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To cite this article: Bo Zhou, JiBin Zhao, Lun Li & Renbo Xia (2016) Double spiral tool-path generation and linking method for complex pocket machining, Machining Science and Technology, 20:2, 262-289, DOI: 10.1080/10910344.2016.1168928

To link to this article: http://dx.doi.org/10.1080/10910344.2016.1168928

Published online: 01 Jun 2016.

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Double spiral tool-path generation and linking method for complex pocket machining

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ABSTRACT
Complex pockets with one or more islands have been widely used in industrial and manufacturing production. In this paper, a new double spiral tool-path generation and linking method are proposed for complex pockets with islands which can be used for high-speed machining (HSM) is used. Taking into account the path interval, step length and other processing parameters, precise milling can be achieved without cutter lifting and retraction motions to guarantee machining accuracy and reduce processing time. The method has been implemented in several simulations and validated successfully through the actual machining of a complicated pocket. The results indicate that this method is superior to other existing machining methods, and it can achieve HSM of complicated shaped pockets based on parametric surface.

Introduction
High-speed machining (HSM) possesses the obvious superiority in complicated surfaces machining with the basic characteristics of high efficiency and high surface quality. It has not only met higher requirements for the lathe, clamp, cutter and so on, but also proposed further requests for the topology geometric shape and dynamics performance of the tool path (Heisel and Gringel, 1996; Lei and Liu, 2002; Schmitz et al., 2001). To effectively avoid sudden direction changes of the velocity vectors, all the transitions between feed, retract, etc., motions need to be as smooth as possible.

At present, the basic forms of tool path which most commercial CAM software systems used are still limited to zig-zag and contour-parallel offset (CPO). The major advantage of zig-zag method consists in its simplicity. As for the CPO method, at each offsetting step, an appropriate algorithm is executed to eliminate self-intersections and global intersections for all offset loops (Hansen and Arbab, 1992). For these methods, cutter lifting and retraction motions always exist during machining process. Obviously, these methods are not suitable for HSM.
Based on characteristics of smooth, no retraction, etc., spiral tool path has become an increasingly adopted tool path planning method for HSM. The existing tool path generation methods are mainly based on CPO, then connecting the offset loops to generate spiral tool path (Li et al., 2013; Lin et al., 2013). Moreover, Hauth and Linsen (2012) proposed an algorithm which created a double spiral by blending adjacent offset curves. When the given offset curves divided into multiple components, the algorithm created multiple smaller spirals and connected them appropriately. However, the complicated self-intersection loop-type judgments and reducing operation were made to eliminate the self-intersection.

There are also several novel algorithms: Held and Spielberger (2009) introduced a new spiral tool-path generation algorithm by interpolating growing disks placed on the medial axis of the 2D pocket without islands; Bieterman and Sandstrom (2003) aimed at solving the boundary value problem with MATLAB™ PDE toolbox and obtaining a series of contours. However, this method was not considered as machining parameters, so there occurred repeat-machining or residual-machining areas. Since undesired artifacts may get even more intensified when there is a high curvature at the start or the end of the spiral tool path, especially inside of the spiral (Hauth and Linsen, 2012). The double spiral tool-path which can effectively reduce the high curvature in the internal spiral tool path may become a superior choice for precision milling. Based on this advantage and self-complementary structure, double spiral tool-path generating and linking method is used.

The remainder of this paper is organized as follows: First, the thermal conductivity model is constructed for the obtainment of isothermal lines, and a dichotomy method is presented to determine the initial isothermal lines. Furthermore, the mapping rules which are between the standard parametric domain and parametric domain are established for double spiral tool-path generation. Moreover, the linking method is conducted between subdomains. To improve machining quality and efficiency, the principles for dividing pocket with islands are presented to handle the complicated pocket. Finally, simulations and experiments are conducted and the conclusions are given.

**Double spiral tool-path generation method**

**Thermal conductivity problem and difference calculation method**

The thermal conductivity model is used to solve the thermal distribution problems for complicated plane, such as complex-shaped boundaries or with complex hollow areas in the domain. The heat equation is a parabolic PDE that describes the distribution of heat (or variation in temperature) in a given region over time:

\[
\frac{\partial T}{\partial t} = K_0 \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(1)

where, \( K_0 = \lambda/\rho C \), \( \lambda \) represents thermal conductivity, \( \rho \) represents density, \( C \) represents specific heat capacity (heat capacity per unit of volume).
Figure 1. Temperature field distribution in a boy-shaped region.

