(n)]^T, \\
B(n) & = [b_1(n) \ b_2(n) \ \cdots \ b_{N_2}(n)]^T \tag{7}
\end{align*}
\]

Assuming that the reference signal vector \(X(n)\) and the control output vector \(Y(n)\) can be represented as follows

\[
\begin{align*}
X(n) & = [x(n) \ x(n-1) \ \cdots \ x(n-N_1 + 1)]^T, \\
Y(n) & = [y(n-1) \ y(n-2) \ \cdots \ y(n-N_2)]^T \tag{8}
\end{align*}
\]

According to equations (7) and (8), two new vectors can be defined as

\[
W = [a_1(n) \ a_2(n) \ \cdots \ a_{N_1}(n) \ b_1(n) \ b_2(n) \ \cdots \ b_{N_2}(n)]^T = [A^T(n) \ B^T(n)]^T \tag{9}
\]

and

\[
U = [x(n) \ x(n-1) \ \cdots \ x(n-N_1 + 1) \ y(n-1) \ y(n-2) \ \cdots \ y(n-N_2)]^T = [X^T(n) \ Y^T(n)]^T \tag{10}
\]

where \(W\) is the weight matrix of the controller and \(U\) is the filtered-U signal.

The real output of system \(y(n)\) consists of two parts: the output \(y_1(n)\) of the feed-forward controller \(A\) and the output \(y_2(n)\) of the feedback controller \(B. \) The real output can be expressed as

\[
y(n) = y_1(n) + y_2(n) = U^T(n)W(n) \tag{11}
\]

The existence of the control path \(H_2\) in Figure 2 has a close relationship with the control performance of the algorithm. The reference signal vector \(X(n)\) and the control output vector \(Y(n)\) cannot be used directly for the updating of controllers \(A\) and \(B. \) \(X(n)\) and \(Y(n)\) need to do convolution computations with \(H_2\) which has the same characteristics of the physical control path \(H_2. \) By doing these operations, the time delay of \(H_2\) could be compensated. \(H_2\) can be written as

\[
\hat{H}_2 = [h_1 \ h_2 \ \cdots \ h_P]^T \tag{12}
\]

where \(P\) is the order of \(\hat{H}_2. \)
Figure 3. Generation of the filtered signals.

\( R_1(n) \) and \( R_2(n) \) in Figure 2 are the filtered signals for the adaptive feed-forward controller \( A \) and adaptive feedback controller \( B \), respectively. The filtered signals \( R_1(n) \) and \( R_2(n) \) can be expressed as

\[
\begin{align*}
R_1(n) &= \begin{bmatrix} r_1(n) & r_1(n-1) & \cdots & r_1(n-N_1 + 1) \end{bmatrix}^T \\
R_2(n) &= \begin{bmatrix} r_2(n-1) & r_2(n-2) & \cdots & r_2(n-N_2) \end{bmatrix}^T
\end{align*}
\tag{13}
\]

The generations of the filtered signals \( R_1(n) \) and \( R_2(n) \) of the FULMS algorithm are shown in Figure 3. It can be seen from Figure 3 that each part of \( R_1(n) \) is obtained by the convolution operation of reference signal \( x(n) \) and \( \mathcal{H}_2 \). The vector of \( R_2(n) \) is obtained by the convolution computation of control output of \( y(n) \) and \( \mathcal{H}_2 \).

Assume that the reference signal sequence \( X_1(n) \) with the dimension of \( N_1 \times P \) can be expressed as

\[
X_1(n) = \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-P+1) \\ x(n-1) & x(n-2) & \cdots & x(n-P) \\ \vdots & \vdots & & \vdots \\ x(n-N_1+1) & x(n-N_1) & \cdots & x(n-N_1-P+2) \end{bmatrix}
\tag{14}
\]

The control output sequence \( Y_1(n) \) with the dimension of \( N_2 \times P \) can be written as

\[
Y_1(n) = \begin{bmatrix} y(n-1) & y(n-2) & \cdots & y(n-P) \\ y(n-2) & y(n-3) & \cdots & y(n-P-1) \\ \vdots & \vdots & & \vdots \\ y(n-N_2) & y(n-N_2-1) & \cdots & y(n-N_2-P+1) \end{bmatrix}
\tag{15}
\]

The filtered signals \( R_1(n) \) and \( R_2(n) \) at the \( n \)th sampling instant can be written as

\[
R_1(n) = \begin{bmatrix} r_1(n) \\ r_1(n-1) \\ \vdots \\ r_1(n-N_1 + 1) \end{bmatrix}
= \begin{bmatrix} x(n) & x(n-1) & \cdots & x(n-P+1) \\ x(n-1) & x(n-2) & \cdots & x(n-P) \\ \vdots & \vdots & & \vdots \\ x(n-N_1+1) & x(n-N_1) & \cdots & x(n-N_1-P+2) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_P \end{bmatrix} \tag{16}
\]

\[
R_2(n) = \begin{bmatrix} r_2(n-1) & r_2(n-2) & \cdots & r_2(n-N_2) \end{bmatrix}^T = X_1(n) \mathcal{H}_2
\]
\[
R_2(n) = \begin{bmatrix}
    r_2(n-1) \\
    r_2(n-2) \\
    \vdots \\
    r_2(n-N_2) \\
    y(n-1) \\
    y(n-2) \\
    \vdots \\
    y(n-N_2) \\
    y(n-N_2-1) \\
    \vdots \\
    y(n-N_2-P+1)
\end{bmatrix}
= \begin{bmatrix}
    y(n-1) & y(n-2) & \cdots & y(n-P) \\
    y(n-2) & y(n-3) & \cdots & y(n-P-1) \\
    \vdots & \vdots & \ddots & \vdots \\
    y(n-N_2) & y(n-N_2-1) & \cdots & y(n-N_2-P+1)
\end{bmatrix}
\]
\[
h_1 \\
h_2 \\
\vdots \\
h_p
= Y_1(n)H_2
\]  
(17)

