Design and modelling of a snake robot in traveling wave locomotion

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Abstract

In this paper starting with the model, simulation and finishing with experiment, a systematic complete description of traveling wave locomotion is presented. The dynamic effects and a detailed model of the friction between the mechanism and the ground are described. And the relationship between the driving forces with the body shape and joint torques are established. The simulation results present the regularity of the joint torques during traveling wave locomotion. The effects of the initial winding angle and friction coefficient on the joint input torques are also depicted. The traveling wave locomotion was achieved in a snake robot. The experiments validated some simulation results and proved that the established kinematics and dynamics of snake robot are reasonable.

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1. Introduction

Snake robots with many degrees of freedom have been studied in the last thirty years [6]. Some types of limbless locomotion were achieved in snake robots [12,13,1]. Two main planar models, serpentine locomotion (horizontal locomotion) and traveling wave locomotion (vertical locomotion) are often adopted in snake robot [8,7,9,2]. The serpentine locomotion is formed in the supporting plane, and its driving forces originate from the different friction coefficients of the body in the tangential and the normal directions with respect to the supporting plane [5,14]. So the locomotors usually need wheels to realize the directional friction and hence the adaptability to the environment was somewhat weakened. On the contrary, traveling wave locomotion developed in vertical plane orthogonal to the supporting plane has more substantial potential for adaptability to the environment without additional consideration of the friction condition. Firstly, in vertical locomotion, its motion width is only approximately equal to its body section. However, in horizontal locomotion, its
motion width is decided by the amplitude of its body shape, which is much wider than the body section. Thus, vertical locomotion has some advantages over horizontal locomotion in a narrow environment. Secondly, in vertical locomotion the snake robot can control not only the body shape but also every joint’s torque to create a proper supporting forces and driving forces. However, in the horizontal locomotion, this can be achieved only by controlling the body shape. Thirdly, by controlling the vertical joints in vertical locomotion, the snake robot can lift its head or some sections of its body to make its head reach the other edge of a gap, and then move forward to cross the gap. Moreover, the authors also found that three-dimensional locomotion may be achieved through the combination of these two types of planar locomotion [3].

However, the traveling wave locomotion is always characterized as a low efficient gaits [4] because till now its dynamics is somewhat obscure [10,11] and only based on kinematic analysis no effective controlling methods can be conducted. In this paper starting with the dynamic model, simulation and finishing with experiment, a systematic complete description of traveling wave locomotion is conducted for a good comprehension and higher efficiently controlling this gait.

2. Controlling of the body shape

Until now, the control of the body shape is more related to the simple mimicking the motion of living snakes or to find the snake’s backbone curve in order to derive the control variables of the body shape [6]. For convenience, in this paper the serpenoid curve is used as the basic body shape of the snake robot during traveling wave locomotion. The serpenoid curve is given by the curvature function as follows:

$$\rho(s) = \frac{2K_n \pi \alpha}{L} \sin \left( \frac{2K_n \pi}{L} s \right)$$

where $L$ is the whole length of snake body, $K_n$ is the number of the wave shape, $\alpha$ is the initial winding angle of the curve, and $s$ is the body length along the body curve, as shown in Fig. 1. The snake robot consists of $n$ links with element of unit length $l$ connected through $n - 1$ joints. Each link is rigid with uniformly distributed mass. Each joint is swung by an electrical motor. It is assumed that the robot holds its shape on the serpenoid curve and the body shape changes with respect to the change of the serpenoid curve while moving in the plane. If the joints are located on the curve, the joint variables, i.e., relative angles $\theta_i(s)$ can be regarded as the shape control variables, which can be derived approximately from the integration of the curvature function:

$$\theta_i(s) = \int_{s+(i-1)l}^{s+il} \rho(u) du = -2\alpha \sin(K_n \pi/n) \cdot \sin(2K_n \pi s/L + 2K_n \pi i/n + k_n \pi/n)$$

$$\dot{\theta}_i(s) = -4K_n \pi \alpha/L \cdot \sin(k_n \pi/n) \cdot \cos(2K_n \pi s/L + 2K_n \pi i/n + k_n \pi/n) \dot{s}$$

$$\ddot{\theta}_i(s) = -4K_n \pi \alpha/L \sin(k_n \pi/n) \cos(2K_n \pi s/L + 2K_n \pi i/n + k_n \pi/n) \dot{s}^2$$