The numerically computing solution of the 2D heat equation is based on the difference calculation method. The calculation of the temperature at any arbitrary location can be achieved (Haberman, 2005). The line joining a sequence of points with the same temperature value will be the isothermal line. In Figure 1, let the temperature value of outer boundaries have the same value, and without other heat sources located inside the domain, isothermal lines of approximately equal temperature to observe the thermal energy invading the interior of this domain are drawn.

Isothermal lines selection in parametric domain

The determinations of step length and path interval play an important role in improving the machining quality and efficiency (Lee, 2003). In this paper, the machining path interval is determined by means of the scallop-height method. As a mature method, simplified equations can be found in some references (e.g., Feng and Li, 2002; Kim and Yang, 2006).

Determinations of scallop height

The tool-path interval $l$ is the distance between the two adjacent paths. If $r \gg h$, $l$ can be replaced with path interval $L$. And $L$ on a CL point and the $R_c$ of a CL surface along the normal direction to a tool path for machining can be expressed as:

$$L = \sqrt{\frac{8hr}{R_c + r}} \quad \text{or} \quad L = \sqrt{\frac{8hr}{R_c - r}}$$

(2)
where $R_c$ is the radius of normal curvature along path interval direction, $h$ is the scallop height, and $r$ donates the ball-end cutter radius. The former expression is for convex surfaces and the latter is for concave ones.

According to this formula, the relationship between scallop height $h$ and path interval $L: h \propto L^2$ can be found. Under ideal conditions where spiral $B$ can divide spiral $A$ equally, the path interval ($L_1$) between tool path A and tool path B is approximately equal to $L/2$. And so is the case with tool path C and tool path B, the path interval ($L_2$) is also approximately equal to $L/2$ (Figure 2). Therefore, after implementing double spiral machining, the ideal scallop height $h_i$ should be approximately equal to $h/4$ ($h_i = (L/2)^2 / (L/2)^2$). In this article, the processing requirements of preset scallop height is $h_i$. And the set of path interval points $P_0', P_1', ..., P_n'$ which are obtained according to scallop-height method are determined by $h$.

**Determinations of guide line and initial isothermal lines**

First, a guide line $C'$ is determined on the surface, on which a set of path interval points $P_0', P_1', ..., P_n'$ according to scallop-height method can be obtained (Figure 3). The principles for determining the guide line $C'$ are to reduce the processing repeat as much as possible and to avoid leaving unprocessed domains residue simultaneously. The densities of isothermal lines in the domain of obtuse and acute boundaries are sparse and dense, respectively, as illustrated in Figure 4. So, in general, the guide line $C'$ shall be the one which start at the midpoint of the longer boundary and end in the center of the blank area. Usually, adequate judgments can be merely made according to visual observation.

If $r'_{(k-1)}$, $r'_{(i)}$ and $r'_{(k+1)}$ are three adjacent isothermal lines, $d_{(k)}$ (distance between $P'_{(k+1)}$ and $P'_{(i)}$) and $d_{(k-1)}$ (distance between $P'_{(i)}$ and $P'_{(k-1)}$) are the
Figure 3. Path interval points $P_0', P_1', \ldots, P_n'$.

intervals, respectively (Figure 5), and points $P_{(i)}'$ and $P_{(k)}'$ are the same one. The isothermal lines with temperature value $T_{(i)}'$ at mapping point $P_{(i)}'$ are generated as shown in the following flowchart: (1) The temperature value of outer boundaries are assumed to be maximum temperature $T_{\text{high}}$, and temperature value of innermost isothermal line is $0^\circ\text{C}$. Let the temperature interval be $T^\circ$C, then $T_{\text{high}}/T + 1$ sets of isothermal lines are generated including outer boundaries. There will always be two adjacent isothermal lines $r'_{(k-1)}$ and $r'_{(k+1)}$ existing and the interval between them is $d'_{(i)}$ (equal to $|P'_{(k+1)}P'_{(k-1)}|$) along the direction of line segment $P'_{(k+1)}P'_{(k-1)}$. Their temperature values are $T_{(k-1)}'$ and $T_{(k+1)}'$, respectively, and the mapping point $P_{(i)}'$ is just located between them, which means that the temperature value of mapping

Figure 4. Distribution of isothermal lines in dense and sparse regions.
Figure 5. Initial isothermal lines. (a) Obtained temperature value corresponding to $P'_i$ and (b) isothermal lines with corresponding temperature values.

