Decentralized Receding Horizon Control for Multiple Unmanned Helicopters Considering Dynamics Model

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Abstract—In this paper, the formation flight of multiple Unmanned Helicopter (UH) systems is researched and a new decentralized receding horizon formation control algorithm is supposed. The formulation of formation control problem is firstly given where a local tracking controller for each helicopter system is supposed and the flight trajectories are taken as optimizing variables. Secondly, full dynamics of UH system is introduced into the formation control algorithm by a new concept of Formation Control Lyapunov Functions (FCLF). Subsequently, the convergence of the proposed formation control algorithm is ensured by combining the concept of FCLF and some constraints with respect to the optimized variables. Finally, in order to verify the feasibility and validity of the proposed algorithm, the formation flight simulations of 3 UH systems are conducted.

I. INTRODUCTION

Unmanned helicopter (UH) has been extensively used in all kinds of applications due to its high maneuverability and flexibility. Thus, the researches about UH system, including the stability augmentation, tracking control, and trajectory planning, have been greatly conducted during past decades. However, when facing a high complicated task, single UH system is usually insufficient in either autonomy or intelligence. While, cooperation/coordination of multiple UH systems is considered as a good substitution of that and has obtained great attentions in most recent.

In the cooperation/coordination of multiple robot systems, the formation control, i.e., multiple systems working together with a fixed geometry configuration, is a top important problem. And many formation control strategies have been published, such as the leader-follower version, behavior based one, and virtual structure method. Most recently, receding horizon control (RHC), also called model based predictive control, has also been employed in formation control because it can achieve a sub-optimal behavior with a simultaneous consideration of the constraints. And how to design convergent and decentralized formation control algorithm are two key topics during the research of receding horizon formation control.

As far as the formation flight control of multiple UH systems is concerned, many existing formation control strategies can be used. For example, in [11], the leader-follower strategy is utilized. Receding horizon formation control of multiple UH systems can also be found in [12]-[15], these algorithms can be divided as centralized methods and decentralized ones. In centralized algorithms, the formation control problem is considered as a controller design problem and the optimal behavior of all UH systems are obtained simultaneously by solving a high-dimensional optimal control problem. While in decentralized versions, the receding horizon strategy is usually used as a tracking controller of each single RH system, and the formation is realized by integrating some neighboring states penalties into the cost function of receding horizon algorithm.

However, the existing algorithms are difficult to be used in real systems due to the following reasons: 1) the centralized algorithm can usually ensure the convergence, but the huge computational burden make it unfit for real applications; 2) the full dynamics is the most direct and complete constraints that the UH system subjects to, however, it is terrific tough to be considered in receding horizon formation control algorithm because of the accompanied huge computational burden; 3) the convergence of decentralized receding horizon formation control algorithm is difficult to be ensured, especially when the dynamics model is considered.

In this paper, a new framework of decentralized receding horizon formation flight control of multiple UH systems is proposed to solve the preceding disadvantages by using a new concept of Formation Control Lyapunov Functions (FCLF) and some constraints with respect to the optimized inputs.

II. DYNAMICS MODEL OF UH SYSTEM

Supposing that each UH system can be modeled as follows,

$$\begin{bmatrix}
\dot{p} \\
\dot{v}^p \\
\dot{\Theta}
\end{bmatrix} = \begin{bmatrix}
\frac{1}{m}R(0 0 -T_w) \\
\Psi \omega^b \\
J^{-1}(r^b - \omega^b \times J\omega^b)
\end{bmatrix}$$

where $p \in \mathbb{R}^3$ and $v^p \in \mathbb{R}^3$ are the position and velocity vector of center of mass in inertia frame; $R \in SO(3)$ is the rotation matrix of the body frame relative to the inertia frame; $\omega^b$ is angular velocity vector; $\Theta = [\phi \ \theta \ \psi]^T$ is Euler angle vector; $m$ and $J$ are respectively the mass and inertia; $\Psi$ is the transformation matrix from angular velocity to angular position; $r^b$ is the moment presented in body frame; $T_w$ is the
quantity of force acting on the main rotor. \( r^k \) and \( T_M \) consists the control input vector.