The error signal at the \(n\)th sampling instant is
\[
e(n) = d(n) - z(n) = d(n) - H_1^T(X_1^T(n)A(n) + Y_1^T(n)B(n))
\]
\[
= d(n) - (X_1(n)H_2)^T A(n) - (Y_1(n)H_2)^T B(n)
\]
\[
= d(n) - R_1^T(n)A(n) - R_2^T(n)B(n)
\]
(18)

The purpose of the adaptive filter operation is to minimize the objective function \(J(n)\). The objective function is
\[
J(n) = E[e^2(n)]
\]
(19)

Actually, it is hard to get the value in equation (19). The instantaneous sample of square error is often used as the approximation. Thus, equation (19) can be written as
\[
J(n) \doteq e^2(n) = (d(n) - R_1^T(n)A(n) - R_2^T(n)B(n))^2
\]
(20)

By performing the differential operation of the objective function \(J(n)\) to the weights of feed-forward controller and feedback controller, one can get the following formulations
\[
\nabla J_A(n) = \frac{\partial e^2(n)}{\partial A(n)} = 2e(n)R_1(n)
\]
\[
\nabla J_B(n) = \frac{\partial e^2(n)}{\partial B(n)} = 2e(n)R_2(n)
\]
(21)

According to the steepest descent method, the iteration expressions of \(A\) and \(B\) are
\[
\begin{cases}
A(n+1) = A(n) + 2\mu e(n)R_1(n) \\
B(n+1) = B(n) + 2\beta e(n)R_2(n)
\end{cases}
\]
(22)

where \(\mu\) and \(\beta\) are the convergence factors of feed-forward controller \(A\) and feedback controller \(B\), respectively. \(\mu\) and \(\beta\) are the key factors for the control performance of the FULMS algorithm. Larger values of \(\mu\) and \(\beta\) will lead to better performance and faster convergence rate. To ensure the stability of the control process, the values should not be chosen too large.

Finally, the iterative process of the FULMS control algorithm can be summarized as follows (Eriksson, 1991)
\[
\begin{cases}
y(n) = U^T(n)W(n) \\
e(n) = d(n) - R_1^T(n)A(n) - R_2^T(n)B(n) \\
A(n+1) = A(n) + 2\mu e(n)R_1(n) \\
B(n+1) = B(n) + 2\beta e(n)R_2(n)
\end{cases}
\]
(23)

Different from the FXLMS algorithm, it can be seen from equation (20) that the MSE function of the adaptive IIR filters is generally not a quadratic function of the control weights and may have multiple local minima. Therefore, the FULMS algorithm may have the possibilities to trap in a local minimum. Figure 4 shows the MSE surfaces of FIR filter and IIR filter.

Remarks. The values of the convergence factors should be chosen between the range of 0 and \(1/\lambda\) in theory. \(\lambda\) is the largest eigenvalue of the autocorrelation matrix of the input reference signal. However, in many references the convergence factors are selected according to the experimental performance due to many uncertainties. Generally speaking, larger values of convergence factors \(\mu\) and \(\beta\) will lead to a better control performance and faster convergence rate. But if the values are chosen too large, instability may happen during the control process.

Adaptive control path identification

It can be seen from Figure 2 that \(H_2\) is needed to get the filtered signals \(R_1(n)\) and \(R_2(n)\) of the FULMS algorithm. Two kinds of system identification methods are usually used, including offline modeling and online modeling. The online modeling method has the advantages of real-time and good controlling performance. At the same time, this method will increase the computation, which will be a great challenge for the
realization of real-time control. The offline modeling method assumes that the system is stable. This method uses the input and output offline data acquisition to model the system according to a specific algorithm. The identified model can be directly used in control experiments.

Since the characteristics of the experimental setup are stable in this investigation, the offline modeling method is utilized. Figure 5 shows the schematic diagram of offline control path identification. The control path in this article includes the lead zirconate titanate (PZT) voltage amplifier, the PZT actuator for control, the error sensor, the charge amplifier, and the analog-to-digital (A/D) converter. As shown in Figure 5, $x(n)$ is the input of the system and $d(n)$ is the actual output of the error sensor; $x(n)$ and $d(n)$ can be obtained by experimental data acquisition. $y(n)$ is the output of the adaptive filter. An FIR adaptive filter is used in this process. $e(n)$ is the difference between $d(n)$ and $y(n)$. The coefficients of the adaptive filter are adjusted according to the LMS algorithm to make $y(n)$ approach $d(n)$. The characteristics of the adaptive filter can be used as the estimation of the control path after the value of $e(n)$ reaches 0. It should be noted that the order of the adaptive filter should be selected appropriately to satisfy not only the precision of the model but also the stability of the control algorithm.