Point $P'_{(i)}$ is just between the range of $T'_{(k-1)} \leq T'_{(i)} \leq T'_{(k+1)}$, $(T'_{(k+1)} - T'_{(k-1)}) = T'\degree{C}$; (2) Judging the relationship between $d'_{(i)}$ and $\varepsilon_d$:

If (1) $d'_{(i)} > \varepsilon_d$: Let $r'_{(k+1)}$ and $r'_{(k-1)}$ be the outer and inner boundaries, respectively, and let the temperature interval be $T'\degree{C}$. By implementing dichotomy method, the above-mentioned interval will gradually be shortened until the condition $d'_{(i)} \leq \varepsilon_d$ can be satisfied. It is assumed that the finally determined two isothermal lines are $r'_{(j)}$, $r'_{(j+1)}$ and their corresponding temperature values are $T'_{(j)}$, $T'_{(j+1)}$, respectively. Moreover, the conditions $T'_{(j)} \leq T'_{(i)} \leq T'_{(j+1)}$ $(T'_{(j+1)} - T'_{(j)}) = T'/(w \times 2)\degree{C}$ should be satisfied, where $w$ is the times of recursive computation.

$r'_{(j)}$ and $r'_{(j+1)}$ intersect the line segment $P'_{(k+1)}P'_{(k-1)}$, respectively, at points $P'_{(j)}$ and $P'_{(j+1)}$, then the temperature value on mapping point $P'_{(i)}$ can be obtained by the following proportion Equation (3):

$$T'_{(P'_{(i)})} = \left| P'_{(j+1)}P'_{(i)} \right| \times T'_{(j)} + \left( 1 - \frac{P'_{(j+1)}P'_{(i)}}{P'_{(j+1)}P'_{(j)}} \right) \times T'_{(j+1)} \quad (3)$$

If (2) $d'_{(i)} \leq \varepsilon_d$, then there is no need to make recursive computation. The temperature value on mapping point $P'_{(i)}$ can be calculated directly by the following proportion Equation (4):

$$T'_{(P'_{(i)})} = \left| P'_{(k+1)}P'_{(i)} \right| \times T'_{(k)} + \left( 1 - \frac{P'_{(k+1)}P'_{(i)}}{P'_{(k+1)}P'_{(k)}} \right) \times T'_{(k+1)} \quad (4)$$

The corresponding isothermal line with temperature value $T'_{(i)}$ is obtained according to the difference calculation method, as illustrated in Figure 5.
If there exist some points located at inner side of the isothermal line with the value 0°C, then let the isothermal line with the value 0°C be the outer boundaries to obtain isothermal lines with negative temperature values. The other procedures can be referenced to the related content of positive temperature values.

Determination of interpolated isothermal lines:
For each isothermal line $r_{(i)}$, the step lengths are calculated by means of the equichord deviation method. Take three adjacent isothermal lines $r_{(i)}$, $r_{(i+1)}$, and $r_{(i-1)}$, e.g., the interpolated number is equal to the larger amount of the corresponding points on the two adjacent isothermal lines, respectively (Figure 6).

**Parametric domain mapping**
Herein, a surface mapping method based on custom mapping rules is proposed.

**Parametric surface mapping**
For a 2D pocket with arbitrary boundaries, the isothermal lines in each subdomain are obtained directly through difference calculation method. As for a 3D pocket, the steps of the algorithm are as follows:

1. Dividing the subdomains: according to surface segmentation method with geometry characteristics, a trimmed surface can be divided into subdomains and corresponding subdomains can be obtained in standard parametric. Then a guide line $L$ is determined on each of the subdomains, and a set of intersections $P_0$, $P_1$, ..., $P_n$ ($P_0$ and $P_n$ are, respectively, the two endpoints of $L$) can be obtained by scallop-height method. The guide line $L$ and points $P_0$, $P_1$, ..., $P_n$ are mapped onto the standard parametric domain to obtain the corresponding mapping line $L'$ and a set of the mapping points $P'_0$, $P'_1$, ..., $P'_n$. The above-mentioned procedure will be recurred and repeated until all the mapping points are determined in each corresponding subdomains.

2. Calculating the initial isothermal lines and (3) interpolating isothermal lines: The procedures can be referenced to the related content in the above-mentioned chapter.

3. Constructing the mapping rules: the mapping...
rules are further established for the standard parametric domain and the parametric surface. The trajectory is planned out in the standard parametric domain, and the corresponding double spiral tool-paths and linking tool paths can be obtained on the parametric surface.

**Constructing mapping rules**

In this paper, for a 2D pocket with arbitrary boundaries, positive $X(Y)$ direction of the workpiece coordinate system is assumed to be positive $u(v)$ parameter direction. The corresponding relationship between $s/t$ directions and $u/v$ directions can be similarly constructed. The initial parametric domain is indicated in Figure 7a. The boundaries in the $u-v$ parametric domain are obtained by projecting the boundaries of parametric surface onto the $UV$ plane (Figure 7b). As for a 3D pocket, the boundaries of the $u-v$ parametric domain are corresponding to the boundaries of parametric surface. By establishing mapping rules, the following is to construct the relationship between parametric domain and standard parametric domain.

