Static Measuring Model and Deadweight Compensation of a Stewart Platform Based Force/Torque Sensor

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Abstract -The static measuring model and the deadweight compensation of a Stewart platform based force/torque sensor have been studied in this paper. Firstly, some preliminary kinematics of this sensor is presented. Coordinates attached to every links are established, and the transforming relations between link frames and other frames are derived. Then, the static measuring model and the deadweight compensation of the sensor are produced. In the modeling, forces on links are composed based on an analytic method, and the resultant forces and torques obtained can be used as the measuring model. The weight need to be compensated of the whole mechanism is divided into two parts: links weights and top platform weight. The compensation of the links weights is involved in static modeling. And the top platform weight is compensated, subsequently. Finally, some experiments are carried out, and the results verify the validity of the static measuring model and the deadweight compensation.

Index Terms - Stewart platform, force/torque sensor, static model, deadweight compensation.

I. INTRODUCTION

A Stewart platform is a 6-DOF parallel mechanism that consists of six links connecting a movable platform to a fixed base[1]. The parallel mechanism offers high load capacity, high structural rigidity and high accuracy. So the Stewart platform can be used as a multi-component force/torque sensor, whose top platform and base platform are connected by six links with single-axis bidirectional force transducers. When a load is applied on the top platform, there will be outputs on the six force transducers. With the six values output from the force transducers, the six dimension resultant force/torque can be calculated. Owing to the high load capacity of the parallel mechanism, the Stewart platform based force/torque sensor can be used for large force/torque measurement.

In recent years, some cases are reported that parallel mechanisms are used for measuring multi-component force/torque. Dwarakanath[2] dealt with the implementation of the Stewart platform based force/torque sensor. Gao[3] also developed a six dimension force sensor with Stewart structure, and the isotropy of the sensor was studied in their paper. Dwarakanath and Gao all used the conventional force Jacobian matrix as the static measuring model, and not considered the influence of the links deadweight. Du and Sun[4,5] proposed a force sensing system for 6-DOF parallel robots by integrating six pull-press sensors into the parallel links. In order to develop a general multi-component force/torque sensor, authors have made a lot of studies. Some problems such as kinematic calibration and parameters identification have been studied by Wang[6,7]. In this paper, a detailed static modeling and the deadweight compensating are done so that the interface of the static measuring model become more transparent and can be used more conveniently.

This paper is organized as follows. Following the introduction, preliminary kinematics is presented in section II. Then static measuring modeling and deadweight compensation are carried out respectively in section III and IV. After theoretical studies, some experiments are done in section V. Through analyzing experimental results, some conclusions are drawn in section VI.

II. PRELIMINARY KINEMATICS

Different from the assumption that the bottom platform is horizontally fixed in paper[4,5], the whole force/torque sensor may be movable when measuring in this paper. So before establishing the static measuring model of the sensor, some kinematic transformations must be derived first of all. The six dimensions Stewart platform based force/torque sensor consists of a top platform, a base platform and six links, as shown in Fig.1. Each link, which contains a single-axis bidirectional force transducer and two link rods, is connected with the two platforms using two spherical joints at the two ends Ai and Bi (i=1~6).

The coordinate system is established as shown in Fig.1. In view of the fact that the sensor may be movable in some applications, an immovable reference frame O0-x0y0z0 is defined. Axis y0 is plumb, and axis x0 and z0 are in the horizontal plane. Frames O0-x0y0z0 and O2-x2y2z2 are attached to the top platform and the base platform, respectively. The rotation transformation from O2-x2y2z2 to O0-x0y0z0 and that

Fig.1 The configuration of the sensor
from \(O_1-x_1y_1z_1\) to \(O_0-x_0y_0z_0\) are denoted by matrices \(\mathbf{R}^0\) and \(\mathbf{R}^1\). And the position of \(O_0\) in \(O_1-x_1y_1z_1\) is denoted by \(\mathbf{r}_{O_0}\). Then the rotation transformation from \(O_1-x_1y_1z_1\) to \(O_0-x_0y_0z_0\) can be expressed as

\[
\mathbf{R}^0 = \mathbf{R}^1 \mathbf{R}^0
\]

And the position vector from \(O_0\) to \(O_1\) can be described in \(O_0-x_0y_0z_0\) as

\[
\mathbf{r}_{O_1} = \mathbf{R}^0 \mathbf{r}_{O_0}
\]