Actually, the identified model of the control path $\hat{H}_2$ cannot be perfectly equivalent to the practical control path $H_2$. There are mainly two aspects of the error between $H_2$ and $H_2$, that is, gain error and phase error. Assuming that the phase and gain of $H_2$ are $0^\circ$ and 1, respectively. The estimation control path $\hat{H}_2$ can be expressed in complex form

$$\hat{H}_2 = h_R + jh_L$$

where $h_R$ is the real part of $\hat{H}_2$ and $h_L$ is the imaginary part of $\hat{H}_2$.

Some researchers have already deduced the relationship between the feed-forward convergence factor and the error of the control path (Snyder and Hansen, 1990)

$$0 < \mu < \frac{\cos(\phi)}{\lambda_{\text{max}} |H_2|}$$

where $\lambda_{\text{max}}$ is the largest eigenvalue of the autocorrelation matrix of the input signal, $|H_2|$ is the modulus of $H_2$, and $\phi$ is the phase error between $H_2$ and $H_2$.

It can be seen from equation (25) that the phase error of the control path will reduce the maximum convergence factor. When the phase error absolute value is greater than $90^\circ$, this algorithm will become unstable. At the same time, the larger gain error of $H_2$ will also lead to that the maximum convergence factor is smaller.

Remarks. By using the offline modeling method, the transfer function of the secondary path is expressed in a difference equation. To get this transfer function, the input and output of the system have to be obtained. In this article, the input is the sinusoidal signal generated by the signal generator. After the vibration is excited, one can get the output of the PZT error sensor. Actually, the time delay between the PZT actuators for control and the PZT error sensor is included in this process. With the values of the input of the signal generator and the output of the PZT sensor, one can get the secondary path including time delay. That is to say, the phase frequency property of the identified transfer function compromises the time delay.

**Experimental results**

**Experimental setup**

The schematic diagram of the clamped–clamped piezoelectric flexible plate experimental system is shown in Figure 6. The piezoelectric sensors and actuators are bonded after optimization. The piezoelectric actuators are bonded on both surfaces of the plate. The piezoelectric sensors are bonded only on the bottom surface. As shown in Figure 6, eight PZT patches bonded on the left end of the clamped plate are connected in parallel as a one-channel piezoelectric actuator to excite the
vibration of the flexible plate. Another eight PZT patches bonded on the right end of the clamped plate are connected in parallel as a one-channel piezoelectric actuator to suppress the vibration of the flexible plate. Signals of the top and bottom PZT patches are reverse-phase connected to get the double control force or excitation force. The dimensions of all the PZT patches are $50 \, \text{mm} \times 15 \, \text{mm} \times 1 \, \text{mm}$. The reference sensor and error sensor are of the same size of $40 \, \text{mm} \times 10 \, \text{mm} \times 1 \, \text{mm}$.

The working principle of the system can be described as follows: The sinusoidal excitation signal generated by the function signal generator is first transmitted to piezoelectric voltage amplifier I and to the computer as well. The amplified voltage is then applied to the PZT actuators at the left end of the clamped plate to excite the vibration. The reference sensor and the error sensor can measure the vibration signals and send them to the charge amplifiers. Through a peripheral external A/D conversion circuit, the continuous analog signals are converted to discrete signals and then transmit to the computer through an ARM control board. The computer calculates the control output and converts the digital signal into an analog signal through the digital-to-analog (D/A) conversion. Finally, the control output is applied to the actuators at the right end of the clamped plate to suppress the vibration after amplification by piezoelectric voltage amplifier II.

The PZT actuators act according to the signal of PZT sensor, as shown in Figure 6. When the value of the sensor goes high, the voltage of the actuators also goes high to suppress the amplitude of vibration. Besides, the voltage of the actuators becomes smaller, while the value of the sensor decreases. The vibration of the system can be suppressed effectively in real-time with the variation in the sensor’s value. An ARM9 board of Mini2440 is used in this article. It is produced by Guangzhou Friendly ARM company. The Samsung S3C2440 CPU is used in Mini2440. The frequency of the processor is 400 MHz and can be maximized to 533 MHz. Since there are many modules integrated in the ARM board, users can use it to develop applications conveniently. In this research, the general-purpose input/output (GPIO) ports of the ARM board are used in communication with the peripheral A/D and D/A circuit.

**Remarks.** The reference signal of the FULMS algorithm can be obtained from the function signal generator or
An active resonant vibration control system of the clamped–clamped piezoelectric flexible plate was developed to validate the feasibility and the control performance of the FULMS control algorithm. The photograph of the clamped–clamped plate experimental setup is shown Figure 7. Figure 7(a) shows the front view and Figure 7(b) shows the back view of the experimental setup.

The experimental setup is composed of several parts: a piezoelectric flexible plate, the resonant excitation system, the measurement, and the control system. The plate is made up of epoxy resin. The dimension of the clamped–clamped piezoelectric flexible plate is 600 mm × 510 mm × 2 mm.

Sinusoidal signals are generated by a function signal generator (SP-F05). The piezoelectric voltage amplifier (APEX PA240CX) is used to amplify the sinusoidal signals from the signal generator to excite the clamped–clamped piezoelectric plate. It can amplify the signal from −5 to + 5 V to a high voltage from −130 to + 130 V. The measured signals by the error PZT sensor and the reference sensor are amplified by two charger amplifiers (YE5850) to the range of −10 to + 10 V.

