Modular Universal Unit for a Snake-Like Robot and Reconfigurable Robots

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Abstract

A modular universal unit (MUU) is developed for snake-like robots, which has 3 d.o.f. with a series of passive rollers around its cylindrical shell. Among those d.o.f., pitching and yawing are actuated by means of differential gears to accomplish a large ratio of propulsion to mass. The series of passive rollers around the cylindrical aluminum shell of the MUU form another large wheel that can be used as a driving wheel of mobile robots. The snake-like robot composed of those MUUs has more powerful propulsion and higher mobility. By connecting MUUs in different forms, we can also realize a connected mobile platform or a manipulator in addition to a snake-like robot. Owing to it having 3 d.o.f., two or more MUUs can be connected to make up many mobile robots or form a manipulator that exhibits high mobility and agility. Some typical reconfigurable robots composed of these MUUs are analyzed for locomotion control. Finally, the locomotion experiments and simulations are given to show the characteristics of this MUU.

Keywords

Modular universal unit, snake-like robot, reconfigurable robot, kinematic model, locomotion

1. Introduction

Snake-like robots present more advantages than conventional mobile robots like legged robots and wheeled robots. For example, stability is always a crucial point for legged robots, but for snake-like robots it is not considered much because of their long slim body contacting the ground all along to keep the center of gravity (CG) lowest. Also, wheeled robots cannot work on rugged ground, in marshes and on sand, while snake-like robots can easily overcome all of above problems.
by changing gaits. Generally, snake-like robots are composed of modular units. Those modular units can be connected to form other reconfigurable robots. The reconfigurable robot usually consists of a variety of interconnected heterogeneous or homogenous robot modules. Those modules are highly integrated for control, sensing, communication and connection. Such robots can change their configuration and move from one place to another through reassembling the position of modules. Reconfigurable robots have various advantages compared with conventional robotic systems. Due to their environmental adaptation and self-repair ability, these robots show more potential applications in unstructured and hazardous environments, such as nuclear power stations, outer space and collapsed buildings.

Many works related to snake-like robots have been published since Hirose’s group built the first snake-like robot using the so-called active cord mechanism (ACM) in the 1970s [1]. The newest snake-like robot by ACM is the ACM-R5, which can perform more three-dimensional (3-D) locomotion [2]. Chirikjian and Burdick developed a 30-d.o.f. hyper-redundant robot with a parallel mechanism [3]. Shan and Koren designed the Michigan snake robot and analyzed the quasi-static locomotion with some links fixed on the ground [4]. Klaassen and Paap developed the GMD-Snake robot by using driving wheels as the snake belly to enhance propulsion [5]. NEC Corporation developed a 12-d.o.f. snake-like robot Orochi with oblique joints [6]. Nilsson has developed a universal serpentine link that is a roll-pitch-roll joint [7]. Ma has also performed many works on snake-like robots, which include the development of two types of snake-like robots [8, 12] and the analysis of locomotion [9–12]. The Omnitread serpentine robot recently developed at the University of Michigan comprises multiple rigid segments with a driving track connected by actuated joints. This robot has strong propulsion and good mobility [15]. On the other hand, reconfigurable robots have attracted many researchers and lots of robot systems have been developed [16–24]. ATRON is latticed-based and is able to self-reconfigure in 3-D environments, but it has limited mobility in unstructured environments due to the constricted motion [16]. The hardware prototypes of modules are able to self-reconfigure, including MTRAN [17], 3D-unit [18], Molecule [19] and I-cubes [20]. Some related motion control and planning works are studied relating to efficient reconfiguration and locomotion [19, 24]. There are also various related works that can be found in Ref. [16].

In this paper, we present a modular universal unit (MUU) with 3-d.o.f. that has been developed for snake-like robots and reconfigurable robots. This MUU equipped with a series of free rollers around its cylindrical shell can be used as a wheel of mobile robots beside being used as a driving joint, which expands the potential applications of the MUU. Figure 1 shows the concept of a snake-like robot: a two-unit connection vehicle, a four-wheel driven omni-directional vehicle and a manipulator. These reconfigurable robots composed of MUUs show the broad applications of this MUU.
2. Considerations on the Joint Mechanism of Snake-Like Robots

Snake-like robots were mostly developed with modular units and a simple sequentially connected structure in order to save design and manufacturing costs. The modular design makes module replacement and repair convenient. Several types of modular units with 1 d.o.f. [11, 12] have been designed by our group for the development of the snake-like robots shown in Fig. 2. The number of d.o.f. and shell shape of the module, which are very important to the movement of robot, have not been considered in detail. In this section, we discuss the d.o.f. number for module design and analyze the shell shape for better performance of 3-D snake-like robots.