Mapping rule (1):
The serial number of the corresponding isothermal line can be obtained by \( t \) value.

\[
u' = [N_t \times t]
\]  

(5)

Per unite length along the \( t \) direction is divided into \( N_t \) aliquots. Therefore, each \( t \) value is an integer multiple of \( 1/N_t \). The \( t \) value varies from small to big corresponding to the isothermal lines distribution from the external to internal. The calculated floor rounding result is used to determine the serial number No. \( u' \) of isothermal lines. By this method, all the isothermal lines will be associated and numbered.

Mapping rule (2):

As mentioned earlier, each isothermal line is a closed polygon, and the additional condition of mapping points \( P'_0 = P'_n \) is fulfilled. The polygon \( P'_n = (P'_0, P'_1, ..., P'_n) = (P'_j)_{j=0}^n \) is the simplest model of difference geometry. Let the component element (line segment) of polygon be \( d_i = P'_{i-1}P'_i \). Now, the length \( L_{u'} = \sum_{i=1}^{n} d_i \) of polygon \( P'_n \) is simply the sum of the line segment lengths. It also indicates the length of the isothermal line whose serial number is \( u' \). Assumed that the mapping point in the standard parametric domain which is corresponding to point \( P(u, v) \) is point \( P(s, t) \), and the point \( P(u, v) \) is located at line segment \( P'_{j-1}P'_j \) whose serial number is \( j \).

Therefore, the following inequality is established:

\[
\sum_{i=1}^{j-1} d_i \leq s \leq \sum_{i=1}^{j} d_i
\]

(6)

And the coordinate value of point \( P'_{j-1} \) can be determined. The establishment of equal conditions are \( P(u, v) = P'_{j-1} \) or \( P(u, v) = P'_j \). Otherwise, according to proportion Equation (6):

\[
\frac{1}{L_{u'}} = \frac{s}{\sum_{i=1}^{j-1} d_i + \lambda}
\]

where \( \lambda \) is the length of \( P'_jP(u, v) \), circle \( C \) with center point \( P'_j \) and radius \( R = P'_jP(u, v) \), the coordinate value of point \( P(u, v) \) is the intersection of \( C \) and line segment \( P'_{j-1}P'_j \). The distribution of the parameter values \( s \) reflects the position of tool cutter location (CL), there are different kinds of parameterization methods (e.g., chord length parameterization method).

By this method, every point in the standardization parametric domain can get a corresponding point in the parametric domain. If there are five trajectories planned out in the standard parametric domain (Figure 7c), five corresponding isothermal lines will be mapped in the parametric domain (Figure 7d).

**Planning the spiral trajectory in parametric domain**

As previously described, the point \( P(u, v) \) is calculated by equi-chord deviation method on each initial isothermal line and the corresponding point \( P(s, t) \) can be obtained in the standard parametric domain. To guarantee that there are same amount of the points located on each of the adjacent isothermal lines, the adjacent
isothermal lines with more points shall connect its mapping point with the nearest point on the other one. After the trajectory is planned out in the standard parametric domain, every point on the planned trajectory is connected as per the $t$ value from big value to small value. The value $t$ of the end of current trajectory is just equal to the start of the next trajectory (Figure 8a); the spiral trajectory will be accordingly achieved in the parametric domain (Figure 8b).

**Double spiral generation method in arbitrary domain**

For the obtainment of spiral $B$, taking the path intervals which are between $A$ and $B$ into account, the path intervals need to be as even as possible. Therefore, the ideal spiral $B$ can be obtained by connecting the equidistant spacing points between every two adjacent loops of spiral $A$. Take three (triplet) adjacent curves (i.e., $a$, $b$, and $c$) which are illustrated in Figure 9a, e.g., equidistant spacing points on the adjacent curves are connected successively. Then the curves $a'$ and $b'$ are obtained as the
mapping trajectories with $s$ value varies from 1 to 0. Along the direction of longitudinal increment, for the same $s$ value, each value $t'$ on these curves can be computed directly. Moreover, according to the $t$ values form both sides of the value $t'$. The proportion equation can be constructed according to the distances from each side of adjacent value $t-t'$. Then a new isothermal line can be interpolated. According to mapping rule (2), a corresponding point in the parametric domain can be obtained. Finally, the spiral $B$ can be generated in the parametric domain, as illustrated in Figure 9b. The path interval between every two adjacent loops of double spiral $AB$ can be guaranteed to just approximate equidistance. As illustrated in Figure 10, the double spiral $AB$ with self-complementary spiral pattern is obtained by linking the centers of interior endpoints of spiral $A$ and $B$.