Tracking control of system (1) is obtainable by some nonlinear control technique, for example, feedback linearization tracking\(^{[10]} \). And (1) can be transformed into the following equations by taking \( \dot{\theta} \) and \( \theta \) as inputs,

\[
\begin{align*}
\dot{\theta} &= f_\theta(p_1, p_2, p_3, p_r) + g_\theta(p_1, p_2, p_3, p_r)u \\
\ddot{\theta} &= f_\theta(p_2, p_3, \dot{\theta}, \ddot{\theta}) + g_\theta(p_2, p_3, \dot{\theta}, \ddot{\theta})u
\end{align*}
\]

(2)

The detailed definition of \( f_\theta, g_\theta, f_\varphi, g_\varphi \) can be found in \([3]\).

And the closed loop can be denoted as following form,

\[
x_i = f_i(x_i,u_i) \\
L_i(x_i,u_i) \subseteq \Pi_i(x_i) \\
u_i = k_i(x_i,y_i^d)
\]

(3)

where \( i = 1, 2, \ldots, N \) denotes the \( i \)-th UH system; \( x_i \in \mathbb{R}^n \), \( u_i \in \mathbb{R}^m \) are state and input vector; \( y_i^d \) is the desired output; \( f_i(\cdot), L_i(\cdot) \) and \( \dot{\cdot} \) (\( \cdot \)) is a system and constraint functions; \( \Pi_i(\cdot) \) is the constraint set; \( k_i(\cdot) \) is controller.

The constraints in Eq. (3) are necessary and important due to the following two reasons: 1) the state and input constraints are both unavoidable in real systems; 2) Eq. (1) is only an approximate model of real helicopter system, and the state constraints must be considered to ensure the validity of the designed tracking controller.

With tracking controller, each UH system can track its desired output trajectory. Thus, the formation can be implemented by each UH system tracking corresponding desired trajectory. Define the desired formation as a set \( \{y_{1,d}(t), y_{2,d}(t), \ldots, y_{N,d}(t)\} \) in \( \mathbb{R}^m \) where \( y_{i,d}(t) \) is the desired output profile of \( i \)-th UH system, and the formation control problem of multiple robot system can be formulated as,

**Problem I:**

Find out the desired output profile \( y_{i,d}(t) \) of each UH system such that the following equation is satisfied,

\[
\lim_{t \to \infty} \|y(t) - y_{i,d}(t)\| = 0
\]

(5)

where \( y(t) \) and \( y_{i,d}(t) \) are the output and desired output of the formation defined as following equations,

\[
y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_N(t) \end{bmatrix} \quad \text{and} \quad y_{i,d}(t) = \begin{bmatrix} y_{i,d}(t) \\ y_{i,d}(t) \\ \vdots \\ y_{i,d}(t) \end{bmatrix}
\]

(6)

III. THE CONCEPT OF TCLF AND FCLF

Taking \( y_{i,d}^d \) \( (i=1,2,\ldots,N) \) as inputs, the formation control problem I is equivalent to the tracking controller synthesis problem. Thus, many nonlinear controller design strategies can be used to solve problem I.

CLF is a new tool for nonlinear control, and it can also find the effectiveness in improving the performance of nonlinear RHC (NRHC)\(^{[17]} \). In this paper, in order to use it in the formation control, the following two new concepts from traditional CLF are firstly given:

**Definition I:**

A continuously differentiable map \( V(x,y_{i,d},t): \mathbb{E} \subset \mathbb{R}^n \to \mathbb{R}^+ \) is a Tracking CLF (TCLF) of system (3)-(4) if: 1) \( V(x,y_{i,d},t)=0 \) if and only if \( y_i = y_{i,d} \) and 2) there at least exists a control input \( u_i \) \( (i=1,2,\ldots,N) \) at each point of \( (x,y_{i,d}) \in \mathbb{E} \) such that

\[
V_x^n f(x,u) + V_{y_{i,d}}^n y_{i,d} + V_t \leq -\sigma_i(V)
\]

(7)

where \( \sigma_i(\cdot) \) is a class \( K \) function.

**Definition II:**

A continuously differentiable map \( F(x,y_{i,d},t): \mathbb{E} \subset \mathbb{R}^{2n} \to \mathbb{R}^n \) is a Formation CLF (FCLF) of formation system (3)-(4) with \( i=1,2,\ldots,N \) if: 1) \( F(x,y_{i,d}) = 0 \) if and only if \( y_i = y_{i,d} \) \( (i=1,2,\ldots,N) \), and 2) there at least exists a control input \( \bar{u}_i \) at each \( (x,y_{i,d}) \in \mathbb{E} \) such that

\[
\sum_{i=1}^{N} F_x^n y_{i,d} + F_y^n y_{i,d} + F_t \leq -\sigma(F)
\]

(8)

where \( \sigma(\cdot) \) is a class \( K \) function.