2.1. Degrees of Freedom of a Unit for Snake-Like Robots

Each type of modular unit has its own characteristics in terms of mechanism or control. A module with 1 d.o.f. is the simplest one in terms of mechanism to construct a snake-like robot through connecting the modules in series. However, for a 3-D snake-like robot consisting of 1-d.o.f. modules, there exists an offset between the pitch axis and yaw axis, which easily leads to motion singularities. Moreover, the ratio of output torque to weight of the modular unit is limited by the actuator performance, even though some improvements were made on it. A module with 2 d.o.f., which generally are 2 rotational d.o.f. including a yaw d.o.f. and a pitch
d.o.f., provides the chance to enlarge the ratio of output torque to weight through a coupled drive mechanism. Moreover, the locomotion singularity could be conditionally avoided by intersecting two axes at one point. For example, Hirose et al. developed the ACM-R2 driven by a coupled drive of wires [25]. In terms of agility and propulsion, the 2-d.o.f. module is better than the 1-d.o.f. module for improving the performance of snake-like robots.

Next, we consider a module with 3-d.o.f. Usually, the 2-d.o.f. module provides the snake-like robot with pitch d.o.f. and yaw d.o.f. to enable it to move in 3-D environments. For the 3-d.o.f. module, the roll d.o.f. is added. The reason we add the roll d.o.f. is that the ball joint of the backbone and the multi-directional muscle generate freely a 3-D motion for natural snakes. In experiments, we have also seen that both the sidewinding motion and the sinus-lifting locomotion of snakes need the roll d.o.f. to adjust their posture. The roll d.o.f. is also necessary to keep the belly of snake always in contact with the ground for efficient movement. Therefore, we conclude that the roll d.o.f. can enlarge the environmental adaptability of snake-like robots. The 3-d.o.f. module for snake-like robots is an appropriate one, even though it would increase the mass and bulk of a module. There is always a trade-off between the addition of third actuation and the increase of the ratio of output torque to weight, which should be efficiently chosen according to the application. In this paper, we investigate 3-d.o.f. module for future possible applications.

2.2. Shell Shape of a Unit for 3-D Snake-Like Robots

The shell shape of snake-like robots has not received much attention, because they are designed to move on the flat with only one surface contacting the ground. In experiments, we find that when the environment is changed or when the environment becomes complicated, the 3-D snake-like robot had better use of the roll d.o.f. to move or adapt to the environment. When the robot needs to roll its unit, the shell shape of the unit will affect the contact surface of other units, as shown in Fig. 3. The rolling motion of the square unit leads to the adjacent units leaving the ground, which affects the motion of the whole snake-like robot. The cylindrical

![Figure 3. Shell shape of 3-D snake-like robots.](image-url)
shape makes all units of the snake-like robot keep position and contact even in the rolling motion. There is no change in the height of the CG in the rolling motion of the cylindrical unit, which saves a great deal of energy for efficient locomotion. As a result, we conclude that the cylindrical shell shape of the unit is the best choice for 3-D snake-like robots. This cylindrical shell shape of the unit combined with the roll d.o.f. provides the snake-like robot with redundancy and much mobility.

3. MUU

A 3-d.o.f. module with a cylindrical shell shape has been developed for realization of a snake-like robot called ‘Perambulator-II’. This enhances the snake-like robot in terms of environmental adaptation and locomotion mobility. In this section, we will describe the MUU design and its mechanism.