3D parametric surface fitting
For complex surfaces (as illustrated in Figure 11a), two sets of opposite boundaries can be swept to generate a complete parametric surface (i.e., “ruled surface”) (Figure 11b). By mapping the boundaries of islands in original parametric surface to standard parametric domain, corresponding boundaries of islands will be obtained (Figure 11c). Then the problem can be solved on the ruled surface for the corresponding points and mapping these points on original pocket for the machining points. Then referring to Chuang and Yang (2007), the mapping relationship between $u-v$ parametric domain and 3D parametric surface can be constructed.

Double spiral tool-paths linking method
According to the number of islands contained within one pocket, a pocket can generally be categorized into either of the following two cases: (1) with a single island or (2) with multiple islands. First, the tool-path generation method was proposed for Case (1). Based on this, the above method was developed to handle the tool-path generation for Case (2). For more complex pocket, take the machining parameters (e.g., path interval, feed rate) into consideration, the domain shall be divided into several subdomains based on the characteristics of complex surface and machining properties. Finally, the tool path of each subdomain is planned and these tool paths are connected sequentially.
Double spiral tool-path for pockets with one island

A pocket with one island inside is illustrated in Figure 12a. First, temperature values of the outer boundary and inner boundary were set as different values to apply difference calculation method (Figure 12b). Then the successive dichotomy procedure is applied to determine initial isothermal lines (Figure 12c). Finally, the double spiral tool-path was generated as illustrated in Figure 12d. Figure 13 is a simplified schematic of this linking method. A and B are spiral tool paths, respectively. The linking trajectory is L. The intersection point of double spiral tool-path AB and the central island is O.

Pockets with multiple islands

As for a pocket with multiple islands, according to the layout of islands, it can be classified geometrically into two primary categories: island chain and island ring, as shown in Figures 14 and 15, respectively. If the islands in a pocket can be roughly connected to form an island chain, this pocket can be divided into two parts and
Figure 12. Double spiral tool-path for pocket with one island. (a) Original domain, (b) isothermal lines generation, (c) initial isothermal lines and (d) double spiral tool-path.

Figure 13. Linking schematic of pocket with one island.
neither of them with an island. Consequently, the algorithm presented in the previous chapter can be directly applied in both parts. If the islands in a pocket can be roughly bridged into a ring, then a free pocket tool path can be constructed by a combination of the procedures given in previous chapters.

**Pockets with multiple islands connected into a chain**

As illustrated in Figure 14a, these three islands can be connected to form an island chain, and each part can be dealt with the case in previous chapter (Figure 14b).

As for the starting point can be present for a convenient determination on anywhere of the isothermal lines, the linking of the two set of double spiral tool-paths shall be very easy. The linking rules behave as follows:

The ending point of current double spiral tool-path and starting point of the adjacent boundary of next domain to be machined shall be as close as possible; The corresponding two tangential directions at above-mentioned two points intersect at one point (i.e., intermediate point). These three points are a set of control points to generate the quadratic Bezier curve, named with $L$. The obtainment details of transition curve are given in Appendix A.
Applying this method, the tool path with a linking sequence of $A - B - L - A' - B'$ can be referred to in Figure 14d and double spiral tool-paths is illustrated in Figure 14c.

**Pockets with multiple islands connected into a ring**

Referring to Figure 15a, the example has four islands located circularly. To obtain a more even tool-path distribution and better machining efficiency, a continuous tool path can be obtained by combining the machining of a single pocket and a pocket with one island. Hence, the entire pocket area is divided into two subdomains. The internal domain of the island ring is considered as a new pocket that
contains no island, while the external domain is treated as a pocket with a single island, as shown in Figure 15b. Each domain can be dealt with the case in previous chapter (Figure 15c). Finally, the double spiral tool-path is obtained, as illustrated in Figure 15d.

In this case, the linking sequence is as follows:

The machining order for completing the internal pocket is $A - B$. The linking trajectory $L_1$ is used for linking the ending point of spiral $B$ and inner point $O'$ of the external island-ring domain. The machining order for completing the external pocket is $B' - A'$ (or vice versa) and the linking trajectory is $L_2$. Figure 16 is a simplified schematic of this linking method with the sequence is $A - B - L_1 - A' - L_2 - B'$ (or $A - B - L_1 - B' - L_2 - A'$). The subdomain within the pink-dashed line is internal pocket.