The vectors denoting the positions of the spherical joints centers \(A_i\) and \(B_i\) (\(i=1\sim6\)) are defined respectively in frame \(O_1-x_1y_1z_1\) and \(O_0-x_0y_0z_0\) as

\[
\mathbf{r}_{Ai} = \begin{bmatrix} x_{Ai} \\ y_{Ai} \\ z_{Ai} \end{bmatrix}^T
\]

\[
\mathbf{r}_{Bi} = \begin{bmatrix} x_{Bi} \\ y_{Bi} \\ z_{Bi} \end{bmatrix}^T
\]

Then the position vector from \(O_0\) to \(A_i\) and \(B_i\) can be described in \(O_0-x_0y_0z_0\) as

\[
\mathbf{r}_{Bi} = \mathbf{R}^0 \mathbf{r}_{Ai}
\]

For the convenience of force analysis of the link, the link frame \(A_i-x_iy_iz_i\) (\(i=1\sim6\)) attached to each link is established as shown in Fig.1. Axis \(z_i\) is along \(A_iB_i\) and points to \(B_i\). And axis \(x_i\) is perpendicular to the plane determined by a plumb line and line \(A_iB_i\). Then axis \(y_i\) is vertical to the plane determined by axis \(x_i\) and \(z_i\). A matrix \(\mathbf{R}^i\) is defined to describe the rotation transform from frame \(A_i-x_iy_iz_i\) to the reference frame \(O_0-x_0y_0z_0\), and usually it is written as

\[
\mathbf{R}^i = [\mathbf{n}_i, \mathbf{o}_i, \mathbf{a}_i]
\]

In fact, \(\mathbf{a}_i\) is the directional vector of the \(i\)th link, which is in the form of

\[
\mathbf{a}_i = \frac{1}{l_i} \begin{bmatrix} x_{Bi} - x_{Ai} \\ y_{Bi} - y_{Ai} \\ z_{Bi} - z_{Ai} \end{bmatrix}
\]

where \(l_i = \sqrt{(x_{Bi} - x_{Ai})^2 + (y_{Bi} - y_{Ai})^2 + (z_{Bi} - z_{Ai})^2}\) is the length of the link.

\(\mathbf{n}_i\) is the directional vector of axis \(x_i\), and can be obtained by cross multiplying the direction vector of axis \(y_i\) and vector \(\mathbf{a}_i\). That is

\[
\mathbf{n}_i = \begin{bmatrix} 0 \\ 1 \times \begin{bmatrix} x_{Bi} - x_{Ai} \\ y_{Bi} - y_{Ai} \\ z_{Bi} - z_{Ai} \end{bmatrix} = \begin{bmatrix} z_{Bi} - z_{Ai} \\ 0 \\ - (x_{Bi} - x_{Ai}) \end{bmatrix} \end{bmatrix}
\]

The unit vector of it is

\[
\mathbf{n}_i = \frac{1}{l'_i} \begin{bmatrix} z_{Bi} - z_{Ai} \\ 0 \\ - (x_{Bi} - x_{Ai}) \end{bmatrix}
\]

where \(l'_i = \sqrt{(x_{Bi} - x_{Ai})^2 + (y_{Bi} - y_{Ai})^2 + (z_{Bi} - z_{Ai})^2}\)

\(\mathbf{o}_i\) is the directional vector of axis \(y_i\), and can be obtained by cross multiplying \(\mathbf{a}_i\) and \(\mathbf{n}_i\), that is

\[
\mathbf{o}_i = \mathbf{a}_i \times \mathbf{n}_i = \frac{1}{l'_i} \begin{bmatrix} - (x_{Bi} - x_{Ai})(y_{Bi} - y_{Ai}) \\ (x_{Bi} - x_{Ai})^2 + (z_{Bi} - z_{Ai})^2 \\ - (y_{Bi} - y_{Ai})(z_{Bi} - z_{Ai}) \end{bmatrix}
\]

Thus the transform matrix \(\mathbf{R}^i\) can be written as

\[
\mathbf{R}^i = [\mathbf{n}_i, \mathbf{o}_i, \mathbf{a}_i] = \frac{1}{l'_i} \begin{bmatrix} l_i dx_i \quad - dx_i dy_i \quad l'_i dz_i \\ 0 \quad l_i^2 \quad l'_i dy_i \\ - l_i dx_i \quad - dy_i dz_i \quad l'_i dz_i \end{bmatrix}
\]