With definition I, the following proposition is obtainable.

**Proposition I,**

If \( k_i(x_i,y_{i,d}^d,t); E \subset \mathbb{R}^n \to \mathbb{R}^m \) is a local continuous feedback tracking controller of system (3), and \( V(x,y_{i,d},t) \) is a Lyapunov function of the closed loop system (3)-(4). Then the following positively definite function

\[
W(x_i,\xi_{i},y_{i,d}^d,t) = V(x_i,\xi_{i},t) + \mathbb{L} (\xi_{i},y_{i,d}^d,t)
\]

(9)

is a TCLF of the following enhanced system,

\[
\dot{x}_i = f_i(x_i,k_i(x_i,\xi_i)) \\
\dot{\xi}_i = h_i (\xi_i) + \nu_i (\xi_i) \\
L_i(x_i,k_i(x_i,\xi_i)) \subseteq \Pi_i (x_i)
\]

(10)

with the desired states

\[
\left( \begin{array}{c} x_i^n \\ \xi_i^n \end{array} \right) \quad \left( \begin{array}{c} x_i^n \\ \xi_i^n \end{array} \right)
\]

(11)

where \( \xi_i \) is a new-defined variable vector with the same dimension as \( y_{i,d}^d \); \( \mathbb{L} (\xi_{i},y_{i,d}^d,t) \) is a TCLF of the second sub-system of (10).

**Proof:**

\[
V(x,y_{i,d},t) \text{ is a Lyapunov function of system (3)-(4) means there exists a class } K \text{ function } \sigma_i(V) \text{ such that}
\]

\[
V_t f(x,k(x,\xi_i)) + V_{\xi_i} \dot{\xi}_i + V_t \leq -\sigma_i(V)
\]

(12)

Similarly, there exists \( v_i \) satisfying inequality,

\[
\mathbb{L} (h_i (\xi_i) + \nu_i (\xi_i)) + \mathbb{L} (\xi_i,\dot{\xi}_i) + \mathbb{L} = -\sigma_i(\mathbb{L})
\]

(13)

Furthermore, we have

\[
W(x_i,\xi_{i},y_{i,d}^d,t) = V(x_i,k(x_i,\xi_i)) + V_{\xi_i} \dot{\xi}_i + V_t \leq -\sigma_i(V) - \sigma_i(\mathbb{L})
\]

(14)

From Eq. (13), we can obtain \( v_i \) such that

\[
W(x_i,\xi_{i},y_{i,d}^d,t) \leq -\sigma_i(V) - \sigma_i(\mathbb{L})
\]

(15)

It is obvious that Eq. (15) is equal to or less than zero, and the equality is satisfied if and only if \( \{x_i = y_{i,d} \} \) and \( \{y_{i,d}^d = y_{i,d}^d \} \). Thus, based on Definition I, (9) is a TCLF of system (10) with the desired states (11) for all \([x_i, \xi_i]\) in the following set,
\[ E_{\infty} = \left[ x^T, \zeta^T \right] W(x, \zeta, \zeta') \leq r^2 \]  
where \( r_{\infty} \) is the maximum \( r \) such that the projection of \( E_{\infty} \) in the state space of system (3) belongs to \( E \).

For the purpose of simplification, (10) can be rewritten as,
\[ \begin{cases} \dot{z}_i = h_i(z_i) + g_i(z_i) v_i \\ \gamma_i = \zeta_i \\ A_i(z_i) \subseteq \Theta_i(z_i) \end{cases} \]
(17)

With Proposition I, we have the following Theorem.

**Theorem I:**
If each single enhanced system (17) has its own local TCLF \( V_i(z_i, y_i^d, t) \) in \( E_{\infty} \), then the following function is a FCLF of the formation control problem,
\[ F(z, y_i, t) = \sum_{i=1}^{N} \beta_i V_i(z_i, y_i^d, t) \]  
where \( z \) and \( y_i \) are state vector and desired output vector being composed of all UH's state vector and desired state vector; \( \beta_i \) are some positive constants.