3.1. Joint Design of the MUU

The MUU is developed with pitch, yaw and roll d.o.f. Three Futaba S3305 DC servomotors with high torque output have been selected as joint actuators in this unit. The pitch and yaw motions are coupled drives through the differential mechanism actuated by two motors, as shown in Fig. 4a. The differential mechanism consists of three bevel gears [12]. This joint unit with a differential mechanism can lift up two units. A reduction ratio of 2:1 is utilized between the motor and the bevel gears. Potentiometers shown in Fig. 4a are fixed on the axes of each d.o.f. to detect the joint angles for the control. Roll motion is generated by another servomotor through a 2:1 reduction radio. A Panasonic lithium battery in the unit supplies power to these three motors. The control module composed of a 16-bit processor manages its own control information. A CAN bus is chosen to communicate among the units. The outer shell of this joint unit is an aluminum cylinder, on which a series of passive rollers are fixed to obtain the necessary frictional properties. This unit can thus slip easily in the direction along the axis of the cylindrical body and has difficulty slipping in the normal direction orthogonal to the body. The cylinder can be used as a large driven wheel to compensate for the lack of propulsion resulting from passive rollers, which makes the robot perform possibly holonomic motion and non-holonomic motion. The motion singularities due to non-holonomic constraints can be avoided. The motion ranges of the pitch, yaw and roll d.o.f. are from $-90^\circ$ to $90^\circ$. All three axes intersect at one point to avoid the singularity, as shown in Fig. 4c. Here, $\alpha$, $\beta$ and $\gamma$ are three variables of these three axes, respectively. The specifications of the MUU are listed in Table 1. Its outer dimensions are 172 mm (long) × 120 mm (circumferential) and its mass is about 1 kg. The hardware of the MUU is shown in Fig. 4d.

3.2. Body Design of the MUU

The body wheel is designed only for contact with the ground to produce effective propulsion. This contact is a crucial property for snake-like robots to perform serpentine locomotion like natural snakes, which has shown that the contact supplied
impetus for the whole body. The passive rollers installed around the body avoid interference between snake body and obstacles to minimize the frictional force. These passive free rollers also form a body wheel to realize contact. The free rollers make the body easy to slide in the tangential direction and difficult to slide in the normal direction. The shape of the roller is a spindle, which thus makes the formation of the body wheel, whose radius is equal to the radius of the spindle in the ver-

### Table 1. Specifications of the MUU

<table>
<thead>
<tr>
<th>D.o.f.</th>
<th>3 (pitch, yaw, roll)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>Futaba S3305</td>
</tr>
<tr>
<td>Dimensions (mm$^3$)</td>
<td>172 × 120 (circumferential)</td>
</tr>
<tr>
<td>Joint workspace (°)</td>
<td>−90 to 90 (pitch, yaw and roll)</td>
</tr>
<tr>
<td>Joint mass (kg)</td>
<td>1.0</td>
</tr>
<tr>
<td>Torque (Nm)</td>
<td>3.56 (pitch, yaw), 1.78 (roll)</td>
</tr>
<tr>
<td>Joint angular velocity (°/s)</td>
<td>150 (pitch, yaw, roll)</td>
</tr>
</tbody>
</table>
tical section as shown in Fig. 5. This body wheel is actuated by the roll actuator of the MUU to make the series of passive rollers form a large driving wheel. In this case, the roll d.o.f. is actuated continuously through changing position control to velocity control. The settings of the rollers enable each part of the whole body to possibly contact the ground for environment adaptation and body wheel actuation. The dimensions of the roller shown in Fig. 5 are considered for continuous rotation of the body wheel and the continuous contact of the roller while contacting the ground. All rollers are embedded in the shell to minimize the unit’s outer dimensions. The roll d.o.f. actuates body wheel rotation, while the force acts on the body in the normal direction, as shown in Fig. 5. The forces in other directions are released by free rollers. From this point of view, this body wheel is an omnidirectional wheel, which benefits the locomotion of robots. All passive rollers are arranged on the middle of the line from the pitch axis to the next pitch axis, as shown in Fig. 6. This location of the rollers mainly comes from the research in Ref. [8], which benefits the serpentine locomotion and robot modeling. The body wheel rotation cannot only be actuated by the roll d.o.f., but can also be actuated by the combination of the pitch d.o.f. and yaw d.o.f. This is an important characteristic of the MUU for the locomotion of a reconfigurable robot. When the pitch d.o.f. is actuated to 90° as shown in Fig. 7, the yaw axis coincides with the axis of the body wheel. Consequently, after the L-shape edge is fixed, the body wheel will be actuated to rotate around the yaw axis by driving the yaw d.o.f. At this configura-
tion, the yaw d.o.f. is also actuated to realize continuous rotation through changing the position control to velocity control for the body wheel to be used as a driving wheel of the vehicle, which increases the motion performance of reconfigurable robots. In the future, position control in continuous rotation will be realized on this unit.