The digital data of the collected signals are obtained through an A/D converter (AD7862). The output signal after D/A converter (AD7847) ranges from −5 to + 5 V. The output driving voltage for the control actuator ranges from −260 to + 260 V after amplification by another piezoelectric amplifier. An ARM board with the corresponding A/D and D/A peripheral expander circuit communicates with a personal computer (PC) in the master-slave communication mode. The PC is used as the master control unit. The sampling period is chosen as 3 ms. The PD feedback control algorithm and the FULMS adaptive feed-forward control algorithm are implemented for resonant vibration control of the clamped–clamped plate.

**Experimental identification and filter design**

In order to excite the flexible plate structure effectively at its first modal frequency, it is necessary to identify the natural frequency of the system. A swept sine (chirp) signal is generated by the signal generator. The starting frequency and the stop frequency are specified as 0.5 and 50 Hz, respectively. The swept time is 50 s. The amplitude of the swept sine signal is 4 V. Figure 8(a) shows the excited sinusoidal signal and the time-domain response excited by the PZT actuator. Figure 8(b) depicts the frequency response of the plate by employing fast Fourier transform (FFT). From Figure 8(b), one can know that the first modal frequency of the clamped–clamped plate is configured as 22.3 Hz.

**Remarks.** The natural frequencies of the practical system will change in different seasons. The possible
reasons that may lead to the change in modal frequencies are listed as follows: (a) the clamped–clamped boundary condition in the experiments may not be totally clamped. The clamped ends may loose as time goes on. And this change will influence the modal frequencies of the system. (b) The change in the environmental temperature will lead to the fluctuation of the modal frequencies.

The first modal frequency of 22.3 Hz is the bending mode. In this research, only the first bending mode of the system is considered. Since there are many noises and disturbances mixed with the error signal, a band-pass Chebyshev filter has to be designed to improve the quality of the actual signal. Otherwise the noises would lead to the instability of the system. Before going into the FULMS algorithm, the reference signal and the error signal are filtered by the same band-pass filter. The higher mode response and noises are all filtered to guarantee the stability and performance of the controller.

In the experiments, the Type I Chebyshev filter is designed to filter out the high-frequency noise or harmonic frequency components of the measured vibration signal by the PZT sensor. A fourth-order Type I Chebyshev band-pass filter is used here for signal processing. The central frequency of the filter is specified as the first modal frequency of 22.3 Hz. The bandwidth of the passband is set as 30 rad/s. The ripple of the passband is 1 dB. The magnitude and phase of the frequency response of the Chebyshev band-pass filter are shown in Figure 9(a) and (c), respectively. Figure 9(b) shows the ripple in the passband of the band-pass filter. From Figure 9(b), one finds that the Type I Chebyshev filter has ripple in its passband. Although the amplitude fluctuates in the passband, Chebyshev filter has the merit of attenuating fast in the transitional zone.

**Experiments on resonant vibration suppression using PD feedback control**

The function signal generator is used to generate a single-frequency sinusoidal signal. The amplitude and frequency are tuned as 4 V and 22.3 Hz, respectively. The PD feedback control algorithm is applied for two cases of experiments. The parameters of the PD controller are chosen as (1) $K_p = 0.85$, $K_d = 0.005$ and (2) $K_p = 0.85$ and $K_d = 0.005$. Different from experiment (1), a shock disturbance is applied to the system around the time of 16 s in (2) to testify the robustness of PD control algorithm. The control voltages in all two cases of the experiment are applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V.

Figure 10 shows the PD control experimental results of case (1). Figure 10(a) shows the time-domain response of resonant vibration suppression using PD control. Figure 10(c) shows the corresponding control voltage. Figure 10(b) and (d) show the time history of vibration response and control voltage in the neighborhood of applying the PD controller, respectively. It can be seen from Figure 10(b) that once the control action is applied, the control voltage reaches the control saturation value rapidly. With the amplitude of the vibration response in Figure 10(a) attenuating to 1.5 V after about 7 s, the control voltage in Figure 10(c) decreased to a stable range. This is because the control value of PD control is equal to the sum of vibration signal and its derivative multiplying the corresponding control gains.
In order to verify the robustness of the PD control algorithm under persistent excitation, a shock disturbance is applied after the system reaches the stable state in case (2). The robust experiments here mean the ability of resisting transient shock disturbance. If the system could automatically adjust itself to the convergence value after the transient disturbance disappears, then one can say this system has good robustness. Figure 11(a) shows the time-domain response of resonant vibration suppression using PD control. The corresponding control voltage is shown in Figure 11(c). The shock disturbance is applied to the system around the time of 16 s. It can be seen from Figure 11(b) that the maximum vibration response signal increases to about 6 V in a short time after the shock disturbance is applied. The control voltage in Figure 11(d) is regulated to reduce the amplitude of the vibration signal. From Figure 11(b) and (d), one can see that the amplitudes of vibration response and control voltage of the system have been regulated to the steady-state values at the time of 16.8 s after the shock disturbance is applied. The robust experimental results demonstrate that the PD control algorithm under persistent excitation is robust to external disturbance.

The experimental results demonstrate that the PD control algorithm can suppress the resonant vibration under persistent excitation effectively. The amplitude of the vibration response can be suppressed to a small value when the control effect is applied. However, the control voltage may also reach saturation with larger coefficients. Therefore, the proportional and
differential coefficients should be chosen appropriately. In addition, the experimental results can prove that the PD control algorithm has a very good robustness to external shocks. The system can eliminate the influence caused by external disturbance in a very short time using the designed PD control algorithm.

