Calculating model for equivalent thermal defocus amount in infrared imaging system

Chengshuo Zhang a,b,c,* , Zelin Shi a,c, Baoshu Xu a,c, Bin Feng a,c

a Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang, Liaoning 110016, China
b University of Chinese Academy of Sciences, Beijing 100049, China
c Key Laboratory of Opto-electronic Information Processing, Chinese Academy of Sciences, Shenyang, Liaoning 110016, China

HIGHLIGHTS

- Analyze the equivalent effect of temperature variation and room-temperature defocus.
- A parameter called equivalent thermal defocus amount (ETDA) is defined.
- Room-temperature defocus has the same imaging effect as temperature changes by ETDA.
- Experiments on focal shift, aberration with temperature and ETDA prove our model.

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ABSTRACT

The main effect of temperature change on infrared imaging system is the focus shift of infrared lenses. This paper analyzes the equivalent influence on imaging between the temperature change and the defocus at room temperature. In order to quantify the equivalence, we define an equivalent thermal defocus amount (ETDA). The ETDA describes the distance of the photosensitive surface shifting at room temperature, which has the same effect on imaging as the temperature changes. To model the ETDA, the expression of the focal shift as a function of temperature is obtained by solving partial differential equations for the thermal effect on light path firstly with some approximations. Then point spread functions of the thermal effect and defocus at room temperature are modeled based on wave aberration. The calculating model of ETDA is finally established by making their PSFs equal under the condition that the cutoff frequency of infrared imaging systems is much smaller than that of infrared lens. The experimental results indicate that defocus of ETDA at room temperature has the same influence on imaging as the thermal effect. Prospectively, experiments at high/low temperature can be replaced by experiments at room temperature with ETDA.

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1. Introduction

The infrared imaging systems for military, industrial, space exploration and security applications are expected to perform satisfactorily at a large temperature scale. However, common infrared optical materials are characterized by larger variations in refraction index with temperature than visible materials. For instance, the change of refraction index respect to temperature for Germanium is over 100 times that of common optical glasses BK7 of the visible spectrum. Other temperature dependent parameters such as expansion coefficients of lens and metallic mounting also have influence on imaging quality when temperature changes. Therefore, it is of great challenge to keep the same performance at a large temperature scale.

In order to solve this problem, athermalization is necessary for optical systems, which includes passively mechanical compensation [1], actively mechanical compensation [2], optical compensation [3,4] and wavefront coding techniques [5,6]. All of these athermalization methods have a common hypothesis, which holds that the main effect of temperature change on infrared imaging systems is the focus of infrared lens shifting. Hence, athermalization can keep the same performance of imaging by maintaining focus of the infrared system over an extended temperature range. The hypothesis has yet been discussed in some literature. Thomas H. Jamieson considered that the variation in temperature caused the focus of single lens shifting and proposed an opto-thermal expansion coefficient.
2. Thermal effect on focal shift

In order to link the focal shift to temperature, equations of light paths are obtained by applying the raytrace method to each surface of an infrared lens as

\[
\begin{align*}
\frac{dV_i}{dT} &= V_i^p \frac{n_i - n_{i+1}}{n_i} \left( c_i - \frac{1}{n_i} \right) + V_i^p \frac{n_i - n_{i+1}}{n_i} \left( c_{i+1} - \frac{1}{n_{i+1}} \right), \\
\frac{dU_i}{dT} &= -dV_i,
\end{align*}
\]

where \( U_i \) is the object point distance of the \( i \)-th surface, \( V_i \) is the conjugate point distance, \( n_i \) is the refractive index of the \( i \)-th optical medium (\( n_0 \) is the refractive index of air), \( c_i \) is the curvature of \( S_i \), \( d_i \) is the distance from the vertex of the surface \( S_i \) to the vertex of the surface \( S_{i+1} \). When \( U_i = +\infty \), the focus position \( V_i \) of the lens can be achieved by making \( V_i = V_0 \). Derivative of Eq. (1) with respect to temperature \( T \) yields

\[
\begin{align*}
\left( \frac{dV_i}{dT} \right) &= \frac{dV_i}{dT} + 2V_i^p \frac{n_i - n_{i+1}}{n_i} \left( c_i - \frac{1}{n_i} \right), \\
\left( \frac{dU_i}{dT} \right) &= -\frac{dV_i}{dT} - d_i, \quad i = 1, \ldots, N,
\end{align*}
\]