The inverse of \(\mathbf{R}^i\) is in the form of

\[
\mathbf{R}^{-1} = \mathbf{R}^i \mathbf{R}^{-i} = \mathbf{R}^i \mathbf{R}^T
\]

III. STATIC MEASURING MODELING

For parallel mechanism, the weights of the links are usually ignored and the links are simplified as two-force members in their static analysis. So the relation between the link forces \(\mathbf{N}\) and the resultant forces \(\mathbf{F}\) are

\[
\mathbf{JN} = \mathbf{F}
\]

where \(\mathbf{J} = \begin{bmatrix} l'_1 & \cdots & l'_6 \\ \mathbf{r}_1 \times l'_1 & \cdots & \mathbf{r}_6 \times l'_6 \end{bmatrix}\) is the force Jacobian matrix.

\(l'_i\) (\(i=1\sim6\)) is the unit directional vector of the \(i\)th link, and \(\mathbf{r}_i\) (\(i=1\sim6\)) is the position vector from \(O_0\) to \(B_i\).
However, the weights of links should not be ignored in the static modeling when the parallel mechanism is used for constructing the force/torque sensor. Especially, if the pose of it is variable when it works, the influence of the links weights must be compensated real time. The weights links compensation can’t be separated from static modeling. In fact, the inertia forces of the links will also affect the output of the force/torque sensor. But in this paper, the moving velocity of the sensor is so trivial that the inertial force can be ignored compared to the self weight of the mechanism.

Selecting a single link as the analyzed object, the force diagram of the link is shown in Fig.2. For the whole link, it is in a static equilibrium state under the action of the gravity of the whole link \( G_i \) and the two joint forces \( ^0F_{Ax} \) and \( ^0F_{Bx} \) respectively on the two ends of the link. Otherwise, the output of the single-axis bidirectional force transducer will be affected by weight of the \( BD_j \) part of the link, because the top platform is the force measuring plane. So for the \( BD_j \) part, it is in a static equilibrium state under the action of the gravity of the \( BD_j \) part \( G_{ij} \), the joint force \( ^0F_{Bi} \) and the output of the force transducer \( N_i \). Provided the mass center of the \( i \)th link is on the point \( C_i \), the length ratio of the \( AC_i \) part and \( CB_i \) part is \( \zeta_i \), namely

\[
\frac{|AC_i|}{|CB_i|} = \zeta_i
\]  

(13)

In fact, the output force value of the force/torque sensor is the resultant force of the six joint forces on point \( B_0 \), so only joint forces from the top platform need to be solved in the force analysis. For convenience, all forces act on the link may be projected in to the link frame \( AxOyOz \) of the link.

\[
^iF_{Bi} = ^0R\cdot ^0F_{Bi} = \frac{0}{\zeta_i + 1}G_i = \frac{0}{\zeta_i + 1}R\cdot ^0G_i
\]  

(14)

where \( ^0F_{Bi} = [^0F_{Bix}, ^0F_{By}, ^0F_{Bz}]^T \) and \( ^iF_{Bi} = \left[ ^iF_{Bix}, ^iF_{By}, ^iF_{Bz}\right]^T \) are the joint forces the top platform acting on the link, respectively in the reference frame and the \( i \)th frame. \( ^0G_i = [0 - G_i 0]^T \) and \( ^iG_i = [^iG_{ix}, ^iG_{iy}, ^iG_{iz}]^T \) are the gravity vectors of the whole link respectively in reference frame and the \( i \)th frame. \( ^0G_i = [0 - G_i 0]^T \) and \( ^iG_i = [^iG_{ix}, ^iG_{iy}, ^iG_{iz}]^T \) are the gravity vectors of the \( BD_j \) part respectively in reference frame and the \( i \)th frame.

When the link is in force equilibrium state, the moments of forces about axis \( x_i \) and axis \( y_i \) respectively equal to zero for the whole link. And the equilibrium equations are

\[
^iF_{Byi}l_i = \frac{\zeta_i}{\zeta_i + 1}G_{iy}l_i = 0
\]  

(15)

\[
^iF_{Bzi}l_i + \frac{\zeta_i}{\zeta_i + 1}G_{iz}l_i = 0
\]  

(16)

Selecting the \( BD_j \) part of the link as the analyzed object, the equilibrium equation of the forces on \( z_i \) direction is

\[
^iF_{Bzi} + ^iG_{iz} = N_i
\]  

(17)

In fact, \( N_i \) is the internal force of the \( i \)th link, and it is a positive value when the link is pulled.