**Proof:**
From definition I, there exists one control input such that,
\[ \begin{bmatrix} \frac{\partial V_i}{\partial z} \end{bmatrix}^T \dot{y}_i^d + \frac{\partial V_i}{\partial z} \dot{z}_i + \gamma_i(t) \leq -\sigma_i(V_i) \]
(19)

Thus,
\[ F_i \dot{y}_i + F_i \dot{z}_i + \sum_{i=1}^{N} \beta_i \sigma_i(V_i) \leq -\sum_{i=1}^{N} \beta_i \sigma_i(V_i) \]
(20)

As a class K function, \( \sigma_i(V_i) \) satisfied the inequality
\[ \beta_i \sigma_i(V_i) \geq \beta_i \sigma_i \left( \frac{E}{\lambda_\infty N} \right) \]
(21)

Substitute (21) into (20), we have
\[ F_i \dot{y}_i + F_i \dot{z}_i + \sum_{i=1}^{N} \beta_i \sigma_i(V_i) \leq -\sum_{i=1}^{N} \beta_i \sigma_i(V_i) \]
(22)

Thus, from Definition II, we can conclude the proof of this theorem. And the valid region of the local FCLF is
\[ \Omega_{\infty} = \{z | F(z) \leq r_{\infty}^2 \} \]
(23)

IV. DECENTRALIZED RH FORMATION CONTROL ALGORITHM

**A. Algorithm Description**
\[ v_i^*(i; z(t_i)) = \arg \min_{v_i} J_i(z(t_i), z_i(t_i), T) \]
for \( i, j \)
\[ s.t. \int_{T_{i-1}}^{T} [q_i(z_i, z(t_i), z_i(t_i), \gamma, \zeta, v_i) \]
(24)

In this section, the new decentralized RHFC will be given. And the optimization problem of it can be denoted as Eq. (24), where \( \delta \) is the time interval between two neighboring optimizations; \( \zeta \) is a positive constant; \( t_i \) is current instant; \( t_{i-1} \) \( \delta \) is the last time instant; \( N \) is the set of the \( i \)th UH's neighbor which can exchange data with it; \( z^*_i(t_r; z(t_i)) \) is the state at time \( t_r \) with control input \( v_i \) and initial state \( z(t_i) \); \( z_r(t_i) \) is the desired state at time \( t_r \) which can be obtained from the desired output, and \( z^* \) in the following sections is the similar definition of \( y^d \); \( z_j \) is the state of the UH belong to \( N \); \( q_i(t) \) is the cost term defined as,
\[ q_i(z_i(t_k), z_i(t_{k-1})); v_i(t_r; z(t_i)), \gamma_i(t_r), \delta_i(t_r)) \]
(25)

where \( \beta_i \) is the desired relative state of the \( i \)th RH system and \( \delta_i \) is a weight satisfying,
\[ \lambda_i \leq \frac{\lambda_i}{\sum_{i=1}^{N} \lambda_i} \]
(26)

Eq. (26) implies that only one UH system knows its desired trajectory and the other UH systems can only optimize their action by the desired relative states with its neighbors; \( \dot{v}_i(t_i; v_i(t_i)) \) called assumed control \[11\], is defined as follows,
\[ \dot{v}_i(t_i; v_i(t_i)) \leq \frac{\dot{v}_i(t_i; v_i(t_i))}{\dot{v}_i(t_i; v_i(t_i))} + \dot{v}_i(t_i; v_i(t_i)) \]
(27)

where \( \dot{z}_i \) is the local tracking controller of system (17) as in reference [17]; \( v_i \) denote the optimal control input profile. And at initialization step, \( \dot{v}_i(t_i; v_i(t_i)) \) and \( z_j(t_i; z_j(t_i)) \) are both set 0.

**B. Convergence**
Before proving the convergence of algorithm (24), we firstly give some definitions and some assumptions,
\[ J_i(z) \leq \sum_{i=1}^{N} J_i(z_i, z_{i-1}, T) \]
(28)

\[ G(x) \leq \{ z | J_i < \delta \} \]
(29)

\[ G(x) \leq \{ z \in G(x) | J_i(x) \leq \delta \} \]
(30)

\[ G(x) \leq \{ z \in G(x) | J_i(x) \leq \delta \} \]
(31)

**Assumption I:**
\[ h(z) \] is Lipschitz in (17), i.e.,
\[ \| h(z_1) - h(z_2) \| \leq K_1 \| z_1 - z_2 \| \]
(32)

**Assumption II:**
The solution of optimization problem (24) is continuous.

**Assumption III:**
The cost term \( q_i(t) \) is defined as,
\[ \begin{bmatrix} q_i(t_j; z_j(t_j)) + \gamma_i(t_j), \delta_i(t_j) \end{bmatrix} \]
(33)

Firstly, the following lemma can be obtained easily.