where \( n_i \) is the material thermal expansion coefficient of the lens to which surface \( S_i \) belongs and \( \mu_i = \frac{dn_i}{dT} \). According to the temperature ranging from \(-20^\circ C\) to \(+60^\circ C\), \( \Delta U_i << U_i \) and \( \Delta V_i << V_i \), implying that \( U_i \) and \( V_i \) can be considered as constants. With the initial condition \( dU_i/dT = 0, U_i = +\infty \), Eq. (2) is simplified to

\[
\frac{dV_i}{dT} = \frac{dV_N}{dT} = T_c.
\]

Eq. (3) shows that the focal shift is directly proportional to the temperature change. In addition, the scaling factor \( T_c \) can be defined as the temperature coefficient of the infrared imaging system. By solving Eqs. (1) and (2), we have:

\[
T_c = P_N + \sum_{i=1}^{N-1} \left( \frac{n_i - n_{i+1}}{n_i} \right) \left( \frac{P_i - d_i}{P_i} \right) \prod_{j=1}^{i} U_j^p.
\]

where \( P_i = V_i^p c_i (n_i - n_{i+1})/n_i + V_i^p (c_{i+1} - \beta n_i/n_{i+1}) (U_{i+1}c_i - 1)/(n_i U_{i+1}) \).

\( T_c \) can describe the sensitivity of an infrared system to temperature. The smaller \( T_c \) means that the infrared system has a wider temperature range in which it can image clearly. Accordingly, the temperature coefficient \( T_c \) can be used to guide infrared imaging system design.

3. ETDA and its calculating model for infrared imaging system

The thermal defocus leads the focus to shift, but the location of the photosensitive surface is almost constant. Meanwhile, the room-temperature defocus can be regarded as misfocus induced by the photosensitive surface shifting, but the location of focus is constant. Both of them are the case that the focal plane and the photosensitive surface are not coincided with each other. Because the point spread function (PSF) is used to predict imaging performance as a common image quality metrics, PSFs are firstly modeled under the two conditions in order to compare the thermal defocus and room-temperature defocus.

According to Fourier optics [10,11], the PSF of optical system can be modeled by wave aberration. If the optical system is ideal or Gaussian optical, the wave front at exit pupil will be spherical surface whose center is on the focal plane. However, the real wave front always deviates from spherical surface because of the aberration. The wave aberration can be denoted by \( W(x,y; h) \) which represents the difference between the ideal spherical and real wave front surface with the aberration. \( x \) and \( y \) are coordinates in the pupil plane. \( h \) denotes image height. Assuming that the monochromatic wave aberration of a focused infrared imaging system is represented by \( W_d(x,y; h; \lambda) \) at room temperature \( T_0 \), the aberrated pupil function of the system is

\[
P(x,y; h; \lambda) = \begin{cases} 
\exp \left( j \frac{2\pi}{\lambda} W_d(x,y; h; \lambda) \right), & (x,y) \text{ in pupil} \\
0, & (x,y) \text{ outside pupil} 
\end{cases}
\]

Based on Fourier optics, the optical transfer function (OTF) is normalized as follows

\[
\text{OTF}(f_x, f_y; h; \lambda) = \frac{P(\lambda df_x, df_y; h; \lambda) \ast P(\lambda df_x, df_y; h; \lambda)}{\int \left| P(\lambda df_x, df_y; h; \lambda) \right|^2 \, dx \, dy}
\]

where \( d \) is the distance from the exit pupil to the focus and \( \ast \) denotes the correlation operation. The normalization amounts scale the OTF to have a value of 1 at the point \((0,0)\). To get the PSF of the system, the Fourier transform of OTF and simplification is taken as follow

\[
\text{PSF}(x,y; u; \lambda) = \frac{1}{C} \int_{-\infty}^{\infty} \left| \text{FT}[\text{OTF}(f_x, f_y; h; \lambda)] \right|^2 \, df_x \, df_y,
\]

where \( C \) is a normalized constant which makes the PSF have a sum value of 1. Regardless of the spectral distribution of light source, the PSF of the infrared imaging system is

\[
\text{PSF}(x,y; h; \lambda) = \frac{1}{C} \int_{-\infty}^{\infty} \text{PSF}(x,y; h; \lambda) R(\lambda) \, d\lambda.
\]

where \( R(\lambda) \) is the normalized spectral responsivity of infrared detector and \([\lambda_1, \lambda_2]\) is the range of the spectral.