$$+ 8k_n^2 \pi^2 \alpha/L^2 \sin(k_n \pi/n) \sin(2k_n \pi s/L + 2k_n \pi i/n + k_n \pi/n) \dot{s}$$

$$i = 1, 2, \ldots, n - 1$$

where $n$ is the number of links and $l$ is the unit length of link. When $s$ is given, using Eqs. (2)–(4), $\theta_i(s)$, $\dot{\theta}_i(s)$ and $\ddot{\theta}_i(s)$ can be deduced, and the absolute value of joint angles can be written

$$\phi_i = \phi_1 + \sum_{k=1}^{i-1} \theta_k$$

$$\dot{\phi}_i = \dot{\phi}_1 + \sum_{k=1}^{i-1} \dot{\theta}_k$$

$$\ddot{\phi}_i = \ddot{\phi}_1 + \sum_{k=1}^{i-1} \ddot{\theta}_k, \quad i = 1, 2, \ldots, n$$

where $\phi_i$, $\dot{\phi}_i$ and $\ddot{\phi}_i$ are the absolute value of joint angles, angle velocity, and angle acceleration with respect to the $x$-axis, respectively. Based on the above equations, $\theta_i(s)$ is controlled to vary $\phi_i$, thus the body shape control is achieved.
3. Kinematics of a snake robot

Consider the snake robot as shown in Fig. 2, each link $i$ has its own local coordinate system $o_i - x_i - z_i$ on the joint. In addition, there is a base frame $O-X-Z$. The snake robot is located on a supporting plane and can move in the vertical plane. In this case, the mechanical system has $n + 2$ degrees of freedom, among which $n - 1$ for the shape, 2 for the position, and 1 for the orientation. If $O-X-Z$ is fixed in the inertial frame, take the $(i + 1)$th link of the snake robot for example, the relationships of the position, the velocity and the acceleration of the $(i + 1)$th joint with respect to that of the $i$th joint are described as follows:

\[
\begin{align*}
  x_{i+1} &= x_i + l \cos \phi_i \\
  z_{i+1} &= z_i + l \sin \phi_i \\
  \dot{x}_{i+1} &= \dot{x}_i - l \sin \phi_i \dot{\phi}_i \\
  \dot{z}_{i+1} &= \dot{z}_i + l \cos \phi_i \dot{\phi}_i \\
  \ddot{x}_{i+1} &= \ddot{x}_i - l \sin \phi_i \dot{\phi}_i^2 - l \sin \phi_i \ddot{\phi}_i \\
  \ddot{z}_{i+1} &= \ddot{z}_i - l \sin \phi_i \dot{\phi}_i^2 + l \cos \phi_i \ddot{\phi}_i
\end{align*}
\]

$i = 1, 2, \ldots, n$
where \( x_i \) and \( z_i \), \( \dot{x}_i \) and \( \dot{z}_i \), and \( \ddot{x}_i \) and \( \ddot{z}_i \) represent the position, velocity and acceleration of \( i \)th joint with the \( X \)-axis and \( Z \)-axis, respectively. Thus the position, velocity and acceleration of gravity center of link \( i \) can be derived as follows in the case of assuming the uniform link:

\[
\begin{align*}
    x_{iG} &= x_i + \frac{1}{2} \cos \phi_i \\
    z_{iG} &= z_i + \frac{1}{2} \sin \phi_i \\
    \dot{x}_{iG} &= \dot{x}_i - \frac{1}{2} \sin \phi_i \dot{\phi}_i \\
    \dot{z}_{iG} &= \dot{z}_i + \frac{1}{2} \cos \phi_i \dot{\phi}_i \\
    \ddot{x}_{iG} &= \ddot{x}_i - \frac{1}{2} \cos \phi_i \ddot{\phi}_i - \frac{1}{2} \sin \phi_i \dddot{\phi}_i \\
    \ddot{z}_{iG} &= \ddot{z}_i - \frac{1}{2} \sin \phi_i \dddot{\phi}_i + \frac{1}{2} \cos \phi_i \dddot{\phi}_i \\
\end{align*}
\]

\[ i = 1, 2, \ldots, n \]