**Lemma I:**
$z(t_f) \in \Gamma^*_f$ implies $z'(t_f \in T, z(t_f)) \in \Omega$, where $z(t_f \in T, z(t_f))$ is the optimal state value for optimization problem (24) at terminal time instant.

**Proof:**
From (31) and (25), $z(t_i) \in \Gamma^*_n$ means that

$$
\sum_{i=1}^{N} J'(z_i(t_i), z^{(i)}_{z(t_i)}, T) = \sum_{i=1}^{N} V'(z_i(t_i) + T, z(t_i)) + \int_{t_i}^{t_{i+1}} q(z_i(t; z(t_i)), z_{\xi(t)}(t; \xi(t)), \nu'(t), z_{j}(t))dT \leq \gamma \tag{34}
$$

Thus,

$$F(z_{*}(t_i + T; z(t_i))) = \sum_{i=1}^{N} W(z_{*}(T; z(t_i))) \leq \gamma \tag{35}$$

i.e., $z_{*}(t_i + T; z(t_i)) \in \Omega$. 

And then, the following lemma by using the RHFC as (24),

**Lemma II:**
If assumption I - III is satisfied, and $V_i$ is a local TCLF of the formation control problem (17) in $E_t$. Thus, for any $z(t_i) \in \Gamma^*_n$ ($r_i$ is defined as Eq. (37)), there exists a positive constant $\xi$ such that the following inequality is satisfied for all $\delta < T$,

$$J_{f}'(z'_{*}(t_i + \delta; z(t_i))) - J_{f}'(z(t_i)) \leq \delta^2 \xi - \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(x) \tag{36}$$

$$z_{*}'(t; z(t_i)), z_{\xi(t)}(t; \xi(t)), \nu'(t), z_{j}(t))dT \leq \gamma \tag{37}$$

where $l_i$ is the potency of $N_i$; $F(z, z_{*}) = \sum_{i=1}^{N} V(z, z_{*})$.

**Proof:**
Let $(\xi(t), \nu(t))$ and $t \in [t_i, t_i+T+\delta)$ be the trajectory obtained by concatenating $(z_{*}, \nu(\xi(t)))$, $t \in [t_i, t_i+T]$, and $(z_{*}, \nu(\xi(t)))$, $t \in [t_i+T, t_i+T+\delta]$, which is the closed loop trajectory at time $t$ starting from $z(t_i)$ and controlled by the CLF based controller $k(x) = [k_1(x); k_2(x); \ldots; k_N(x)]$.

Consider the cost of using $\nu$ for $T$ seconds beginning at an initial state $z(t_i + \delta; z)$, $\delta \in [t_i, t_i+T]$. For each single system,

$$J_{f}'(z(t_i + \delta; z(t_i))) - J_{f}'(z(t_i)) \leq \int_{t_i}^{t_{i+1}} q_i(x, z(t_i + \delta))z^{(i)}_{\xi(t)}(t)z^{(i)}_{\xi(t)}(t) \tag{38}$$

Thus,

$$J_{f}'(z_{*}(t_i + \delta; z(t_i))) - J_{f}'(z(t_i)) \leq \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(x, z(t_i + \delta))z^{(i)}_{\xi(t)}(t)z^{(i)}_{\xi(t)}(t) \tag{39}$$

Based on the definition of $(\xi(t), \nu(t))$, we have,

$$z_{*}'(t_i + T; z(t_i)) = \bar{\xi}(t_i + T; z(t_i)) \tag{40}$$

And, from (37),

$$\sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(x, z(t_i + \delta), z_{\xi(t)}(t; \xi(t)), \nu'(t)), z_{j}(t))dT \leq \gamma \tag{41}$$

Thus, (39) can be rewritten as,

$$J_{f}'(z_{*}(t_i + \delta; z(t_i))) - J_{f}'(z(t_i)) \leq \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(x, z(t_i + \delta)) \tag{42}$$