When temperature changes, as shown in Fig. 1, the focus of the infrared system moves from the photosensitive surface position A (real line) to B (dotted line). Thus, the wave aberration at temperature \( T \) can be written as

\[
W_T(x,y; h; \lambda) = W_0(x,y; h; \lambda) + W_{df}(x,y; h; \lambda),
\]
where $\Delta T = T - T_0$ and $W_{AT}$ is the wave aberration caused by temperature change. The focus shift with temperature is the main factor and the aberrations hardly change with temperature [9], which is verified by experiments in Section 4. Therefore, $W_{AT}$ is the difference between the spherical wave front $w_A$ (blue dotted line) which converges into the old focus $V_A$ and the spherical wave front $w_B$ (red real line) which converges into the new focus $V_B$, as shown in Fig. 1. We build a left-hand coordinate system with its origin at the center of the exit pupil and with $Z$-axis along the principal ray $V_AV_B$ in Fig. 1. The marginal rays focusing at $V_B$ intersect $Y$-axis at $M_1$ and $M_2$. The wave aberration caused by temperature change $W_{AT}$ is

$$W_{AT} = w_B - w_A = \frac{d + \Delta V_f - (d^2 + \Delta V_f^2 + 2d\Delta V_f \cos \beta)^{1/2}}{\cos \alpha}, \quad (10)$$

where $\alpha = \arctan(h/d)$, $\beta(x,y) = \arctan(\cos \alpha, \sqrt{x^2 + y^2/d})$ and $\Delta V_f = T_c \Delta T$.

Since we do not use the normal coordinate system whose $Z$-axis is along the optical axis, the shape of exit pupil changes in our coordinates. Assuming that the shape change of pupil with temperature can be neglected and the pupil is circular with radius $r$, the shape of the pupil in our coordinate system will be elliptical. Fig. 2 illustrates the diagram of the ellipse parameters calculation. First, a line is drawn, which is perpendicular to the optical axis and through the midpoint $M$ of line $M_1M_2$. The line intersects the marginal rays at points $N_1$ and $N_2$. Second, a circle is drawn whose center is the midpoint of $N_1N_2$. Third, a line is drawn, which is perpendicular to $N_1N_2$ and through $M$. The line intersects the circle

Fig. 1. Diagram of wave aberration caused by temperature change.

Fig. 2. Diagram of calculating ellipse parameters.

Fig. 3. Diagram of wave aberration caused by photosensitive surface shifting.

Fig. 4. 2D layout of a real medium wave lens by ZEMAX.

Fig. 5. Assembly of 5-dimension translation stages, infrared lens and detector.

\footnote{For interpretation of color in Fig. 1, the reader is referred to the web version of this article.}
at points $B_1$ and $B_2$. Finally, the length of the short axis is the length of $B_1B_2$ and the length of the major axis is the length of $M_1M_2$. Therefore, the equation of the ellipse is

$$\frac{x^2}{a^2} + \frac{(y-y_0)^2}{b^2} = 1$$

(11)

where

$$a = \left(r\left(h^2 + d^2\right)^2 - r^2 h^1\right) / \left(h^2 + d^2 - rh\right)\left(h^2 + d^2 + rh\right),$$

$$b = \left(rd\left(h^2 + d^2\right)\sqrt{h^2 + d^2}\right) / \left(h^2 + d^2 - rh\right)\left(h^2 + d^2 + rh\right),$$

$$y_0 = \left(r^2 dh\sqrt{h^2 + d^2}\right) / \left(h^2 + d^2 - rh\right)\left(h^2 + d^2 + rh\right).$$

The pupil function of the infrared imaging system at $T$ is

$$P_T(x,y;h) = \begin{cases} P(x,y;h;\lambda) \exp(j \frac{2\pi}{\lambda} W_{\lambda}(x,y;h;\lambda)), & \frac{x-x_0}{a} + \frac{(y-y_0)^2}{b^2} \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