3.3. Decoupling of Input Variables

Each MUU shows 3 d.o.f., among which the pitch d.o.f. and yaw d.o.f. are driven in coupling, while the roll d.o.f. leads to another motion. All three inputs, thus, must be resolved first for generating the rotation of the pitch, yaw and roll d.o.f. The decoupling of three input variables is given as:

$$\begin{bmatrix}
\alpha \\
\beta \\
\gamma
\end{bmatrix} = \frac{1}{4} \begin{bmatrix}
1 & -1 & 0 \\
1 & 1 & 0 \\
0 & 0 & 2
\end{bmatrix} \begin{bmatrix}
\nu_1 \\
\nu_2 \\
\nu_3
\end{bmatrix},$$

(1)

where $\alpha$, $\beta$ and $\gamma$ are the angles to define the unit posture (pitch, yaw and roll) as shown in Fig. 4c, and $\nu_1$, $\nu_2$ and $\nu_3$ are the output angles of the servomotors, respectively. The reduction ratio 2:1 has been considered in this equation.

When the roll d.o.f. is actuated by the servomotor, it will lead the accessional motion of the pitch axis and yaw axis, i.e., the variables $\alpha$ and $\beta$ change their position, which changes the link position. As shown in Fig. 8, the ellipse is perpendicular

![Figure 7. Body wheel configuration.](image)

![Figure 8. Accessional motion led by rolling.](image)
to the roll axis at the intersection of the three axes. When the roll axis rotates an angle $\gamma$, the yaw and pitch axes will rotate the same angle around the roll axis. Link OB will rotate from the initial position B to the final position B' around the roll axis with the same angle. This accessional motion led by the roll d.o.f. must be geometrically counteracted by actuating the yaw and pitch d.o.f. to maintain the robot posture. Therefore, the variables of yaw and pitch are resolved to keep the link at the initial position. It is expressed as follows [12]:

$$\alpha = \arctg(-\sin \gamma \tan \zeta) \quad \beta = \arctg(-\sin \alpha \cot \zeta),$$

(2)

where $\zeta = \angle AOB$ is the joint bend angle resulting from the combined action of the initial $\alpha$ and $\beta$.

From (1) and (2), we can decouple the coupled motion between servomotor inputs and accessional motion comes from the roll d.o.f.

4. Snake-Like Robot Realized by MUUs

A snake-like robot ‘Perambulator-II’ has been realized by the developed MUUs. In this section, we discuss the system of this 3-D snake-like robot ‘Perambulator-II’, and adopt the serpentine motion and twist locomotion on this robot.

4.1. System of the 3-D Snake-Like Robot

Our snake-like robot shown in Fig. 9 consists of seven joint units, which are chained in series. It exhibits mobility and dexterity with a total of 21 d.o.f. The video camera equipped for capturing the video information and GPS for detecting the position of snake are fixed in the head of the robot. The control structure is shown in Fig. 10. This snake-like robot system adopts a distributed control method. A central computer sends a control order to the snake head. Then the head sends its control information to each MUU through the CAN bus. Finally, each MUU processes its information to drive the DC motors and sends the feedback information to the central computer. The specifications of the snake-like robot are listed in Table 2.

Figure 9. A snake-like robot ‘Perambulator-II’.
4.2. Generation of Serpentine Locomotion

Serpentine locomotion is the most efficient locomotion among all types of locomotion observed from natural snakes. The serpenoid curve has been popularly used as the body wave of snake-like robots. Our snake-like robot ‘Perambulator-II’ can also generate serpentine locomotion, while using the serpenoid curve as the snake head tracking path. The serpenoid curve is given by the curvature function:

\[ \kappa(s) = -\frac{2K_n\pi\delta_0}{L} \sin\left(\frac{2K_n\pi}{L}s_p\right) \],

(3)

where \( K_n \) gives the number of the S-shape, \( \delta_0 \) is the initial winding angle, \( L \) is the whole length of one cycle, \( s_p \) is the body length along the serpenoid curve and \( s \) is the displacement of the tail along the serpenoid curve path, respectively. This can realize the serpentine locomotion, which has been validated in Ref. [9]. In the serpentine locomotion, the input variables are expressed as:

\[ \alpha_i = 0 \]
\[ \beta_i = -2\delta_0 \sin(K_n\pi/n) \sin(2K_n\pi s/L + 2K_n\pi i/L) \]
\[ \gamma_i = 0, \]

(4)

where \( \alpha_i, \beta_i \) and \( \gamma_i \) express the variables \( \alpha, \beta \) and \( \gamma \) of the \( i \)th joint, \( i = 1, 2, \ldots, n - 1 \), respectively.