where \( x_{iG} \) and \( z_{iG} \), \( \dot{x}_{iG} \) and \( \dot{z}_{iG} \), \( \ddot{x}_{iG} \) and \( \ddot{z}_{iG} \) represent the position, velocity and acceleration of gravity center of the \( i \)th link along \( X \)- and \( Z \)-axis. Using Eqs. (14)–(19), the position, velocity and acceleration of gravity center of the snake body can be obtained as follows:

\[
\begin{align*}
    X_G &= \frac{\sum_{i=1}^{n} m_i x_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} x_{iG} \\
    \dot{X}_G &= \frac{\sum_{i=1}^{n} m_i \dot{x}_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} \dot{x}_{iG} \\
    \ddot{X}_G &= \frac{\sum_{i=1}^{n} m_i \ddot{x}_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} \ddot{x}_{iG} \\
    Z_G &= \frac{\sum_{i=1}^{n} m_i z_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} z_{iG} \\
    \dot{Z}_G &= \frac{\sum_{i=1}^{n} m_i \dot{z}_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} \dot{z}_{iG} \\
    \ddot{Z}_G &= \frac{\sum_{i=1}^{n} m_i \ddot{z}_{iG}}{M} = \frac{1}{n} \sum_{i=1}^{n} \ddot{z}_{iG} \\
\end{align*}
\]

where \( X_G \) and \( Z_G \), \( \dot{X}_G \) and \( \dot{Z}_G \), \( \ddot{X}_G \) and \( \ddot{Z}_G \) represent the position, velocity and acceleration of gravity center of the snake body along \( X \)- and \( Z \)-axis, respectively.

Fig. 3. The trace of snake robot in traveling wave locomotion.
When the value of $s$ is given, the position of each link in the inertial coordinate frame can be obtained from Eqs. (5), (8) and (9). Therefore, the trace of snake robot in traveling wave locomotion can be drawn, as shown in Fig. 3. It can be seen that the trace of every link is along the serpenoid curve, but the displacement of the whole body is a straight line along the $X$-axis.

### 4. Dynamics of a snake robot

The dynamics of the snake robot can be viewed as a combination of mechanism dynamics and environment constraints. The objective of mechanism dynamics is to model the functional relationship between the joint torques and the robot locomotion. Whereas, the interaction force between the body and the environment can be determined by environment constraints.

#### 4.1. Mechanism dynamics

The force diagram of the $i$th link is illustrated in Fig. 4, where $N_{i,i+1}$ and $F_{i,i+1}$ are the supporting force and friction force of the $i$th link at the $(i + 1)$th joint, $T_i$, $f_i$, $m_i$ and $I_i$ represent the torques, internal forces, mass and moment of inertia of the $i$th link, respectively. Therefore, based on the principle of the Newton, the motion for the $i$th link with respect to the $(i + 1)$th link can be described:

$$
\begin{align*}
    f_{ix} - f_{i+1x} + F_{i,i} + F_{i,i+1} &= m_i \ddot{x}_G \quad (26) \\
    f_{iz} - f_{i+1z} + N_{i,i} + N_{i,i+1} - mg &= m_i \ddot{z}_G \\
    \tau_i - \tau_{i+1} + (f_{ix} + f_{i+1x} + F_{i,i} - F_{i,i+1})l \sin \phi_i/2 - (f_{iz} + f_{i+1z} + N_{i,i} - N_{i,i+1})l \cos \phi_i/2 &= I_i \ddot{\phi}_i \\
\end{align*}
$$

Since the head and tail of the snake robot are free, there are two equations:

$$
\begin{align*}
    f_{1x} &= f_{1z} = f_{n+1x} = f_{n+1z} = 0 \\
    \tau_1 &= \tau_{n+1} = 0 \\
\end{align*}
$$

From Eqs. (26), (27) and (29), we can obtain

$$
\begin{align*}
    \sum_{i=1}^{n} m_i \ddot{x}_G &= \sum_{i=1}^{n} F_i \\
    \sum_{i=1}^{n} m_i \ddot{z}_G &= 0 \\
\end{align*}
$$