(13)

According to Eqs. (7) and (8), the PSF of the infrared imaging system at $T$ is

$$\text{PSF}_T(x,y;h) = \frac{1}{\mathcal{C}} \int_{S_z} \left| \text{FFT}[P_T(\xi dx_y, \xi dy_y; h; \lambda)] \right|^2 R(\lambda) d\lambda.$$

(14)

In a similar way, the photosensitive surface moves from the focus position A to the defocus position C at room temperature as shown in Fig. 3. Since the focal length does not change, the real wave front is also $W_A$, while the ideal wave front becomes $W_C$. Therefore, the wave aberration is written as

$$W_M = W_0 + W_{\lambda\lambda},$$

(15)

where $W_{\lambda\lambda}$ is the wave aberration caused by the photosensitive surface shift, as

$$W_{\lambda\lambda} = W_A - W_C = \frac{d + \Delta d \cos \beta - \left(d^2 + \Delta^2 \cos^2 \beta + 2d \Delta d \cos \beta \cos 2\phi\right)^{1/2}}{\cos \beta}.$$ 

(16)

The equation of the pupil shape is the same as Eq. (11). The pupil function of the infrared imaging system is

$$P_M(x,y;h,\lambda) = \begin{cases} P(x,y;h;\lambda) \exp(j \frac{2\pi}{\lambda} W_{\lambda\lambda}(x,y;h;\lambda)), & \frac{x-x_0}{a} + \frac{(y-y_0)^2}{b^2} \leq 1, \\ 0, & \text{otherwise} \end{cases}$$

(17)

The PSF of the infrared imaging system with photosensitive surface moves $\Delta d$ transforms to

$$\text{PSF}_M(x,y;h) = \frac{1}{\mathcal{C}} \int_{S_z} \left| \text{FFT}[P_M(\xi dx_y, \xi dy_y; h; \lambda)] \right|^2 R(\lambda) d\lambda.$$

(18)
Next we discuss whether thermal defocus and defocus at room temperature have the same influences on imaging according to their PSFs and then equivalent thermal defocus amount (ETDA) is defined for infrared imaging system. The cutoff frequency of an infrared lens is

$$ f_{\text{optical-cut}} = \frac{2r}{\lambda d}. \quad (19) $$

The sampling frequency $f_{\text{sample}}$ of the applied infrared imaging system is 33.33–66.67 cycles/mm, because the size of infrared detector pixel is about 15–30 μm. Usually, the cutoff frequency of an infrared lens is much higher than the sampling frequency. Therefore, we can get

$$(df_x)^2 + (df_y)^2 \leq (df_{\text{sample}}/2)^2 << (r/d)^2 < 1. \quad (20)$$

According to approximations below:

$$ \cos \beta (\lambda df_x, \lambda df_y) = \frac{1}{\sqrt{2^2[(x_2-x_1)^2 + \lambda^2]} \cos \alpha + 1} $$

$$ \approx 1 - \frac{1}{2} x^2 (f_x^2 + f_y^2) \cos \alpha. \quad (21) $$

$$ W_{\text{AT}}(\lambda df_x, \lambda df_y; h) $$

in Eq. (13) is simplified to

$$ W_{\text{AT}}(\lambda df_x, \lambda df_y; h) = \frac{x^2(f_x^2 + f_y^2)\Delta V_f \cos \alpha}{2(d + \Delta d)}, \quad (22) $$

and $W_{\Delta d}(\lambda df_x, \lambda df_y; h)$ in Eq. (17) is simplified to

$$ W_{\Delta d}(\lambda df_x, \lambda df_y; h) = -\frac{x^2(f_x^2 + f_y^2)\Delta \dd \cos \alpha}{2(d + \Delta d)}. \quad (23) $$