Then forces the links acting on the top platform, which are the counterforces of those the top platform acting on the links, can be obtained

\[
\begin{align*}
^iF_{Bzi} &= \frac{\zeta_i}{\zeta_i + 1}G_{iz} \\
^iF_{Bzi} &= \frac{\zeta_i}{\zeta_i + 1}G_{iz} \\
^iF_{Bzi} &= ^0G_{iz} - N_i
\end{align*}
\]  

(18)

Rewriting Eq.(18) into a vector format and substituting Eq.(14) into it yields

\[
^iF_{Bi} = \left[ \begin{array}{c}
\frac{\zeta_i}{\zeta_i + 1}G_{ix} \\
\frac{\zeta_i}{\zeta_i + 1}G_{iy} \\
\frac{\zeta_i}{\zeta_i + 1}G_{iz} - N_i \\
\end{array} \right]
\]  

(19)

Projecting \( ^iF_{Bi} \) into the reference frame \( 0x'0y'0z_0 \) and shifting it to the origin of the top platform frame \( O_0 \), we obtain

\[
\begin{align*}
^0F_{Bi} &= ^0R\cdot ^iF_{Bi} = [n, o, a]^T F_{Bi} \\
&= \frac{\zeta_i}{\zeta_i + 1}(n, o)^T G_i + a_n^T G_i \cdot a_i N_i \\
&= K_{ii}^0G_i + K_{ii}^0G_i + K_{ii}^0N_i \\
^0M_{Bi} &= K_{ii}^0G_i + K_{ii}^0G_i + K_{ii}^0N_i,
\end{align*}
\]  

(20)

where the coefficient matrices are in the forms of

\[
K_{ii} = \frac{\zeta_i}{\zeta_i + 1} \begin{bmatrix}
\ddot{x}_i + \frac{\omega_i^2}{l_i^2} - \dot{x}_i \ddot{y}_i - \dot{y}_i \ddot{z}_i \\
- \dot{x}_i \ddot{y}_i - \dot{y}_i \ddot{z}_i - \dot{z}_i \ddot{y}_i \\
\dot{x}_i \ddot{z}_i - \dot{z}_i \ddot{x}_i - \dot{y}_i \ddot{z}_i \\
\end{bmatrix}
\]  

(21)

\[
\begin{align*}
K_{ii} &= a_n a_i^T = \frac{1}{l_i^2} \begin{bmatrix}
\ddot{x}_i^2 + \ddot{y}_i^2 + \ddot{z}_i^2 \\
\ddot{x}_i \ddot{y}_i + \ddot{y}_i \ddot{z}_i + \ddot{z}_i \ddot{x}_i \\
\ddot{x}_i \ddot{z}_i + \ddot{z}_i \ddot{x}_i + \ddot{y}_i \ddot{z}_i \\
\end{bmatrix}
\end{align*}
\]  

(22)

\[
K_{iii} = -a_i = -\frac{1}{l_i} \begin{bmatrix}
\ddot{x}_i & \ddot{y}_i & \ddot{z}_i
\end{bmatrix}^T
\]  

(23)

\[
K_{ii} = x_n x_i^T, \quad K_{ii} = x_n x_i^T, \quad K_{ii} = x_n x_i^T
\]  

(24)

\[
^0r = ^0r_{Bx} - ^0r_{Bx}
\]  

(25)
Thus the output of the force/torque sensor is the sum of the six link forces. So combining the six link forces and simplifying them yields

$$\begin{bmatrix} F_x \\ M_z \end{bmatrix} = \sum_{i=1}^{6} \begin{bmatrix} 0 F_{li} \\ 0 M_{li} \end{bmatrix} = K_1^0 G_i + K_\Pi^0 G_{li} + K_{III}^0 N \quad (22)$$

where

$$K_1 = \begin{bmatrix} \ldots & \frac{\xi_i}{l_i^2} - dx_i dy_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} - dx_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} - dx_i dy_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \end{bmatrix}_{6x6}$$

$$K_\Pi = \begin{bmatrix} \ldots & \frac{\xi_i}{l_i^2} - dx_i dy_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} - dx_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} - dx_i dy_i & \ldots \\ \ldots & \frac{\xi_i}{l_i^2} + dy_i dz_i & \ldots \end{bmatrix}_{6x6}$$