Experiments on resonant vibration suppression using FULMS algorithm

Before using the FULMS control algorithm, an offline identification of the control path between the actuator for excitation and the error sensor was performed. The data acquisition of the input sinusoidal signal and the output signal measured by the PZT error sensor is implemented by using the ARM controller with the peripheral expander circuit. The amplitude and frequency of the input sinusoidal signal are set as 3.5 V and 22.3 Hz, respectively.

The dimension of the identified model has a significant impact on the control performance of the FULMS algorithm. The identified model should describe the characteristics of the control path accurately. At the same time, the dimension should be selected as low as possible. Higher order of the identified model will increase the amount of calculation, which will make real-time control implementation difficult. Due to the gain and phase error between the identified control path model and the physical control path, the control process may become unstable if the dimension of the model is selected too high. The dimension of the identified control path model is chosen as 12 after several times of tests.

First of all, a sinusoidal excitation signal is applied to the actuators for control. While the vibration of the clamped–clamped plate is excited, one can get the vibration signal from the PZT error sensor. By collecting the input sinusoidal signal and the error sensor signal, the
The offline model of the secondary path can be obtained. The experimental apparatuses included in the offline modeling are the PZT voltage amplifier, PZT actuator for control, error sensor, charge amplifier, and A/D converter. The FIR model of the control path based on the LMS algorithm is illustrated in Figure 12. The actual output measured by the PZT error sensor and the output of the adaptive filter are shown in Figure 12(a) and (b), respectively. From Figure 12(b), one can find that the output of the adaptive filter gradually tracks the actual output of Figure 12(a). The output of the adaptive filter remains stable after 0.7 s. Figure 12(c) shows the identified model error between the actual output and the output of the adaptive filter. It can be seen from Figure 12(c) that the error almost decreased to 0 after 0.7 s. This means that the output of the adaptive filter is basically the same as that of the PZT error sensor. The transfer parameters of the identified control path are illustrated by Figure 12(d) to (f), respectively. From Figure 12(d) to (f), one finds that the transfer parameters gradually increase to constant values after 0.7 s. After that, the constant transfer parameters can be used as the estimation of the control path approximately.

The estimated model of the control path could be finally written as follows

\[
\hat{H}_2 = 0.205463180700830 + 0.0519248796517833z^{-1} - 0.10968953777248z^{-2} - 0.251484805255340z^{-3} - 0.349152091771307z^{-4} - 0.386156629206711z^{-5} - 0.356543954398766z^{-6} - 0.265901130435789z^{-7} - 0.1303031222486706z^{-8} + 0.026545474863246z^{-9} + 0.177472672040696z^{-10} + 0.29656599323981z^{-11}
\]
The sinusoidal signal generated by the signal generator is first used as the reference signal to evaluate the control performance of the FULMS algorithm. First, the convergence factors of the feed-forward controller and the feedback controller are chosen as $\mu = 1.4e-5$ and $\beta = 1.0e-6$, respectively. In this case, $\mu$ is set as a

![Graphs showing experimental identification of control path transfer parameters.](image)

**Figure 12.** Experimental identification of the control path transfer parameters: (a) actual output measured by the PZT sensor, (b) the output of adaptive filter, (c) identified model error of the control path, (d) control path transfer parameters $h_1$–$h_4$, (e) control path transfer parameters $h_5$–$h_8$, and (f) control path transfer parameters $h_9$–$h_12$. 
Figure 13. Resonant vibration suppression response using FULMS control with $\mu = 1.4e-5$ and $\beta = 1.0e-6$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 4 to 7 s; (c) control voltage; (d) zoom in on the time axis from 4 to 7 s; (e) control voltage of feed-forward controller; (f) control voltage of feedback controller; (g) feed-forward controller weights of $a_1$, $a_2$, and $a_3$; (h) feed-forward controller weights of $a_4$, $a_5$, and $a_6$; (i) feedback controller weights of $b_1$, $b_2$, and $b_3$; and (j) feedback controller weights of $b_4$, $b_5$, and $b_6$. 
large value and $\beta$ is set as a small value to study the control effect of feed-forward controller. The orders of the adaptive filters $A$ and $B$ are selected as $N_1 = 6$ and $N_2 = 6$, respectively. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V. The corresponding experimental results of the FULMS control method are shown in Figure 13.

Figure 13(a) shows the time-domain response of resonant vibration suppression using the FULMS control algorithm. Figure 13(c) shows the corresponding control voltage. Figure 13(b) and (d) show the time history of vibration response and control voltage at the moment of applying the FULMS control, respectively. Figure 13(e) and (f) depict the control voltage of feed-forward controller and feedback controller, respectively. Figure 13(g) and (h) illustrate the self-regulating process of the feed-forward controller weights. The self-regulating process weights of the feedback controller are illustrated in Figure 13(i) and (j). Although the transient response time of the designed FULMS algorithm in Figure 13(b) is longer than that of the applied PD control, it can be seen from Figure 13(a) that the vibration almost attenuates to 0 eventually. Therefore, the control performance of the FULMS algorithm is much better than that of the PD algorithm in the long run. The control voltage in Figure 13(c) is the superposition of the feed-forward controller and feedback controller. As shown in Figure 13(d), the control voltage gradually increases with the online adjustment of the corresponding weights. In the process of dynamic adjusting, the control weights are convergent to constant values gradually as shown in Figure 13(g) to (j) after several times of fluctuation.