If we let Eq. (22) equal to Eq. (23) as

$$ W_{\Delta d}(\lambda df_x, \lambda df_y; h) = W_{\text{AT}}(\lambda df_x, \lambda df_y; h), $$

the PSF [Eq. (13)] of the infrared imaging system at $T$ is equal to the PSF [Eq. (17)] when the photosensitive surface moving $\Delta d$ at room temperature. Therefore, $\Delta d$ can be defined as equivalent thermal defocus amount (ETDA). It represents the distance corresponding to the photosensitive surface shift at room temperature, which has the same effect on imaging as temperature change. The calculating model for ETDA can be solved easily from Eq. (24) and it is illustrated as below

$$ \text{ETDA}(T) = -\frac{d}{d + 2\Delta V_f} \Delta V_f = -\frac{d}{d + 2fi dT} T \Delta T. \quad (25) $$

ETDA includes three parts: (1) the minus which shows that the direction of ETDA is always opposite to the direction of shift, (2) the focus shift $T \Delta T$, (3) the correction factor $d/(d + 2fi dT)$.

For small temperature change or $T \Delta T < d$ ETDA is approximated to $-T \Delta T$. When temperature change is dramatic, there will be a huge difference between ETDA and $-T \Delta T$. Therefore, ETDA needs to be modified with a correction factor to be modified. In addition, ETDA applies to the whole infrared imaging system, not to the infrared lens. That is because that Eq. (20) is not valid for the infrared lens. According to the physical interpretation of ETDA, the high-low temperature testing experiments of infrared system can replace the defocus testing experiments at room temperature. Thus, ETDA and its calculating model can reduce the complexity and the cost of

**Table 2**

<table>
<thead>
<tr>
<th>Temperature</th>
<th>-2 °C</th>
<th>8 °C</th>
<th>18 °C</th>
<th>28 °C</th>
<th>38 °C</th>
<th>48 °C</th>
<th>58 °C</th>
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<tbody>
<tr>
<td>Focusing location (μm)</td>
<td>16891.14</td>
<td>16845.32</td>
<td>16800.39</td>
<td>16755.22</td>
<td>16714.08</td>
<td>16657.21</td>
<td>16620.33</td>
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<tr>
<td>Focus offset (μm)</td>
<td>90.75</td>
<td>44.93</td>
<td>0.00</td>
<td>-45.17</td>
<td>-86.31</td>
<td>-143.18</td>
<td>-180.06</td>
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**Table 3**

<table>
<thead>
<tr>
<th>Temperature</th>
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<th>38 °C</th>
<th>48 °C</th>
<th>58 °C</th>
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</thead>
<tbody>
<tr>
<td>RMSE of MTF</td>
<td>0.1023</td>
<td>0.0682</td>
<td>0.1399</td>
<td>0.3246</td>
<td>0.4413</td>
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</table>

**Table 4**

<table>
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<th>Temperature</th>
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<th>38 °C</th>
<th>48 °C</th>
<th>58 °C</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE of MTF</td>
<td>0.0016</td>
<td>0.0062</td>
<td>0.0040</td>
<td>0.0065</td>
<td>0.0162</td>
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</table>
high-low temperature testing experiments. Particularly, low temperature testing experiments, which may dew inside the infrared imaging system. Otherwise, defocus experiments at room temperature can make outdoor image testing more easily without temperature-holding cabinet.

4. Experiments

In order to verify ETDA and its calculating model, we design a normal middle-wave infrared lens without athermalization whose 2D layout is shown in Fig. 4. The physical experimental platform includes 5-dimension translation stages, temperature cabinet, blackbody, knife-edge target, collimator, IR detector, computer and other devices. The main part of the 5-dimension translation stages is a motorized linear stage made by Zaber company’s technology Ltd., which can move along the optical axis. A middle-wave infrared lens is fixed in front of the metal holders before the motorized linear stage and the IR detector is put on the top of motorized linear stage as shown in Fig. 5. The computer can control and acquire the position of the motorized linear stage to set and read the photosensitive surface position of the IR detector. The motorized linear stage and the infrared imaging system are put inside the temperature cabinet to keep the environment temperature constant. There is a germanium window on the sidewall of the temperature cabinet in order to take knife-edge images at constant temperature as shown in Fig. 6. Main parameters of physical experimental device are shown in Table 1. The room temperature is 18 °C and the blackbody temperature is 30 °C.