$$K_{III} = \begin{bmatrix} \ldots & -dx_i & \ldots \\ \ldots & -dy_i & \ldots \\ \ldots & -dz_i & \ldots \\ \ldots & dx_i & \ldots \\ \ldots & dy_i & \ldots \\ \ldots & dz_i & \ldots \end{bmatrix}_{6x6}$$

$$^0 G_i = \begin{bmatrix} -^0 G_{i1} \\ \vdots \\ -^0 G_{i6} \end{bmatrix}$$

$$^0 G_{li} = \begin{bmatrix} -^0 G_{li1} \\ \vdots \\ -^0 G_{li6} \end{bmatrix}$$

$$^0 N = \begin{bmatrix} N_1 \\ \vdots \\ N_6 \end{bmatrix}$$

Comparing Eq.(12) and Eq.(22), it is easily to learn that $J$ in Eq.(12) and $K_{III}$ in Eq.(22) are in the same form. This means that the two equations will be identical if the weights of the links are ignored.

IV. DEADWEIGHT COMPENSATION

In section III, the static model of the force/torque sensor has been established. But in applications, what the sensor measures must only be the external load applied on it. So the self deadweight, not only the weights of links but also that of the top platform, must be compensated. In fact, the deadweight compensation of the links has been involved in the static modeling. In this section, the main task is to compensate the deadweight of the top platform.

In the top platform frame $O_t-x_ty_tz_t$, the position vector of mass center of the top platform is defined as $^t r_c = [^t x_c \; ^t y_c \; ^t z_c]^T$, and it can be expressed in reference frame $O_0-x_0y_0z_0$ as

$$^0 r_c = ^0 R_c^t r_c \quad (23)$$

Writing the gravity of the top platform into a vector format $^0 \mathbf{G}_t = [0 \; -G_t \; 0]^T$, we obtain the following equation by shifting it to the origin of the top platform frame $O_t$

$$\begin{bmatrix} F_{Gr} \\ M_{Gr} \end{bmatrix} = \begin{bmatrix} ^0 r_c \times ^0 \mathbf{G}_t \end{bmatrix} = \begin{bmatrix} 0 \; -G_t \; 0 \; ^0 z_c \times ^0 \mathbf{G}_t \; 0 \; ^0 x_c \times ^0 \mathbf{G}_t \end{bmatrix} \quad (24)$$

If an external load $\begin{bmatrix} F \\ M \end{bmatrix}$ is applied on the top platform of the sensor, the equilibrium equation can be obtained

$$\begin{bmatrix} F_s \\ M_s \end{bmatrix} + \begin{bmatrix} F_{Gr} \\ M_{Gr} \end{bmatrix} + \begin{bmatrix} F \\ M \end{bmatrix} = \theta \quad (25)$$

Thus the measured external load can be expressed

$$\begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} F_s \\ M_s \end{bmatrix} - \begin{bmatrix} F_{Gr} \\ M_{Gr} \end{bmatrix} \quad (26)$$

If necessary, it can be projected into the top platform frame of the sensor

$$\begin{bmatrix} ^1 F \\ ^1 M \end{bmatrix} = \begin{bmatrix} ^0 R_c^t & ^0 R_c^t \end{bmatrix} \begin{bmatrix} F \\ M \end{bmatrix} = \begin{bmatrix} ^0 R_c^{-1} & ^0 R_c^{-1} \end{bmatrix} \begin{bmatrix} F \\ M \end{bmatrix} \quad (27)$$
V. EXPERIMENT AND ANALYSIS

To verify the static measuring model and the deadweight compensation, some numerical computations and experiments are to be done based on the theoretical studies in the above sections. A six dimension force/torque sensor system and an experimental device have been designed. The force/torque sensor system consists of six single-axis bidirectional force transducers and amplifiers, a data acquisition system and an embedded computer based on PC104 bus (as shown in Fig. 3). The signals from the strain force transducers are amplified respectively by six amplifiers, and then they are sampled into the embedded computer through an A/D converter. After being conditioned in the embedded computer, the signals are transmitted through a CAN bus to a superior computer. After calculating the theoretical measuring model presented in the above sections, the resultant force/torques is outputted in the upper computer. In order to apply force and torque on the top platform of the multi-component force/torque sensor and measure them, an experimental device which is composed of a set of workbenches, pulleys, steel belts, poises and accessories has been designed.