Now, based on Assumption II, there exists a positive constant $K_2$ such that the following inequality is satisfied,

$$|z_{*}'(t_i + \delta; z(t_i)) - z_{*}'(t_i; z(t_i))| 
leq K_2 \gamma \tag{43}$$

Based on comparison lemma (Lemma 3.4 in reference [18]), inequality (44) means that,

$$|z_{*}'(t_i + \delta; z(t_i)) - z_{*}'(t_i; z(t_i))| \leq K_2 \gamma \tag{45}$$

Thus, Eq. (43) can be changed into the following form,

$$J_{f}'(z_{*}(t_i + \delta; z(t_i))) - J_{f}'(z(t_i)) \leq \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(x, z(t_i + \delta))z^{(i)}_{\xi(t)}(t)z^{(i)}_{\xi(t)}(t) \tag{46}$$

Define

$$\xi = \frac{\alpha K_2 \gamma}{K_1} \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} e^{2\kappa_{i}T} + e^{2\kappa_{i}T} + T \tag{47}$$

lemma II is proved.

And now, we show that the cost function $J_{f}'(\cdot)$ is monotonously decreasing by select a small $\delta$.

**Lemma III:**

Under assumption I - III, for any positive constant $\delta$, there exists a $\delta(\z(t_0)) > 0$ such that

$$\sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} q_i(z'_i(t; z_i(t_i)), z_{i(\alpha-1)}(t; z_{i(\alpha-1)}(t_i)), v'_i(t), z_j(t)) \, dt \geq \delta^2 \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i - z_{i,d}\| + \kappa \|v'_i(t)\| \right) \, dt$$

(48)

Further, if $z(t_0) \neq z_{i,d}$ for all $i = 1, 2, \ldots, N$ is satisfied, then $\delta(z(t_0)) = 0$.

**Proof:**

Given $t = t_0$, the following condition is right,

$$z'_i(t; z_i(t_i)) > z_i(t)$$

(49)

Thus, from assumption III, when at least one agent $i$ satisfies $z(t_0) \neq z_{i,d}$, we have

$$\sum_{i=1}^{N} q_i(z'_i(t; z_i(t_i)), z_{i(\alpha-1)}(t; z_{i(\alpha-1)}(t_i)), v'_i(t), z_j(t)) \geq 2(\tau - t_0) \delta^2 + \|z'_i(t) - z_{i,d}\|_0^2 + \kappa \sum_{i=1}^{N} \|v'_i(t)\|$$

(52)

Thus, we have,

$$\int_{t_0}^{t_{i+1}} \sum_{i=1}^{N} q_i(z'_i(t; z_i(t_i)), z_{i(\alpha-1)}(t; z_{i(\alpha-1)}(t_i)), v'_i(t), z_j(t)) \, dt$$

$$> \delta^2 \delta + \int_{t_0}^{t_{i+1}} \left[ \|z'_i(t) - z_{i,d}\|_0^2 + \kappa \sum_{i=1}^{N} \|v'_i(t)\| \right] \, dt$$

(53)

is satisfied for any $\delta \leq \delta(z(t_0))$.

Based on the preceding analysis, we can conclude the following result about the convergence of the formation algorithm (24)-(25).

**Theorem II:**

Under assumption I - III, the RHFC algorithm (24)-(25) is asymptotically convergent if $z(0) \in \Gamma_{z,\infty}$.

**Proof:**

The proof of this theorem is divided into the following two steps,

1) $\Gamma_{z,\infty}$ is a positively invariant set of the closed loop;

2) $\eta(\Theta_{\beta,0}(t); \nu_{\beta,0}(t); z(0)) \rightarrow 0$ as $t \rightarrow \infty$.

**Step-I:**

$z(t_0) \in \Gamma_{z,\infty}$ implies that

$$J_z(z(t_0), z_{(\alpha-1)}(t_0), T) \leq r_0^2$$

(54)

Based on Lemma II and Lemma III, we have

$$J_z(z(t_0) + \delta; z(t_0)) - J_z(z(t_0))$$

$$\leq \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i - z_{i,d}\| + \kappa \|v'_i(t)\| \right) \, dt < 0$$

(55)

That means $z(t_0 + \delta) \in \Gamma_{z,\infty}$, and thus $\Gamma_{z,\infty}$ is a positively invariant set of the closed loop system.