From Figure 13(a), one finds that the amplitude of the vibration sometimes varies from a small value to a large one. That is because the control weights require a learning process to get the best values. The control voltage in Figure 13(e) and the final values of the weights in Figure 13(g) and (h) indicate that the feed-forward
Figure 14. Resonant vibration suppression response using FULMS control with $\mu = 1e-6$ and $\beta = 2e-5$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 4 to 7 s; (c) control voltage; (d) zoom in on the time axis from 4 to 7 s; (e) control voltage of the feed-forward controller; (f) control voltage of the feedback controller; (g) feed-forward controller weights of $a_1$, $a_2$, and $a_3$; (h) feed-forward controller weights of $a_4$, $a_5$, and $a_6$; (i) feedback controller weights of $b_1$, $b_2$, and $b_3$; and (j) feedback controller weights of $b_4$, $b_5$, and $b_6$. 
controller plays a leading role in this control process. It is chiefly because the convergence factor of the feed-forward controller is much larger than that of the feedback controller.

To evaluate the effect of the feedback controller of the FULMS algorithm on the system, the convergence factors of the feed-forward controller and the feedback controller are chosen as $\mu = 1.0e-6$ and $\beta = 2.0e-5$, respectively. In this case, $\mu$ is set as a small value and $\beta$ is set as a large value. The dimensions of the adaptive filters $A$ and $B$ are selected as $N_1 = 6$ and $N_2 = 6$, respectively. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V. The corresponding experimental results are shown in Figure 14.

Comparing Figure 14(a) with Figure 13(a), one can find that the vibration is attenuated slowly in Figure 14(a), especially at the beginning of the moment applying the FULMS control. The vibration amplitude in Figure 14(b) decreased to 6 V at the time of 7 s. However, the amplitude is only about 2.6 V in Figure 13(b) at that time. The control voltage is shown in Figure 14(c). The time history of control voltage at the moment of applying the FULMS control is illustrated in Figure 14(d). From Figure 14(c) and (d), it can be known that the control voltage is increased much more slowly as compared with those in Figure 13. This is because the convergence factor of the feedback controller is selected much larger than that of the feed-forward controller. It can be seen that the control voltage of feedback controller in Figure 14(f) is much larger than that in Figure 13(f). Therefore, one can come to the conclusion that the feedback controller plays the leading role in this experiment. Actually, the feedback controller uses the control voltage as the reference signal to update its weights. Since the control voltage is small at the beginning, the control voltage of the feedback controller is increased very slowly during this period. With time goes on, the control voltage of the feedback controller gradually increases to a larger value. After 12 s, the control voltage of the feed-forward controller reaches 45 V. The control voltage of the feedback

Figure 14. (continued)
Figure 15. Resonant vibration suppression using FULMS control with $\mu = 1.4e - 5$ and $\beta = 2e - 5$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 4 to 7 s; (c) control voltage; (d) zoom in on the time axis from 4 to 7 s; (e) control voltage of the feed-forward controller; (f) control voltage of the feedback controller; (g) feed-forward controller weights of $a_1$, $a_2$, and $a_3$; (h) feed-forward controller weights of $a_4$, $a_5$, and $a_6$; (i) feedback controller weights of $b_1$, $b_2$, and $b_3$; and (j) feedback controller weights of $b_4$, $b_5$, and $b_6$. 
controller is obtained by the convolution operation between the adaptive filter $B$ and the control voltage.

The control voltage of the feedback controller reaches to a relatively large value of 80 V at 13 s. After that, the feedback controller could suppress the vibration in a very short time. Since the convergence factor of the feed-forward controller is selected small, the control weights in Figure 14(g) and (h) increase very slowly before 10 s and the ultimate weights are much smaller than those in Figure 13(g) and (h). However, the ultimate weights of feedback controller illustrated in Figure 14(i) and (j) are much larger than those in Figure 13(i) and (j) with larger selection of convergence factor $\beta$.

And then, the convergence factors of the feed-forward controller and the feedback controller are chosen as $\mu = 1.4e^{-5}$ and $\beta = 2.0e^{-5}$. These two factors are both chosen as 0.4 to evaluate the control performance of the FULMS algorithm. The dimensions of the adaptive filters $A$ and $B$ are selected as $N_1 = 6$ and $N_2 = 6$, respectively. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V. The experimental results are shown in Figure 15.

From Figure 15(a), it can be seen that the vibration response decays to a very small value in a very short period. The vibration response at the moment of applying the FULMS control illustrated in Figure 15(b) attenuates to a much smaller value of 2 V at the time of 7 s compared with Figures 13(b) and 14(b). From Figure 15(b), one can find that the vibration first reduces to a rather small value of 1.8 V. However, the vibration amplitude gradually gets larger after that. Actually, this is because the control weights are not adjusted to the optimum values. After several adjustments, the vibration almost attenuates to 0 after 20 s. The feed-forward and feedback control voltages shown in Figure 15(c) and (f) increase to very large values due to large convergence factors $\mu$ and $\beta$. The two control voltages all tend to reach steady states after several
Figure 16. Robust experimental results of resonant vibration control using FULMS control with $\mu = 1.4e - 5$ and $\beta = 2e - 5$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 15 to 25 s; (c) control voltage; (d) zoom in on the time axis from 15 to 25 s; (e) control voltage of the feed-forward controller; (f) control voltage of the feedback controller; (g) feed-forward controller weights of $a_1, a_2,$ and $a_3$; (h) feed-forward controller weights of $a_4, a_5,$ and $a_6$; (i) feedback controller weights of $b_1, b_2,$ and $b_3$; and (j) feedback controller weights of $b_4, b_5,$ and $b_6$. 
adjustments. The control voltage in Figure 15(c) is the superposition of Figure 15(e) and (f).