Because it is difficult to get a point light source in lab, the precise PSF of the infrared imaging system cannot be directly obtained. The modulation transfer function (MTF) is the module of the Fourier transform of PSF, so MTF can be used to evaluate and optimize an optical system too. The knife-edge method [12] can measure MTF accurately. Thus, knife-edge images under different conditions are obtained by the physical experimental platform. Otherwise, the focus level of knife-edge images also needs to be evaluated. As the motorized linear stage fixed on the mechanical vibration isolation platforms, the location of the knife-edge does not change in image. This paper chooses an area of the knife-edge as indicated in the red box in Fig. 7. We use the sum of the absolute gray gradient in the focus evaluation area to evaluate the focus level of image. The focus evaluation function is

\[
M(I) = \sum_{(x,y) \in D} \left( |I(x,y) - I(x-1,y)| + |I(x,y) - I(x,y+1)| \right)
\]

where \( D \) is the focus evaluation area. The focus position is the point that makes \( M(I) \) maximum. At last, we use the method [12] to get MTFs under different conditions. In order to improve the test accuracy and reduce the influence of noise, 100 images are averaged to improve the signal noise ratio.
4.1. Experiment on focal shift with temperature

The ETDA calculating model (25) needs the temperature coefficient $T_c$, so Eq. (4) of $T_c$ should be verified. Firstly, the middle-wave infrared lens is simulated by ZEMAX. We use ZEMAX Programming Language (ZPL) to obtain the locations of the focus from $C_{010}$ to $C_{176}$. The result is shown in Fig. 8. The least squares relationship of simulated results is

$$D_{V_f} = -4.58\Delta T - 0.1568.$$  \hspace{1cm} (27)

The R-square is 1, and the root-mean-square error (RMES) is 0.215. Secondly, we measure the real focus shift at different temperature. Locations of focused knife-edges are recorded by the motorized linear stage at different environment temperatures. Focus offsets are the difference between focal locations at different temperatures. Results are listed in Table 2. The least squares relationship of physical results is

$$D_{V_f} = -4.55\Delta T - 0.04286.$$  \hspace{1cm} (28)

The R-square is 0.9985, and the RMSE is 3.769.

Fig. 8 shows that the focal shift is directly proportional to the change of temperature. The simulated temperature coefficient is $-4.59/\degree C$, our calculated temperature coefficient is $-4.62/\degree C$. The error between them is only $0.03/\degree C$. Even if the temperature change is $70\degree C$, the error is smaller than the wavelength. However, the measured temperature coefficient is $-4.55 \mu m/\degree C$. The error is $0.07 \mu m/\degree C$, which comes from assembly error of the middle-wave lens and the motorized linear stage’s accuracy.

4.2. Experiment on aberration with temperature

Further experiments on aberration at different temperature are designed to find out whether variation of aberrations are negligible when temperature changes. First, the infrared imaging system is focused by the motorized linear stage at the room temperature ($18\degree C$). At the same time, the location of the motorized linear stage is fixed. Second, six knife-edge images are obtained when the environment temperature has been constant at $-2\degree C$, $18\degree C$, $28\degree C$, $38\degree C$, $48\degree C$ and $58\degree C$. Fig. 9 shows MTFs at each temperature calculated by their knife-edge images. The differences of the six MTFs in Fig. 9 originate from the focal shift and aberrations with temperature. In order to eliminate the influence of focal shift, we obtain focused knife-edge images at different temperatures by the motorized linear stage. Focused MTFs at different temperatures are shown in Fig. 10. The differences of the six focused MTFs in Fig. 10 only come from the aberration variation with temperature. At the same time, RMSEs of MTFs in Fig. 9 between room temperature and other temperatures are listed in Table 3 and the RMSEs of focus MTFs in Fig. 10 between room temperature and other temperatures are listed in Table 4.

It can be seen that MTFs in Fig. 10 are much closer to each other than MTFs in Fig. 9 and the RMSEs in Table 3 is 10 times’ larger.
than the RMSEs in Table 4. Our experiment results confirm the evidence that the effect of aberration variation on MTF can be negligible compared to the effect of focus shift when temperature changes.

4.3. Experiments on ETDA and its calculating model

From the parameters of physical experimental devices listed in Table 1, the cutoff frequency of the middle-wave infrared lens is about 119 cycles/mm and the cutoff frequency of the infrared system is 16.67 cycles/mm. It is convincing that the cutoff frequency of infrared imaging system is much smaller than that of infrared lens. In order to verify ETDA and its calculating model, this paper designed a physical experiment as follows:

(1) To make the middle-wave infrared imaging system focus by the motorized linear stage at room temperature. At the same time, to record the detector’s photosensitive surface location denoted by $L$ and to fix the motorized linear stage.