4.3. Generation of Twist Locomotion

We know that a flexible cord in a plane can be driven sidewise with the same shape by rolling it with a hand to change the curvature of each side face. Similarly, for a snake-like robot, by driving the curvature of the snake-like robot periodically according to the curvature of the flexible cord, the flexible body can move sidewise.
The twist locomotion needs 2 d.o.f. in the perpendicular direction to drive the links in coupled action, which requires the snake-like robot with 3-D motion ability. The input function of twist locomotion is expressed as:

\[
\begin{align*}
\alpha_i &= -\xi \sin(\nu) \\
\beta_i &= \xi \cos(\nu) \\
\gamma_i &= 0,
\end{align*}
\]

where \(\xi\) is the bend angle of snake-like robots and \(\nu\) is the absolute angle of body rotation related to the ground. This function was presented in Ref. [13], which shows the detailed derivation of twist locomotion.

5. Reconfigurable Robots Using MUUs

The developed MUU is mainly for a 3-D snake-like robot and a modular manipulator. Of course, as a modular unit, it can form some other desired configurations. If we add the connection mechanism and some sensors onto the MUU, it can connect or disconnect to another robot to complete self-repair or self-reconfiguration, which will be studied in future works. Here, we just show some typical reconfigurable robots generated by the MUUs. Generally, a reconfigurable robot needs many modular units to compose a configuration or change its configuration. This self-reconfiguration ability is mainly decided by the locomotion performance of the MUUs. Two or more MUUs can be connected to move under control, which makes MUUs convenient to reform the configuration. The kinematics is analyzed for assurance of the controllability of MUUs including the two-, three- and four-MUU combinations.

5.1. Two-MUU Motion

The vehicle with two MUUs can move like a car with a minimum turn radius, which can be controlled to move from point to point. In this case, one MUU is used as a drive wheel and another MUU is used as a caster. Two MUUs have three contact points to obtain stable posture. From Fig. 11, we can see that both the passive roller and the cylinder edge of the rolling MUU are in contact with the ground, which prevents any sliding along the body. Otherwise, the two-MUU connection can be

Figure 11. Car-like movement of two MUUs.
free around a point. For example, if there is no contact edge between the MUU and ground to prevent sliding along the body, the connection vehicle will rotate freely around \( p \) point in Fig. 12. From Fig. 12, we have the model of the two-MUU vehicle:

Geometric constraints:

\[
\begin{align*}
x_2 &= x_1 + l_1 C(\theta) + l_2 C(\theta + \beta_2) \\
y_2 &= y_1 + l_1 S(\theta) + l_2 S(\theta + \beta_2).
\end{align*}
\]  

Velocity constraints:

\[
\begin{align*}
\dot{x}_1 S(\theta) - \dot{y}_1 C(\theta) &= 0 \\
\dot{x}_2 S(\theta + \beta_2) - \dot{y}_2 C(\theta + \beta_2) &= 0.
\end{align*}
\]  

Therefore, the basic equation for locomotion control can be written as:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{y}_1 \\
\dot{\theta}
\end{bmatrix} =
\begin{bmatrix}
C(\theta) & S(\theta) \\
S(\theta) & S(\beta_2) \\
l_1 C(\beta_2) + l_2 & 0
\end{bmatrix}
\begin{bmatrix}
R \dot{\beta}_1 \\
0 \\
l_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
l_1 C(\beta_2) + l_2
\end{bmatrix} \dot{\beta}_2,
\]

where \((x_i, y_i)\) are the coordinates of a reference point on the MUU, \( R \) is the radius of this kind of mobile robot, subscript \( i \) means the unit number and \( \theta \) is its orientation. \( C(\bullet) := \cos(\bullet), S(\bullet) := \sin(\bullet) \) and \( l_1 \) and \( l_2 \) express the distance between rotational axis of the drive wheel and the joint of caster, the distance between the joint and the passive roller of caster, respectively. From (8), we can find that the two-MUU vehicle is controllable, and through controlling the variables \( \dot{\beta}_1 \) and \( \dot{\beta}_2 \), the vehicle can be driven to anywhere. This car-like configuration is very useful to complete movement and reconfiguration.