When the vibration amplitude gets larger in Figure 15(a), the controller weights are tuned online to make the control voltage larger. Figure 15(d) depicts the time history of control voltage at the moment of applying the controller. The weights of feed-forward controller and feedback controller illustrated in Figure 15(g) and (h) and Figure 15(i) and (j) are adjusted adaptively online to achieve the best values gradually. Since the estimated model of the control path cannot be totally equal to the practical control path, it should be mentioned here that the convergence factors cannot be selected too big. Otherwise, control divergence may happen in the experiment.

From the experiments described above, one can draw the following conclusion: the control performance of the FULMS feed-forward algorithm is much better than that of the PD feedback algorithm. The control performance of the FULMS algorithm is dependent on the selection of the convergence factors of the feed-forward controller $\mu$ and the feedback controller $\beta$. Larger values of $\mu$ and $\beta$ will lead to better performance of the FULMS control algorithm. In addition, the decay rate of the vibration at the beginning of the control process is mainly dependent on the value of $\mu$. The vibration attenuates faster at the beginning of the control process with larger value of $\mu$. Since the FULMS algorithm is based on IIR filter structure, its stability is not unconditionally ensured. Therefore, the values of $\mu$ and $\beta$ should be set appropriately.

In order to verify the robustness and anti-interference ability of the FULMS control algorithm, a shock disturbance is applied to the system. The convergence factors of the feed-forward and the feedback controllers are chosen as $\mu = 1.4e - 5$ and $\beta = 2e - 5$, respectively. The dimensions of the adaptive filters $A$ and $B$ are selected as $N_1 = 6$ and $N_2 = 6$, respectively. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V.

**Figure 16.** (continued)
Figure 17. Robust experimental results of resonant vibration control using FULMS control with $\mu = 2e - 5$ and $\beta = 1e - 5$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 15 to 25 s; (c) control voltage; (d) zoom in on the time axis from 15 to 25 s; (e) control voltage of feed-forward controller; (f) control voltage of feedback controller; (g) feed-forward controller weights of a1, a2, and a3; (h) feed-forward controller weights of a4, a5, and a6; (i) feedback controller weights of b1, b2, and b3; and (j) feedback controller weights of b4, b5, and b6.
The shock disturbance is applied to the system around the time of 16 s. Its maximum value is about 6 V.

The experimental results are illustrated in Figure 16. Figure 16(a) shows the self-regulating process of the system after the shock disturbance is applied. From Figure 16(a), one can find that once the shock disturbance is applied, the system could automatically adjust itself to suppress the vibration effectively and eventually reaches the convergence value after several adjustments. Figure 16(c) shows the control voltage. Figure 16(e) and (f) depict the control voltages of the feed-forward and the feedback controllers, respectively. It can be seen from Figure 16(e) and (f) that once the shock disturbance is applied, the control voltages become larger to suppress the vibration caused by external disturbance. After the vibration caused by the shock is eliminated, the voltages will go back to the original convergence value. Figure 16(b) and (d) show the time history of vibration response and control voltage at the moment of applying the shock disturbance, respectively. Figure 16(g) and (h) show the self-regulating process of the feed-forward controller weights. Figure 16(i) and (j) show the self-regulating process of the feedback controller weights. From Figure 16(g) to (j), one finds that once the shock disturbance is applied, the weights of the feed-forward and feedback controllers suddenly deviate from their original paths. The weights are tuned automatically to get larger control voltage in Figure 16(c) so as to eliminate the vibration caused by the shock disturbance. After several times of fluctuation, the vibration is suppressed to the convergence value and the weights converge to their original states eventually.

However, it is found that the weights of the feedback controller may not always return to the original convergence value in some occasions. One of these cases is illustrated in Figure 17. The convergence factors of the feed-forward controller and the feedback controller are chosen as $\mu = 2e - 5$ and $\beta = 1e - 5$, respectively. It can be seen from Figure 17(i) and (j) that the updating...
Figure 18. The time-domain response of resonant vibration control using FULMS control with $\mu = 2e - 5$ and $\beta = 1e - 6$: (a) resonant vibration before and after control; (b) zoom in on the time axis from 4 to 7 s; (c) control voltage; (d) zoom in on the time axis from 4 to 7 s; (e) control voltage of the feed-forward controller; (f) control voltage of the feedback controller; (g) feed-forward controller weights of $a_1$, $a_2$, and $a_3$; (h) feed-forward controller weights of $a_4$, $a_5$, and $a_6$; (i) feedback controller weights of $b_1$, $b_2$, and $b_3$; and (j) feedback controller weights of $b_4$, $b_5$, and $b_6$. 
direction of the weights is changed. All these weights do not return to their original track after the shock disturbance is applied. And the control voltage in Figure 17(f) is also changed. Actually, the weights of the feed-forward controller are updated according to the input sinusoidal signal. Since the input sinusoidal signal remains invariant during the process, the weights of the feed-forward controller can always go back to the original convergence value. However, the feedback controller uses the control voltage as the reference signal to update its weights and the control voltage is time-varying during the control process. Therefore, the weights of the feed-forward controller may not be able to come back to their original convergence value sometimes. Figure 17(a) shows that the vibration is suppressed effectively. Therefore, the FULMS algorithm shows great robustness to external disturbance.