**Divided pocket with islands**

The shapes of island chains and rings used here are very loosely defined, and they naturally cover a wide range of possibilities. Moreover, if the layout of islands is really complicated and cannot reasonably be classified into a single island chain or island ring type, our algorithm can be easily extended to deal with multiple chains and rings type.

As shown in Figure 17a, the problem of uneven distribution of tool-path interval will be amplified if a pocket has some narrow cross-sections (or bottlenecks). And it will also occur when islands in a pocket cause comparatively narrow tool-path passages. The solution for this problem is to re-divided subdomain to be machined, which can further obtain evenly distributed tool path, shown in Figure 17b, in favor of the determination of path interval.

Furthermore, Figure 18 is a simplified schematic of this linking method in which there are a number of double spiral tool-paths. The inner and outer pink-dashed lines together constitute a ring domain. Then the island ring domain with
multiple islands is divided into four parts, and the linking sequence is $A_1-B_1-L_1-\ldots A_4-B_4-L_4-A-B$.

Referring to Figure 19, the domain to be machined from the foregoing example can be dealt with the internal domain of the island ring is considered as a new pocket that contains no island, while the external domain is treated as four subdomains.
Simulation and verification

This method has been implemented and verified by simulation on a Windows XP+CPU 1.84GHz+RAM 2 GB hardware platform with Microsoft Visual C++ 2005+ACIS+Hoops. To prove the validity of this method, the following examples are given.

Example 1. Complex plane

For the given mickey mouse-shaped pocket in Figure 20, according to the topology geometric shape, the pocket can be divided into several subdomains. For each
Figure 21. Double spiral tool-paths linking for mask surface. (a) Standard parametric domain divided and (b) double spiral tool-paths linking.

subdomain, the thermal conductivity model is constructed and PDEs are solved for the obtainment of isothermal lines, respectively. Applying the successive dichotomy procedure and the result of this step is illustrated in Figure 20a. Table 1 shows the cutting parameters and tool geometries. In this case, the corresponding cutting tool is a flat-end cutter.

The trajectories are planned out in the standard parametric domain, respectively. Then the double spiral tool-paths and the linking tool-paths will be obtained (Figure 20b).

Example 2. Complex surface

For the complex mask surface, it can be divided into several subdomains according to proposed previously principles and topology geometric shape (Figures 11a and 21a). Table 2 shows the cutting parameters and tool geometries. In this case, the corresponding cutting tool is a ball-end cutter. First, the domain is divided into four simple subdomains and calculate the initial isothermal lines, respectively (Figure 21a). Then the trajectories are planned out in the standard parametric subdomains, respectively, four double spiral trajectories can be generated and linked in the corresponding parametric domains, let the machining sequence be from I to IV

<table>
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<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
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<td>Scallop height (mm)</td>
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<tr>
<td>Chord approximation error (mm)</td>
<td>0.2</td>
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<tr>
<td>Tool radius (mm)</td>
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Table 2. Cutting Parameters and Tool Geometries.

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<tr>
<td>Cutter taper (°)</td>
<td>4</td>
</tr>
</tbody>
</table>

(Figure 21a). Finally, these double spiral trajectories are mapped onto the parametric surface, then four double spiral tool-paths are obtained (Figure 21b).

**Curvature analysis**

The simulation results are analyzed as follows: curvatures of the corresponding discrete cutter location (CL) points can be evaluated (Coeurjolly and Svensson, 2003) and the curvature values are marked in Figure 22. The maximum curvature of each subdomain is located at the center of the corresponding subdomain.

The curvature values of each CL point are illustrated in Figure 23. The maximum curvature occurred at the forehead (No. I subdomain) with a value of only 1.01449. Other maximum values of curvature occurred at the left cheek (No. II subdomain, with a maximum value of 0.80155) and right cheek (No. III subdomain, with a maximum value of 0.83114). These two parts are amplified, respectively, as illustrated in Figure 24.

The mean curvature of each tool path is calculated as illustrated in Figure 25. The values of mean curvature are small, meaning that the double spiral tool-paths are very smooth. As seen from the simulation result (Figure 21b), there are no retractions, cracks, wrinkles, distortion, or other signs of instability, and an excellent double spiral tool-path has been generated by adopting the new method.