![Fig. 4. Scheme of forces acted on the $i$th link.](image)
Substituting Eqs. (18) and (19) into Eqs. (31) and (32) we obtain

\[
\begin{align*}
n\ddot{x}_1 &= \sum_{i=1}^{n} \left[ \sum_{k=1}^{n-1} \left( l \cos \phi_k \dot{\phi}_k^2 + l \sin \phi_k \dot{\phi}_k \right) + \frac{l}{2} \cos \phi_i \dot{\phi}_i^2 + \frac{l}{2} \sin \phi_i \dot{\phi}_i \right] = \sum_{i=1}^{n+1} F_i \\
n\ddot{z}_1 &= \sum_{i=1}^{n} \left[ \sum_{k=1}^{n-1} \left( l \sin \phi_k \dot{\phi}_k^2 - l \cos \phi_k \dot{\phi}_k \right) + \frac{l}{2} \sin \phi_i \dot{\phi}_i^2 - \frac{l}{2} \cos \phi_i \dot{\phi}_i \right] = 0
\end{align*}
\]

Eqs. (33) and (34) represent the relation between the acceleration of head (link 1) \(\ddot{x}_1, \ddot{z}_1\) and the angular acceleration \(\ddot{\phi}_1\).

Using the recursive formulas in Eqs. (26) and (27), from the tail to the head the relation between the internal force and external force of every link are obtained:

\[
\begin{align*}
f_{xi} &= \sum_{j=i}^{n} m\ddot{x}_j - \sum_{j=i+1}^{n+1} F_j - F_{i,i} \\
f_{zi} &= \sum_{j=i}^{n} m\ddot{z}_j - \sum_{j=i+1}^{n+1} N_j - N_{i,i} + (n + 1 - i)mg
\end{align*}
\]

Substituting Eqs. (35) and (36) into torques Eq. (28), we have

\[
\tau_i - \tau_{i+1} + \left( 2 \sum_{j=i}^{n} m\ddot{x}_j - 2 \sum_{j=i+1}^{n+1} F_j - m\ddot{x}_i \right) I_s \dot{\phi}_i / 2
- \left( 2 \sum_{j=i}^{n} m\ddot{z}_j - 2 \sum_{j=i+1}^{n+1} N_j - m\ddot{z}_i + (2n + 1 - 2i)mg \right) I_c \dot{\phi}_i / 2 = I_i \ddot{\phi}_i
\]

where \(F_j\) and \(N_j\) represent the friction force and supporting force of the \(j\)th joint and they meet the following equations:

\[
\begin{align*}
F_j &= F_{j-1,j} + F_{j,j} \\
N_j &= N_{j-1,j} + N_{j,j}
\end{align*}
\]

where \(F_i\) and \(N_i\) can be calculated from the environment constraints in the next section.

From Eq. (37) we obtain

\[
\sum_{i=1}^{n} \left\{ \left( 2 \sum_{j=i+1}^{n} m\ddot{x}_j - 2 \sum_{j=i+1}^{n+1} F_j + m\ddot{x}_i \right) I_s \dot{\phi}_i / 2 - \left( 2 \sum_{j=i+1}^{n} m\ddot{z}_j - 2 \sum_{j=i+1}^{n+1} N_j + m\ddot{z}_i + (2n + 1 - 2i)mg \right) I_c \dot{\phi}_i / 2 \right\}
= \sum_{i=1}^{n} I_i \ddot{\phi}_i
\]

Eq. (40) is a linear equation with one unknown variable \(\ddot{\phi}_1\). Solving Eq. (40), the rotation acceleration of the first joint \(\ddot{\phi}_1\) can be obtained, and then substituting it into Eqs. (33) and (34), the linear acceleration of head (link 1) \(\ddot{x}_1, \ddot{z}_1\) can be correspondingly derived. The rotation velocity and angle, and moving velocity and position of first joint can be obtained through integration. Substituting these values into Eq. (37), the joint torques required to generate the robot motion are obtained.

4.2. Dynamics with environment constraints

Main subject of dynamics with environment constraints is to calculate the interaction forces between the environment and the snake body. Because the supporting force and friction force from the environment on the snake body is a function of the body shape in the traveling wave locomotion, a snake robot with the \(n = 16\) segments and the shape number of the body \(K_n = 2\) is taken as example.
Fig. 5 shows the traveling wave in one segment, where it has at least two supporting points with the ground. The joints contacting the ground are set as $u$ and $u'$. If the gravity center of the body is located within the supporting points, the wave shape would be stable.