Generally speaking, it is not easy to get the excitation signal in engineering applications. The reference signal is often taken from the sensor near the excitation source. However, this reference signal is affected by the feedback path, which could be bad for the control performance in some occasions. The FULMS algorithm is based on the IIR filter structure. With its zero-pole structure, the FULMS algorithm can effectively eliminate the control feedback effect of the system. Therefore, the experiment that reference signal extracted directly from the reference PZT sensor is conducted.

The reference PZT sensor in Figure 7(b) is used to obtain the reference signal in the FULMS algorithm. The error signal PZT sensor in Figure 7(b) is used to measure the vibration of the system. The convergence factors of the feed-forward controller and the feedback controller are chosen as $\mu = 2e - 5$ and $\beta = 1e - 6$, respectively. The dimensions of the adaptive filters $A$ and $B$ are selected as $N_1 = 6$ and $N_2 = 6$, respectively. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V. The corresponding experimental results of the FULMS
algorithm are shown in Figure 18. From Figure 18(a), one finds that the vibration response is suppressed to a small value of 1 V at the time of 7 s. The control voltage in Figure 18(c) is obtained from the additive operation of feed-forward control voltage in Figure 18(e) and feedback control voltage in Figure 18(f). Figure 18(b) and (d) show the time history of the vibration response and control voltage at the moment of applying the FULMS controller, respectively. The self-regulating processes of the feed-forward controller weights are shown in Figure 18(g) and (h). Figure 18(i) and (j) show the self-regulating process of the feedback controller weights. Although the weights of the feed-forward controller and feedback controller do not converge to the best values, the FULMS still shows a good performance in suppressing resonant vibration under persistent excitation due to the zero-pole structure of its controller.

The structure of the IIR filter is much more complicated with respect to the FIR filter. The feedback controller directly uses the output as its input to adjust the poles of the system. This operation can make the system unstable to some degree. Also, the convergence of the algorithm analysis is hard to carry out. Small feedback convergence factor should be taken to ensure the stability of the control process when the feedback path is presented.

Figure 18(a) to (f) show the different case of reference signal for the FULMS control algorithm. Different from the former cases, the reference signal is extracted from the reference PZT sensor. The convergence of the algorithm is complicated and hard to analyze. However, it can be found in many references that the FULMS algorithm has better performance than FXLMS even if the algorithm is not convergent to the global optimal value. Figure 18(f) shows the control voltage of the feedback controller. The feedback controller keeps diverging mainly to compensate for the decreasing value of the feed-forward controller to maintain the stability of the system.

In order to compare the FULMS algorithm with the FXLMS algorithm, the experiment of reference signal extracted from the PZT reference sensor is also conducted for the FXLMS algorithm. The convergence factor of the feed-forward controller is chosen as $\mu = 2 e^{-5}$. The dimension of the adaptive filter $W$ is selected as $N = 12$. The convergence factor and the dimension of the FXLMS controller are the same with the FULMS algorithm. The control voltage is applied at 4.5 s after the vibration response reaches the stable amplitude of 6.5 V. The corresponding experimental results of the FXLMS algorithm are shown in Figure 19. Figure 19(a) shows the time-domain response of resonant vibration suppression using FXLMS control. Figure 19(b) shows the corresponding control voltage. Comparing Figure 19 with Figure 18, one can see that the FXLMS controller gradually diverges after 20 s. This is because that the reference signal from the PZT sensor is polluted. The FXLMS cannot tune itself effectively to suppress the vibration.

The performance of the FULMS algorithm mainly depends on the convergence factors of $\mu$ and $\beta$. According to the source of the reference signal, two occasions can be separated. For the reference signal extracted from the signal generator, large value of convergence factors will lead to fast convergence rate. If the value of the feed-forward controller convergence factor $\mu$ is selected larger than the feedback controller convergence factor $\beta$, the feed-forward controller will take the leading role during the process. If $\beta$ is chosen larger than $\mu$, the feedback controller will take the leading role. However, if $\mu$ and $\beta$ are both chosen large.
enough, the convergence rate will be faster than the above two cases.

From the experiments of the reference signal extracted from the reference PZT sensor, the authors found that the feedback convergence factor $\beta$ should not be selected larger than the feed-forward convergence factor $\mu$. Otherwise, the control process may become unstable.

Remarks. Since only the vibration suppression of the first bending mode is researched in this article, graphs presented are all filtered graphs. The authors also did the experiments of the unfiltered response. However, it is hard for the controller to trace all the frequencies due to wide band of the unfiltered response. The effect of the noises in the unfiltered signal would also lead to the instability of the system during the control process.

Conclusion

This article presents the experimental results for resonant vibration control of a piezoelectric clamped-clamped plate. The natural frequency of the system is identified by using the sweep frequency excitation method. A fourth-order Type I Chebyshev band-pass filter with ripples in passband is designed to process the signals measured by the error PZT sensor and the reference PZT sensor. The experiments are carried out to investigate the performance and robustness of the designed PD control algorithm and the FULMS feed-forward control algorithm. The experimental results demonstrate that both the two algorithms can suppress the excited resonant vibration effectively and have good robustness for external disturbance. The FULMS feed-forward control algorithm shows much better control performance in suppressing the resonant vibration than the designed PD feedback control. Moreover, the control performance of the FULMS feed-forward control algorithm is much relevant to the convergence factors.

Declaration of Conflicting Interests

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