**Step-II:**

The process in Step-I and Lemma I show that terminal state is in $\Omega_{z,\infty}$ if $z(0) \in \Gamma_{z,\infty}$. Thus, inequality (55) is satisfied for all the time, i.e.,

$$J_z(t_0) \leq J_z(t_0) - \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i - z_{i,d}\| + \kappa \|v'_i(t)\| \right) \, dt$$

(56)

$$J_z(t_0) \leq J_z(t_0) - \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i - z_{i,d}\| + \kappa \|v'_i(t)\| \right) \, dt$$

From Eq. (56), the following inequality is satisfied,

$$J_z(t_0) \leq J_z(t_0) - \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i - z_{i,d}\| + \kappa \|v'_i(t)\| \right) \, dt$$

(57)

Since the last term of inequality (57) is always less than zero except for the point $z = 0$, the statement in 2) is obviously satisfied, and the closed loop system is obviously asymptotically stable.

Finally, we give the following proposition to show that the convergence region of the closed loop can be extended by using the RHC strategy.

**Proposition II:**

Under assumption I - III and the RHFC, $\Omega_{z,\infty} \subset \Gamma_{z,\infty}$ is satisfied at the initialization step.

**Proof:**

Let $(\tilde{z}(t), \tilde{v}(t))$ be the trajectory controlled by the TCLF based tracking controller, thus,

$$J_z(z(0)) = \sum_{i=1}^{N} \int_{t_i}^{t_{i+1}} \left( \|z'_i(t; z_i(t_i)), 0 \nu'_i(t), z_j(t) \right) \, dt$$

(59)

Eq. (59) means $z(0) \in \Gamma_{z,\infty}$, and this complete the proof of this proposition.

V. SIMULATIONS

In this section, we will list the results of simulations conducted with respect to the formation flight of 3 UH systems. We assume that the 1st UH system (shown as blue line in Fig.1) know its own desired trajectories and others UH system can only track the desired relative position with their neighboring. The tracking controller $k(x, \dot{x})$ and the TCLF $\nu(x, \dot{x})$ can be obtained by using the feedback linearization method[3]. Thus, a FCLF can be obtained by Theorem I. And the left design parameters of the proposed algorithm are as following equations,
\[ q_i = \alpha \left\| z_i - z_{id} \right\| + \beta \left\| \dot{z}_i - \dot{z}_{id} \right\| + \gamma + \delta \left\| \ddot{z}_i - \ddot{z}_{id} \right\| + \epsilon \left\| \dot{\gamma} \right\| + \zeta \]
\[
\left( z_i - z_{id} \right)^T \left( z_i - z_{id} \right) + \kappa \nu^T
\]
\[
1 0 0 \\
0 1 0 \\
0 0 1
\]
\[
u = \begin{bmatrix}
-4\alpha + \zeta & -2\beta & -2\gamma \\
-2\beta & 2\beta & 0 \\
-2\gamma & 0 & 2\gamma
\end{bmatrix}
\]

The simulation results are listed as Fig.1, where the red dashed lines are the desired trajectories of each UH system, and blue solid lines are the real trajectories of each UH system. From Fig.1, we can see that when the desired flight state is changed in the 10th second, the desired velocity of the formation is changed from \([1;0;0]\) to \([0;0;-1]\). Due to the coupling of the UH systems, the positions in \(x\) axis and \(z\) axis are influenced by each other, but the formation can be kept always. This shows that the new proposed decentralized formation algorithm of this paper is feasible and the formation flight is realized.

![Fig. 1 position and euler angle of 3 UH systems](image-url)

**VI. CONCLUSION**

In this paper, a new decentralized receding horizon formation controller was proposed. It has the following three advantages: 1) The full dynamics model of the unmanned helicopter system, instead of the kinematic model, is considered; 2) The local stability of each unmanned helicopter system is ensured by a fast tracking controller, and whose performance can be sufficiently considered to decentralize the formation control algorithm by combining the receding horizon optimization strategy with the concept of Formation Control Lyapunov Functions; 3) The convergence of the formation control algorithm can be ensured by increasing an additive constraints with respect to the predicted trajectory. Finally, the simulations with respect to formation systems consisting of three UH systems are conducted and the results are presented and analyzed to verify the validity of the algorithm.

**REFERENCES**