External forces acting on the snake body are the gravity force $G$, supporting force $N$ and friction force $F$. Gravity force $G$ and supporting force $N$ are balanced, so the resultant force is the friction force, which is the driving force. Based on the force and moment balance, we can obtain

$$
N_u = \frac{1}{\lambda}[(G + M\ddot{Z}_G)(x_G - x_u) - M\dot{X}_Gz_G] \tag{41}
$$

$$
N_u' = G - N_u \tag{42}
$$

where $\lambda$ is the distance between the two supporting points, $M\ddot{X}_G$ and $M\ddot{Z}_G$ are the inertial forces of the body along the $X$-axis and $Z$-axis. In this study the viscous friction is neglected, and the coulomb friction is used to depict the environment dynamics:

$$
F_i = -\mu \cdot \text{sgn}(v) \cdot N_i \tag{43}
$$

where $\mu$ is the friction coefficient between the contacting joint and the supporting plane. From Eqs. (43), (42) and (41) we can see that the driving force, i.e., the friction force is related with the position and acceleration of the robot body. So the snake robot can be controlled by its body shape and joint torques according to its kinematics and mechanism dynamics jointly. In next section we will give some simulation results on the relationship of the joint torques, the body shape and the environmental coefficient.

5. Simulation analysis

In this section the traveling wave locomotion along a straight line is shown by simulation. The variations of the joint torques with time are drawn, and the effect of friction coefficient and the initial winding angle of the body shape on the joint torques are discussed.

Simulation parameters are set as follows: $L = 1.6$ m, $n = 16$, $l = 0.1$ m, $m = 0.1$ kg, $I = 0.0001$ kg m$^2$, $g = 9.8$ N/kg and $K_n = 2$. The body shape changes with respect to the displacement of the tail along the serpentine curve, the acceleration of $s$ is given as follows:

$$
\ddot{s} = \begin{cases} 
a & 0 \leq t < T/10 \\
0 & T/10 \leq t < 9T/10 \\
-a & 9T/10 \leq t < T/10 \end{cases}
$$

where $a = 0.0625$ m/s$^2$, locomotion time $T = 32$ s, initial positions are selected as $x_1 = 0, z_1 = 0$, initial velocities are set as $\dot{x}_1 = 0, \dot{z}_1 = 0, \dot{\phi}_1 = 0$, and the initial winding angle $\phi_1 = \alpha$.

When the friction coefficient $\mu = 0.3$, the initial winding angle of body shape $\alpha = \pi/6$, the changes of the torques in joint 3, 7, 9, 13 are shown in Fig. 6. From Fig. 6 we can see: (1) the required torque for each joint in traveling wave locomotion is periodic, (2) torques in edge joint 3 and joint 13 are small, in joint 7 is larger and in central joint 9 is the biggest, that is because joint 3 and 13 have a longer distance from gravity center than joint 7 and joint 9. The total $n$ is 16, and the joint 9 has the nearest distance from the gravity center, thus
the biggest joint torque output of joint 9 is the biggest torque output of the snake robot. In a word, the input torque of each joint will decrease while the distance between the joint and the center of gravity of the body increases, and the biggest input torque is that of the central joint.

Variations of symmetrical joint torques can be seen from Fig. 7. Symmetrical joint 3 and 15, 7 and 11 have the same distance from the gravity center, and except a phase difference the amplitude variations of the input torques are the same.

Joint 9 is taken to study the effect of variation of the friction coefficient with the environment \(\mu\) on the joint torques. As seen in Fig. 8, when \(\alpha = \pi/6\), and \(\mu\) is chosen as 0.1, 0.3, 0.5 and 0.7 respectively, keeping the initial angle invariable, the joint input torques are increased with the increasing of friction coefficient with the environment. By the way, it can be found that there is difference of torque characteristics for \(\mu = 0.5\) with others. It is because that the friction coefficient is closely related to energy consumption in vertical locomotion. The average power consumption per unit distance \(E\) can be calculated using Eq. (45) as

\[
E = \sum_{i=1}^{n} \int_{0}^{T} |\tau_i/\omega_i|dt/L_{\text{dist}}
\]

\[(45)\]
in which $L_{\text{dist}}$ is the linear distance moved in a same period $T$. The calculating results when $\mu$ is specified as 0.1, 0.3, 0.5, 0.7 and 0.9, respectively are summarized in Table 1. It is can be found from Table 1 that the snake robot has the lowest energy consumption when $\mu = 0.5$, which makes its torque characteristic different from others.