Although the MUU has a large workspace to minimize the turn radius, a singularity of the two-MUU vehicle would appear at the condition of the minimum turn
radius. As shown in Fig. 13, the direction of movement is not under control any-
more when the angle $\beta_2$ of the rotation joint is set at $90^\circ$, because the directions
of the caster and the drive are perpendicular in the parallel configuration. In fact,
because the body wheel cannot manage the relation of contact edge rotation speed
and body wheel rotation speed, there is a slip between the MUU and the ground,
which will be more obvious with increasing the turn angle of the joint. Therefore, in
order to guarantee the performance of movement, we set the joint angle ranges from
$-45^\circ$ to $45^\circ$. When two MUUs are at the singularity of turn motion, the connec-
tion vehicle can shift its locomotion method from one drive wheel and one caster to
two drive wheels, as shown in Fig. 14. There are still three contact points including
passive rollers and one contact edge to support the vehicle on the ground, which
prevent slipping along the body. When two MUUs are driven to rotate in the same
direction at the same speed, the vehicle would move forward. When two MUUs are
driven to rotate in opposite directions at the same speed, the vehicle would move
around the geometrical center $P$. Generally, the vehicle would move around a point
with a turn radius. We, thus, conclude that the connection of two MUUs can be
controlled to move to any position.

5.2. Three-MUU Locomotion

The body wheel of the MUU can propel sidewise while the passive rollers remove
the frictional force along its body. If three MUUs are connected through a platform,
an omni-directional mobile robot can be developed, as shown in Fig. 15. In Fig. 15, each MUU would be used as a universal wheel, which releases the frictional force
along the rotational axis and supplies actuation force normal to the rotational axis.
Figure 15. Omni-directional mobile robot consisting of three MUUs.

Figure 16. Geometrical model of the omni-directional mobile robot.

For each body wheel shown in Fig. 16, the geometric constraint is expressed as:

\[
\begin{align*}
x_i &= x_v + L_i C(\delta_i + \theta_v) \\
y_i &= y_v + L_i S(\delta_i + \theta_v),
\end{align*}
\]

(9)

where \((x_i, y_i)\) is the center of each body wheel, \((x_v, y_v, \theta_v)\) is the coordination of the omni-directional robot, and \(L_i, \sigma_i, \epsilon_i\) and \(\delta_i\) are the geometric constants, as shown in Fig. 16. \(S(\bullet)\) and \(C(\bullet)\) denote \(\sin(\bullet)\) and \(\cos(\bullet)\), respectively.

The velocity constraint of each body wheel is given by:

\[
\dot{x}_i C(\epsilon_i) + \dot{y}_i S(\epsilon_i) = R \dot{\beta}_i.
\]

(10)

The completed model of the three-wheeled robot can then be written as:

\[
J \dot{\omega} = R \dot{\beta},
\]

(11)

where:

\[
J = \begin{bmatrix}
C(\epsilon_1) & S(\epsilon_1) & -L_1 S(\sigma_1) \\
C(\epsilon_2) & S(\epsilon_2) & -L_2 S(\sigma_2) \\
C(\epsilon_3) & S(\epsilon_3) & -L_3 S(\sigma_3)
\end{bmatrix},
\]

\[
\omega = [x_v \ y_v \ \theta_v]^T, \quad \beta = [\beta_1 \ \beta_2 \ \beta_3]^T.
\]
Equation (11) shows that the wheeled robot can be driven in any direction by actuating three body wheels.