![Curvature values of corresponding CL points.](image_url)
Example 3. Experimental validation

To validate the described double spiral tool-path generation and linking method, the machining of a complex-shaped pocket (the bounding box range is $54.95 \times 39.45 \times 18.11$) in aluminum (1050) is analyzed. The rough and finish machining are performed in the HNC-210B 3-axis machining center which is equipped with an embedded industrial PC NC unit. During the experiment, specific oil is also applied to minimize the friction between tool and aluminum sheet.

Combining the segmentation method which is based on mean curvature (Figure 26a) partitioning with geometry characteristics of this trimmed surface, the schematic diagram of subdomains can be obtained (Figure 26b). The corresponding finishing tool paths are illustrated in Figure 26c.

For comparison purposes, the corresponding results of the chosen commercial CAM software, in which the zig-zag and contour-parallel strategies are provided...
here and the machining parameters are just the same as the proposed method (Figure 27). To clearly show contrastive tool paths, the error criterion (i.e., scallop height) is set to be much greater than those in real cases. As the figures show, both zig-zag and contour-parallel tool paths have to implement cutter-lifting motions. And for contour-parallel tool path the removing of intersections between offsetting paths is needed. Another problem of conventional tool-path generation methods is that they preserve sharp corners, which limits the feed rate when approaching these corners. And thus, machining efficiency is reduced and tool wear is intensified. However, the method proposed rounds these corners automatically and gradually.

The cutting parameters and tool geometries of finish machining process is shown in Table 3. The rough machining tool paths are repeated several times until the whole subdomains of mask is machined and finish cutting parameters are shown in Table 4. The machining results are illustrated in Figure 28a ((1) zig-zag, (2) double spiral and (3) contour parallel) and Figure 28b is a drawing of partial enlargement of double spiral tool-path (right side face). The total cutting time of finish machining are 4 h

![Figure 25](image1.png)

**Figure 25.** Mean curvature values of each tool path.

![Figure 26](image2.png)

**Figure 26.** Schematic diagram of subdomains and corresponding tool paths. (a) Colormap of mean curvature, (b) schematic diagram of subdomains and (c) corresponding tool paths.
Figure 27. Tool paths generated by commercial CAM software. (a) Zig-zag tool path and (b) contour-parallel tool path.

13 min 14 s, 3 h 11 min 18 s and 3 h 35 min 26 s, respectively. In summary, our method, which indicate that efficiency, smoothness and no retraction are superior to existing methods.

**Surface roughness measurement**

The finish machining result is evaluated by the surface roughness measurement. The actual values (experiment results) of $R_a$ are observed on the above-mentioned selected cutting conditions. The amount of standard surface roughness parameter (arithmetic average deviation from the mean line $R_a$) is performed using surface roughness tester (SRT-6200, produced by Starmeter Instruments Co., Ltd, China). Results of surface roughness are analyzed with the use of repeated measurements analysis of variance. Finally, the experiment results of $R_a$ are all about $0.9 \pm 0.085 \mu m$.

**Table 3. Machining Parameters for Finish Machining.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scallop height (mm)</td>
<td>0.01</td>
</tr>
<tr>
<td>Chord approximation error (mm)</td>
<td>0.02</td>
</tr>
<tr>
<td>Tool radius (mm)</td>
<td>2</td>
</tr>
<tr>
<td>Cutter taper (°)</td>
<td>4</td>
</tr>
<tr>
<td>Cutter length (mm)</td>
<td>50</td>
</tr>
</tbody>
</table>

**Table 4. Cutting Parameters for Finish Machining.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of cut (mm)</td>
<td>0.4</td>
</tr>
<tr>
<td>Spindle speed (r.p.m)</td>
<td>7000~7500</td>
</tr>
<tr>
<td>Feed rate (mm/min)</td>
<td>1300</td>
</tr>
</tbody>
</table>
However, zig-zag finishing is best used to machine the shallow parts (apex of nose and forehead partitions) of the mask (about 0.903 \( \mu \text{m} \)) but it does a relatively poor job (about 0.973 \( \mu \text{m} \)) on the steep parts (chin and double sided faces partitions). Conversely, contour parallel can definitely get good coverage on the steep part (about 0.921 \( \mu \text{m} \)). But for shallow parts, contour-parallel tool path gets very little coverage there, especially at apex of nose, the cutting procedures have to increase three times higher than the other two methods for getting the contour profile, even though the experiment result shows poor performance (Figure 28a) and machining time is prolonged. So contour parallel finishing is good for steep parts but bad for shallow parts. Our method can get relatively good performances both at shallow parts (about 0.943 \( \mu \text{m} \)) and steep parts (about 0.935 \( \mu \text{m} \)), for it seems to combine the properties of both tool-path methods.

