Reliability Analysis of Kinematic Accuracy of a Three Degree-of-Freedom Parallel Manipulator

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\textbf{Abstract}. The reliability of kinematic trajectory of manipulators describes the ability that manipulators keep kinematic accurate. It is an important parameter to evaluate the performance of manipulators. The kinematic accuracy of manipulators can be improved when piezoelectric material are used as a transducer to suppress the vibration of flexible manipulators. First, a 3 degree-of-freedom parallel manipulator system and its dynamic equations are introduced. The theory and experiment of a vibration suppression system are then presented. The calculation method of both error and reliability of kinematic trajectory of manipulator is further implemented. Finally, the reliability of kinematic accuracy are calculated and analyzed for the 3 degree-of-freedom parallel manipulator with or without vibration suppressing control. The results show that the reliability of kinematic accuracy is improved using vibration suppressing control.

\textbf{Introduction}

The reliability of kinematic accuracy of manipulators is an important indicator to evaluate the accuracy of manipulator motion [1]. In manipulators, light weight linkages are employed to achieve high speed and acceleration motions for better performance. However, the light weight linkage will result in inherent structural vibration, and the structural vibration leads to inaccurate kinematic trajectory of manipulators.

Different methods have been proposed to reduce the vibration of the flexible linkages [2-3]. Over the past decade, the “smart” structure has been widely used in the vibration control of flexible manipulators. The piezoelectric materials have a various applications such as smart sensors and actuators in vibration control. The most popular piezoelectric materials are lead-zirconatetitanate (PZT). PZT requires lower actuation voltages, and can be applied to a wide range of frequency. So it is extensively used both as sensors and actuators. Strain rate feedback (SRF) control strategy is a feedback method. It has a much wider active damping frequency region, and therefore possesses bigger frequency range of vibration suppression [4]. The vibration of flexible manipulators can be efficiently suppressed when PZT active vibration control system is used. Therefore, the accuracy of the kinematic trajectory of the manipulator is improved.

Inspired the above issues and works, this paper focuses on the reliability of kinematic accuracy of manipulators. It is organized as follows. Section 2 shows the coordinate system and the dynamic model of a three degree-of-freedom parallel manipulator with three flexible linkages. The dynamic equations are given as well. The theory and experiment of PZT vibration control are presented in Section 3. Section 4 addresses the reliability of kinematic accuracy of the manipulator. Finally, conclusions are discussed in Section 5.
Dynamic Model of Manipulator

A three degree-of-freedom (3DOF) parallel manipulator (shown in Fig. 1) is composed of three symmetrical closed-loop linkages, each of which consists of one prismatic joint and two revolute joints. Light weight linkages are used for better performance with high speed and high acceleration motion [5]. However, light weight linkages are more likely to vibrate, and its accuracy of kinematic trajectory is lower.

![Fig. 1 The 3DOF parallel manipulator](image1)

The architecture and coordinate system of the manipulator with underformed and deformed linkages are shown in Fig. 2. The position of the center point of the moving coordinate system is located at P. Point O is the origin of the fixed coordinate system. A DC brushless servo motor and linear slider guide mechanism is used as the actuator for each active prismatic joint located at point \( A_i \), \( i = 1, 2, 3 \). The center of each linear slider guide is the origin point of \( \rho_i \). The moving platform has the shape of an equilateral triangle (\( C_1, C_2, C_3 \)). The variable \( F_{ai} \) is defined as the driving force.

Using Lagrange’s method, the dynamic equations of motion for the 3DOF system is given as

\[ M\ddot{\mathbf{q}} + K\ddot{\mathbf{q}} = -M\dddot{\mathbf{q}} - M\mathbf{\beta}\dddot{\mathbf{q}} + F_k, \]

where \( M \) is the modal mass matrix of the system, \( K \) is the structural modal stiffness matrix, \( F_k \) is the modal force, and \( -M\mathbf{\beta}\dddot{\mathbf{q}} \) reflects the effect of elastic vibration of the flexible linkages due to the rigid body motion.

Theory and Experiment PZT Vibration Suppression

PZT materials can be used as sensors to measure the vibration of flexible linkages, and they can be used as actuators to suppress the vibration of linkages too, due to their direct and converse piezoelectric effect [6]. The input of the feedback controller is determined according to the output of vibrations measured from PZT sensors in the feedback path. SRF has a wider active damping frequency range, and hence can stabilize multiple modes simultaneously. Moreover, the strain rate is readily available in practice through differentiation of voltages obtained from PZT sensors.

A smart beam is constructed by bonding PZT patches to the two opposite sides of the linkage. The PZT patches on one side of the beam are used as actuators, while the PZT patches on the opposite side are used as sensors. One actuator and one sensor constitute a PZT control pair, and are located at the same location along the beam.
The strains in a PZT sensor generate a charge signal according to the direct piezoelectric effect. A charge signal is converted to a voltage signal when an impedance converter is used \cite{7}. Therefore, the voltage produced by the sensor located at \( x_k \) on the \( i^{th} \) linkage can be expressed as

\[
V_{si}^k = k_i^k \left( \frac{\partial w_e}{\partial x^2} (x_k, t) \right) = \frac{E_p d_{33} w_p}{C_p} (h_p + h_i) \sum_{j=1}^{n} \eta_j(t) \psi_j^e(x_k), \tag{2}
\]

where \( k_i^k \) is the sensor constant, \( E_p \) is Young’s modulus of PZT materials, \( d_{33} \) is the piezoelectric constant, \( w_p \) is the width of the PZT sensor, \( h_i \) is the half height of the linkage, \( h_p \) is the height of the PZT sensor, and \( C_p \) is the capacitance of the sensor.