The omni-directional wheeled robot enlightens us to configure three MUUs and a platform into another form, as shown in Fig. 17. In this form, the MUU can swing its body to propel the vehicle to move. In order to control this vehicle, the kinematic model is built. As shown in Fig. 18, if each one of three wheels undulates, the motion of the passive rollers around the body is constrained in the direction normal to the MUU’s body. First, the geometric constraint is given as:

\[
\begin{align*}
    x_i &= x_v + RC(\vartheta_i) + LC(\theta_i) \\
    y_i &= y_v + RS(\vartheta_i) + LS(\theta_i),
\end{align*}
\]  

(12)

where \( \theta_i = \vartheta_i + \beta_i \).

Then, the velocity constraint from the passive rollers is written as:

\[
\begin{align*}
    \dot{x}_iS(\theta_i) - \dot{y}_iC(\theta_i) &= 0.
\end{align*}
\]  

(13)

Thus, from (12) and (13), we have a function:

\[
A\dot{\omega} = B\dot{\beta},
\]  

(14)
where $L$ is a geometrical constant and:

$$
A = \begin{bmatrix}
S(\theta_1) & -C(\theta_1) & -L - RC(\beta_1) \\
S(\theta_2) & -C(\theta_2) & -L - RC(\beta_2) \\
S(\theta_3) & -C(\theta_3) & -L - RC(\beta_3)
\end{bmatrix}, \quad
B = \begin{bmatrix}
L & 0 & 0 \\
0 & L & 0 \\
0 & 0 & L
\end{bmatrix},
$$

$$
\omega = [x_v, y_v, \theta_v]^T, \quad \beta = [\beta_1, \beta_2, \beta_3]^T.
$$

Equation (14) shows that the robot with three undulatory units is controllable.

### 5.3. Four-MUU Locomotion

Finally, the developed MUU can be used as the driving wheels of a four-wheel-driven vehicle, which is an omni-directional vehicle. As shown in Fig. 19, the vehicle with four supporting wheels can be driven by four wheels to perform omni-directional motions. The kinematic model to show the omni-directional movement is described as follows.

As shown in Fig. 20, the geometric constraint of MUU 1 is given by:

$$
x_1 = x_v + l_1 C(\theta_v) + l_2 S(\theta_v)
$$

$$
y_1 = y_v + l_1 S(\theta_v) - l_2 C(\theta_v),
$$

(15)
and its velocity constraint is given by:

$$\dot{x}_1 C(\theta_1) + \dot{y}_1 S(\theta_1) = R \dot{\beta}_1.$$  \hspace{1cm} (16)

Similarly, we can obtain the constraint function of other MUUs. The function of the four-wheel-driven vehicle is thus given by:

$$\begin{bmatrix}
C(\theta_1) & S(\theta_1) & a_1 C(\theta_1) + b_1 S(\theta_1) \\
C(\theta_2) & S(\theta_2) & a_2 C(\theta_2) + b_2 S(\theta_2) \\
C(\theta_3) & S(\theta_3) & a_3 C(\theta_3) + b_3 S(\theta_3) \\
C(\theta_4) & S(\theta_4) & a_4 C(\theta_4) + b_4 S(\theta_4)
\end{bmatrix}
\begin{bmatrix}
\dot{x}_v \\
\dot{y}_v \\
\dot{\theta}_v
\end{bmatrix} = R
\begin{bmatrix}
\dot{\beta}_1 \\
\dot{\beta}_2 \\
\dot{\beta}_3 \\
\dot{\beta}_4
\end{bmatrix},$$  \hspace{1cm} (17)

where $a_i$ and $b_i$ are the wheel position parameters that are related to parameters $l_1$ and $l_2$, $\theta_i$ and $\dot{\beta}_i$ are the angle parameters of the vehicle body and the angular velocity of the wheels, respectively, subscript $i$ describes the wheel number, $(x_v, y_v, \theta_v)$ are the vehicle body position and posture, respectively, and $R$ is the radius of body wheel.

From (17), we can see that the system is over-constrained because three wheels can make the vehicle controllable. The fourth wheel is just added for the stability of vehicle, whose motion must be in harmony with the motion of the other three wheels. It is known from (17) that this vehicle can perform omni-directional motion and can move along the $x$- or $y$-axis and around its center.

From the above analyses, we know that the vehicle with four wheels is a controllable omni-directional robot. The body wheel cannot only rotate around the yaw axis, but can also swing a cone path, which enables the vehicle to get over some obstacles, as shown in Fig. 19. In this case, the MUU can be regarded as the leg of the robot, which can lift up to get over obstacles to drive the vehicle.