(2) To make the incubator temperature constant at $T^\circ C$. Then an image $AT$ of knife-edge at this temperature is obtained.

(3) To make the incubator temperature constant at room temperature again. Move the photosensitive surface in the ETDA direction by the step of 0.5 $\mu m$. After each movement of the detector’s photosensitive surface, take a knife-edge image until the knife-edge is blurred heavily. The knife-edge images taken each time are denoted by $f_{Ai}$.

(4) To compute $MTF_T$ from knife-edge image $AT$ and $f_{MTF_i}$ from knife-edge images $f_{Ai}$ using the method of literature [12].

(5) To compute $RMSE_i$ between $MTF_T$ and $f_{MTF_i}$. According to the definition of ETDA, the measured value of ETDA is

$$TED_{measured}(T) = 0.5 \times \arg \min_i \{RMSE_i\}. \tag{29}$$

(6) To take outdoor images at temperature $T^\circ C$ and under the ETDA of $T^\circ C$.

Experiments are performed at $-2^\circ C$, $38^\circ C$, and $58^\circ C$. The experimental results are shown in Figs. 11–13. In each figure, (a) shows the curve of RMSE between $MTF_{58}$ and $MTF_i$; (b) shows the MTF at $T^\circ C$ and the MTF under ETDA ($T^\circ C$); (c) and (d) respectively show the outdoor image at $T^\circ C$ and outdoor image under ETDA ($T^\circ C$) respectively.

Experimental results show that there are some errors between the calculated ETDA and the measured ETDA in Table 5. However, the relative errors are smaller than 1.5%. The errors may include the error of the temperature coefficient $T_c$ and the measurement

<table>
<thead>
<tr>
<th>Temperature ($^\circ C$)</th>
<th>$-2^\circ C$</th>
<th>$38^\circ C$</th>
<th>$58^\circ C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measured values of ETDA ($\mu m$)</td>
<td>-87.0</td>
<td>87.5</td>
<td>177.0</td>
</tr>
<tr>
<td>Calculated values of ETDA ($\mu m$)</td>
<td>-85.7</td>
<td>87.2</td>
<td>175.9</td>
</tr>
<tr>
<td>Relative error (%)</td>
<td>1.49</td>
<td>0.34</td>
<td>0.62</td>
</tr>
</tbody>
</table>

Table 5: Measured ETDAs, calculated ETDAs and relative error of them at different temperatures.
error. From Figs. 11b, 12b and 13b, the MTF at \( T/\)\(^{C} \) and the MTF under ETDA (\( T/\)\(^{C} \)) have the same distributions and the RMSE is smaller than 0.008. From the outdoor images at each temperature, the image at \( T/\)\(^{C} \) has the same sharpness as the image under ETDA (\( T/\)\(^{C} \)). Therefore, the thermal effect on imaging quality is equivalent to that of defocus at room temperature and their effects on imaging are consistent. Moreover, from curves of RMSE between MTF\(_{7} \) and MTF\(_{8} \), it can be seen that ETDA can take place of \( T, AT \) when temperature change is small such as \(-2 \)\(^{C} \) and \(38 \)\(^{C} \). However, there is a big difference between ETDA and \( T, AT \), when temperature change is dramatic.

5. Conclusion

In this paper, we define ETDA to equivalently represent thermal effect on infrared imaging, and the calculating model of ETDA is proposed. Experimental results indicate that the ETDA makes defocus at room temperature have the same effect on imaging as temperature change. Meanwhile the calculating model of ETDA is validated. The ETDA proposed in this paper allows testing experiments at room temperature instead of the high/low temperature testing experiments of infrared system. ETDA and its calculating model can reduce the complexity and the cost of high/low temperature testing experiments. Particularly, low temperature testing experiments may dew inside the infrared imaging system. Otherwise, experiments by ETDA at room temperature can make outdoor image testing easier without temperature cabinet.

Conflict of interest

There is no conflict of interest.

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