A strain is produced in the PZT actuator based on the converse piezoelectric effect when apply a voltage on a PZT actuator. Therefore, the action resulting from the converse piezoelectric effect is represented by two concentrated bending moments applied at both ends of the actuator.

Using SRF control strategy, the derivative of a voltage from a PZT sensor is multiplied by a negative control gain \(-k_i^k\) and fed back to the corresponding actuator. Therefore, the voltage applied on the actuator is obtained as

\[
V_{ai} = -k_i^k V_{si} = -k_i^k k_i^k \sum_{j=1}^{n} \eta_j(t) \psi_j^e(x_k). \tag{3}
\]

![Fig. 3 Responses of linkage 3 at the quarter point](image)

To the experiment of the 3DOF manipulator with PZT vibration control, three PZT sensors and actuators are bonded to every linkage at its quarter point, midpoint and three-quarter point, respectively. The actuators and sensors are selected as BM532, and \( d_{33} = -270 \times 10^{-12} \text{ C/N} \), \( E_p = 6.3 \times 10^{10} \text{ N/m}^2 \). The vibration responses of linkage 3 (B3C3) with and without vibration control at the quarter point are shown in Fig. 3. It shows that the vibration is reduced significantly when PZT vibration control is used. The position error of the moving platform can be calculated by measuring output voltages of the PZT sensors. The position errors can be calculated as

\[
\sigma(l, t) = \int_{0}^{l} \frac{1}{2} (w'(x, t))^2 \, dx = \int_{0}^{l} \frac{1}{2} \left( \sum_{i=1}^{n} \phi_i(x) \eta_i(t) \right)^2 \, dx, \tag{4}
\]

\[
V_s = K_v \left( \frac{\pi}{l} \right)^2 \sum_{i=1}^{n} \phi_i^2(x) \eta_i(t). \tag{5}
\]
The position errors of the 3DOF manipulator with and without PZT vibration control in X and Y direction are shown in Fig. 4-Fig. 7. These four figures clearly show that the position error of the manipulator is significantly reduced when the PZT vibration control strategy is used.

Reliability of Kinematic Trajectory

The accuracy of kinematic trajectory is influenced by the vibration while the 3DOF parallel manipulator movement. The stronger the vibration is, the less accurate the trajectory will be. The vibration of linkages is reduced when PZT vibration control is used. At the same time, the accuracy of kinematic trajectory of the 3DOF manipulator is improved. The reliability of kinematic accuracy of manipulator can be used as an indicator of the accuracy of trajectory.

Reliability analysis of kinematic accuracy of the 3DOF manipulator belongs to mechanism kinematic reliability. The mechanism kinematic reliability expresses the ability to accurately, timely and harmoniously complete the planned mechanical movement under the conditions and time of motion. The reliability can be presented by the probability, which the kinematic output errors are less than the maximal permissible error.

Some same parameters of kinematics are involved no matter there is PZT vibration control or not, such as the kinematic path, speed and acceleration. The state of movement is point to point. In other words, the state of movement of every displacement is independent. The errors between the output value and target value are caused due to the random factors, such as the vibration. These errors are not cumulative. The reliability of kinematic accuracy of the 3DOF manipulator can be obtained
according to the values of experiments with and without PZT vibration control [8].

The cumulative number of the displacement errors, \( n_f(s) \), and the cumulative number of non-errors, \( n_s(s) \), are counted from \( n \) displacement values of the experiment according to the error tolerance \( \varepsilon \). The reliability and the cumulative failure probability of kinematic accuracy can be expressed as

\[
R(t) \approx \frac{n_r(s)}{n}, \quad F(t) \approx \frac{n_f(s)}{n}.
\]

The parameters of kinematics with and without PZT vibration control are the same, the sample size \( n \) is 1000, and the value of error tolerance \( \varepsilon = 0.001 \) mm. According to experiment data, the parameters of reliability of kinematic accuracy of the 3DOF manipulator are shown in Table 1.

<table>
<thead>
<tr>
<th>Axis</th>
<th>( n )</th>
<th>( n_f(s) )</th>
<th>( n_s(s) )</th>
<th>( R(t) ) [%]</th>
<th>( F(t) ) [%]</th>
</tr>
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<tbody>
<tr>
<td>X without control</td>
<td>1000</td>
<td>344</td>
<td>656</td>
<td>65.6</td>
<td>34.4</td>
</tr>
<tr>
<td>X with control</td>
<td>1000</td>
<td>34</td>
<td>966</td>
<td>96.6</td>
<td>3.4</td>
</tr>
<tr>
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<td>669</td>
<td>331</td>
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<tr>
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<td>968</td>
<td>96.8</td>
<td>3.2</td>
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</table>

As illustrated in Table 1, the kinematic accuracy is improved while PZT vibration control is used. The reliability of kinematic accuracy is improved, and the failure probability is reduced.

Conclusion

The theory and experiment of PZT vibration control strategy are proposed. The experiment result presented that the vibration of the 3DOF manipulator is suppressed significantly while PZT vibration control is used. The theory of reliability for kinematic accuracy is discussed. The reliability and failure probability of the 3DOF manipulator with and without PZT vibration control are analyzed. The results show that the kinematic accuracy of the 3DOF manipulator is improved when PZT vibration control is used.

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