6. Experiments

To validate the mobility of the developed MUU, we have executed some experiments of the robots consisting of these MUUs.

6.1. Snake-Like Robot

Some basic experiments have been performed on the snake-like robot ‘Perambulator-II’. The serpentine locomotion and the twist locomotion as well as the slope-climbing locomotion have been realized to validate the mobility of this robot.

Figure 21 shows an example of serpentine locomotion of the snake-like robot while $\delta_0 = -\pi/6$, $K_n = 1$, $L = 1.2$ m and $\dot{s} = 0.1$ m/s. The polymer material of the roller gives the robot enough frictional force. The side-slide of the roller, which has always appeared in other snake-like robots due to lack of a supporting force, does not appear on ‘Perambulator-II’. The snake-like robot can move at a velocity about 0.1 m/s.

The twist locomotion is a practical movement, by which the snake-like robot can move fast and climb a slope of 15°. Figures 22 and 23 show the results when
Figure 21. Serpentine locomotion of ‘Perambulator-II’.

Figure 22. Twist locomotion of ‘Perambulator-II’.

Figure 23. Twist locomotion on a slope of ‘Perambulator-II’.

\[ \xi = \frac{\pi}{7} \text{ and } \dot{\upsilon} = 0.8 \text{ rad/s.} \]

In the twist locomotion, all passive rollers on the unit drive the MUU in the forward direction and the sidewise frictional force is released by the free rollers.

6.2. Two-MUU Vehicle

As a 3-d.o.f. unit, the MUU exhibits many characteristics of locomotion. As basic unit of reconfigurable robots, one MUU is not controllable, because the motion direction of the MUU is affected by the fixed edge that is contacting ground. Under some conditions, the fixed edge will rotate around the body axis that makes the MUU stop. A two-MUU vehicle can move to any position like a car, as shown in Fig. 24. In the parallel configuration, two MUUs can be used as drive wheels to complete the differential drive, as shown in Fig. 25.

6.3. Three-MUU Vehicle

As an omni-directional mobile robot, the three-wheeled platform can move anywhere with an assigned angle. From (11), we can obtain the angle information to control the MUU to rotate. Figure 26 shows the simulation results where a three-
wheeled robot moves in different ways. In Fig. 26a, the robot moves with the velocity $(1, 0, 0)$ while MUU 1 and MUU 2 rotate with the same angle. In Fig. 26b, the robot moves with velocity $(0, 1, 0)$ while MUU2 does not rotate. When the platform rotates at the origin with the velocity $(0, 0, 1)$ in Fig. 26c, all three MUUs rotate with the same angle. Moving with the velocity $(1, 1, 1)$, three MUUs rotate with different angles. In an undulatory form, three MUUs will converge to a singularity and make the robot uncontrollable. To overcome the singularity, the angle of the mobile robot is set to change according to an undulatory function such as a sine or cosine function. Figure 27 shows that the mobile robot moves without singularity. All three MUUs are undulatory to move to the desired position with avoidance of $\det(A)$ equal to zero. In this condition, $\theta_v = \sin(2t), r = 0.06$ m,
Figure 27. Simulation results of a three-linked robot.

Figure 28. Motion of a three-MUU robot.

Figure 29. Turning motion of a four-wheel-driven vehicle.

$L_1 = L_2 = L_3 = 0.1$ m, origin $(0, 0, 0)$ and velocity $(0.5, 0.5, \theta_v)$. Figure 28 shows the movement of a robot consisting of three-wheeled MUUs.

6.4. Four-MUU Vehicle

In Fig. 29, a four-wheel-driven vehicle can turn around its body center. Figure 30 shows that the vehicle can move in a given direction. All experiments show that the MUU has some advantages in terms of agility, mobility and reconfigurability.

7. Conclusions

We have described a unit named MUU for a snake-like robot and reconfigurable robots. This MUU has 3 d.o.f., among which the pitch and yaw d.o.f. are driven by
a differential mechanism. There are many configurations that the MUUs can form, such as a snake-like robot, a car-like robot, a three-unit vehicle and a four-wheeled vehicle. The mathematic models of some reconfigurable robots consisting of MUUs are given for control. Experiments and simulations have been performed to validate the mobility of the MUU and its agility.

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References


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