Also joint 9 is taken to study the effect of variation of the initial winding angle on the joint torques. As seen in Fig. 9, when $\mu = 0.5$, and $\alpha$ is chosen as $\pi/12$, $\pi/6$, $\pi/4$ and $\pi/3$ respectively, keeping the friction coefficient with the environment invariable, the joint input torques are decreased with the increasing of the initial winding angle. So if friction coefficient with the environment is big, and the maximum joint torque exceeds the maximum input torque value of the motor, accordingly the initial winding angle can be increased to reduce the input torque required for locomotion.

![Fig. 8. Effect of environment on the joint torques.](image)

Table 1

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\mu = 0.1$</th>
<th>$\mu = 0.3$</th>
<th>$\mu = 0.5$</th>
<th>$\mu = 0.7$</th>
<th>$\mu = 0.9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$ [N m]</td>
<td>430</td>
<td>380</td>
<td>320</td>
<td>500</td>
<td>700</td>
</tr>
</tbody>
</table>

![Fig. 9. Effect of initial winding angle on the joint torques.](image)
6. Experiment of the traveling wave locomotion

6.1. Mechanical platform

To validate the simulation results a prototype snake robot is designed as shown in Fig. 10. The whole robot consists of nine modules and one head equipped with power and controller, each set with two one-DOF modules connected perpendicularly to form a two-DOF joint. Eight joints allow bending in two mutually orthogonal planes. Four joints with their axes lying in a vertical plane allow bending of the snake robot in the horizontal plane; the other four joints are devoted to bending in the vertical plane. From the combination of the rotation of these joints, three-dimensional motions can be generated.

6.2. Experimental validation

Some experiments were conducted under different environments. Fig. 10a shows the robot going through a narrow space on a flat carpet, where the friction coefficient is 0.3, the initial winding angle \( \alpha \) is 0.4 rad, the maximum joint output torque is 0.5 N m. Fig. 10b shows the robot climbing on the same carpet with a slope of 20\(^\circ\), which means increasing the friction coefficient to \( \mu = 0.7 \), on the same condition of the initial winding angle \( \alpha = 0.4 \) rad, the maximum joint output torque is increased to 0.82 N/m. This validates the simulation results that keeping the initial angle invariable, the joint input torques are increased with the increasing of friction coefficient. Fig. 10c shows the robot is crossing a gap. If the initial winding angle is kept as \( \alpha = 0.4 \) rad, the robot can not provide the output torque high enough to lift its head to reach the other edge of gap, but when the initial winding angle is increased to \( \alpha = 0.75 \) rad, the maximum width of gap that can be crossed is 0.14 m. This also is well in accordance with the simulation results that increasing the initial winding angle can reduce the maximum joint torque. Furthermore, a three-dimensional lateral rolling locomotion obtained through combination of the traveling wave locomotion and serpentine locomotion as shown in Fig. 10d.

Fig. 10. Traveling wave locomotion of snake robot (a) going through a narrow space, (b) climbing a slope, (c) crossing a gap, (d) combined lateral rolling locomotion.
7. Conclusion

In this paper starting with the model, simulation and finishing with experiment, a systematic complete description of traveling wave locomotion is presented. The dynamic effects and a detailed model of the friction between the mechanism and the ground are described, the relationship between the driving forces with the body shape and joint torques are established. According to the simulation, the required torque for each joint in traveling wave locomotion is periodic. The input torque of each joint will decrease while the distance between the joint and the center of gravity of the body increases, symmetrical joints based on the gravity center of the body have the same amplitude variation of input torques but with a phase difference, and the biggest input torque is that of the central joint. Furthermore, when keeping the initial winding angle unchanged, the bigger the friction coefficient with the environment is, the bigger is the joint input torque for each of joints. Whereas, while keeping the friction coefficient with the environment unchanged, the bigger the initial winding angle is, the smaller is the joint input torque for each of joints.

The traveling wave locomotion is achieved in a snake robot. The experimental results, which are well in accordance with the simulation results, show that the established kinematics and dynamics of the snake robot are reasonable. This paper can give some information for the further improvement of its locomotion efficiency and closed loop control.

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