**Geometrical and dimensional tolerances**

A coordinate measuring machine (CMM; Zeiss CMM CS100-2828-18) is used to obtain the measurement data of dimensional and geometrical tolerances (GD&T). After deleting the useless extra data, the measurement data is output as an STL file. The Geomagic Qualify is a software application oriented to virtual metrology on point clouds. This software is used to compare the deviations between the reconstructed surface and original CAD model. Moreover, GD&T deviations were analyzed in this comparison. Finally, through the shape deviation analysis and visualization, Figure 29 shows the 3D deviations. It can be observed that
the 3D matching was very good and the maximum 3D deviation did not exceed 30 μm.

**Errors analysis**

Actually, the comparatively large distortion only appears at the tool paths which are very close to boundary corners or corner areas of the islands. According to the experiments conducted, the number of these tool paths is 3–5. This is rather conservative. By adopting smaller scallop height value at these 3–5 tool paths close to corner areas, local machining accuracy of these corner areas can be effectively controlled.
Furthermore, the concept called bisection error, $\varepsilon_b$, is defined. It is used to evaluate the bisection level of spiral Tool path $A$ divided by spiral Tool path $B$. After calculating path intervals $L_1$ and $L_2$, scallop height $h_1$ and $h_2$ is computed by the above-mentioned equation, respectively. And $h_f$ can be evaluated as average value of $h_1$ and $h_2$ ($h_f = \frac{h_1 + h_2}{2}$). The bisection error $\varepsilon_b$ is evaluated by $\varepsilon_b = \frac{|h_1 - h_2|}{h_1 + h_2}$.

To facilitate the comparison and analysis, a couple of approximately symmetrical tool paths of mask are compared: the colormap of left-side mask is illustrated as original scallop height errors $h_f$; the colormap of right-side mask is illustrated as scallop height errors $h_f$ which have been optimized. After adopting smaller scallop height value at three tool-paths close to corner areas, the machining accuracy of the corner areas can be effectively promote (Figure 30). Figure 31 shows a colormap of bisection errors (located at side face). Therefore, the bisection error is around 0.8%.

**Conclusion**

This paper proposes a new double spiral tool-path generation and linking algorithm for HSM to ensure high efficiency and high quality.

1. This method reasonably constructs the thermal conductivity model and determines boundary conditions to obtain initial the isothermal lines which will satisfy the actual machining parameters;
2. This method construct mapping rules between standard parametric domain and parametric for generating double spiral tool-path. The applicability can be very advantageous in machining complex pocket with islands;
3. The presented double spiral tool-paths have a self-complementary structure with starting and ending point both located on the boundary of the work-piece, and it has the characteristic of low curvature. The tool path is smooth,
continuous, without tool retractions and reduces the probabilities of tool damage;

(4) The linking method is flexible for machining complex pocket with multiple islands. The Bezier curve is adopted as the transition trajectory. It can guarantee the linking trajectory which is between two adjacent machining domains to be smooth and no retraction.

**Funding**

This research was supported by National Program on Key Basic Research Project of China (973 Program) under Grant No. 2011CB302400 and National Natural Science Foundation of China (NSFC) under Grant No. 51175495.

**References**


Appendix A

The linking trajectory generation method of the Bezier curve \( L \) which is described in previous chapter is as follows:

\( L_i \) and \( L_{i-1} \) are the adjacent boundaries of current machining domain and next machining domain, respectively. First, the \( L_i \) is offset with an offset distance \( e \) which is obtained by scallop-height method, as illustrated in Figure A1. Let us take, e.g., domain I to describe the generation method.

![Figure A1](image)

**Figure A1.** Linking trajectory generation. (a) Offset the outer boundaries and (b) Bezier curve generation.

The corresponding offset curve is \( L'_i \) and the original curve is \( L_i \). \( P'_i \) is the endpoint of \( L_i \). \( P_{IN} \) is the end point of the double spiral tool-path in current subdomain. And let \( L'_i \) be the start point of next machining domain. \( T_1 \) and \( T_2 \) are the tangential directions at point \( P_{IN} \) and \( P'_i \), respectively. \( P_i \) is the intersection point of \( T_1 \) and \( T_2 \). So \( P_i \) is the intermediate point.

In this paper, the quadratic Bezier curves is determined by three control points \( P_{IN}, P_i, P'_i \) as shown in Figure A1b. The parametric quadratic Bezier curves \( L \) is given by:

\[
B(t) = (1 - t)^2 P_{IN} + 2t (1 - t) P_i + t^2 P'_i, \quad t \in [0, 1]
\]

The two-terminal tangential directions of Bezier curve are equal to \( T_1 \) and \( T_2 \), respectively.