![Fig.3 Schematic diagram of the experimental system](image)

The parameters of the sensor are listed as follows:

\[
\begin{align*}
\|^b r_a_1 & = \begin{bmatrix} 0.09 & 0.5 \sin 355^\circ & 0.5 \cos 355^\circ \end{bmatrix}^T \\
\|^b r_a_2 & = \begin{bmatrix} 0.09 & 0.5 \sin 245^\circ & 0.5 \cos 245^\circ \end{bmatrix}^T \\
\|^b r_a_3 & = \begin{bmatrix} 0.09 & 0.5 \sin 235^\circ & 0.5 \cos 235^\circ \end{bmatrix}^T \\
\|^b r_a_4 & = \begin{bmatrix} 0.09 & 0.5 \sin 125^\circ & 0.5 \cos 125^\circ \end{bmatrix}^T \\
\|^b r_a_5 & = \begin{bmatrix} 0.09 & 0.5 \sin 115^\circ & 0.5 \cos 115^\circ \end{bmatrix}^T \\
\|^b r_a_6 & = \begin{bmatrix} 0.09 & 0.5 \sin 5^\circ & 0.5 \cos 5^\circ \end{bmatrix}^T \\
\|^b r_b_1 & = \begin{bmatrix} -0.09 & 0.5 \sin 335^\circ & 0.5 \cos 335^\circ \end{bmatrix}^T \\
\|^b r_b_2 & = \begin{bmatrix} -0.09 & 0.5 \sin 265^\circ & 0.5 \cos 265^\circ \end{bmatrix}^T \\
\|^b r_b_3 & = \begin{bmatrix} -0.09 & 0.5 \sin 215^\circ & 0.5 \cos 215^\circ \end{bmatrix}^T \\
\|^b r_b_4 & = \begin{bmatrix} -0.09 & 0.5 \sin 145^\circ & 0.5 \cos 145^\circ \end{bmatrix}^T \\
\|^b r_b_5 & = \begin{bmatrix} -0.09 & 0.5 \sin 95^\circ & 0.5 \cos 95^\circ \end{bmatrix}^T \\
\|^b r_b_6 & = \begin{bmatrix} -0.09 & 0.5 \sin 25^\circ & 0.5 \cos 25^\circ \end{bmatrix}^T
\end{align*}
\]

\[
\begin{align*}
\|^b G_1 & = [-40.93 -40.93 -40.76 -40.99 -40.94 -40.93]^T \\
\end{align*}
\]

\[
\begin{align*}
\|^b R & = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\|^b r_0 & = [0.4 -0.0 0.0]^T \\
G_t & = 1008.35 \\
\|^b r_3 & = [-0.024 0.0 0.0]^T
\end{align*}
\]

\[
\begin{align*}
\varepsilon_t & = 1.3, (i = 1 \sim 6) \\
\|^b R & = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\
\|^b r_0 & = [0.4 -0.0 0.0]^T \\
G_t & = 1008.35 \\
\|^b r_3 & = [-0.024 0.0 0.0]^T
\end{align*}
\]

The parameters listed above are some constant values which are only related to the sensor itself. And they are independent to pose variation of the sensor. Otherwise, \(^b R\) is the rotation matrix which can be used for described the pose of the sensor relative to the reference frame \(O_0-x_0y_0z_0\).

Two experiments have been done for verifying the theoretical studies. In the one, the sensor is mounted with one vertical and two horizontal poses, respectively. The three poses can be described by the rotation matrix as

\[
\begin{align*}
\text{Pose 1: } & ^0 R = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \\
\text{Pose 2: } & ^0 R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\
\text{Pose 3: } & ^0 R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\end{align*}
\]

<table>
<thead>
<tr>
<th>Pose</th>
<th>Link Forces Vector (\vec{N}(N))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Computational ([-313.1 -313.1 -313.1 -313.2 -313.2 -313.2]^T)</td>
</tr>
<tr>
<td></td>
<td>Experimental ([-311.4 -310.8 -312.3 -311.7 -310.5 -312.9]^T)</td>
</tr>
<tr>
<td>2</td>
<td>Computational ([682.3 10.6 -671.7 671.7 -10.6 -682.3]^T)</td>
</tr>
<tr>
<td></td>
<td>Experimental ([651.3 72.6 -666.6 671.4 -5.7 -645.9]^T)</td>
</tr>
<tr>
<td>3</td>
<td>Computational ([-682.3 -10.6 671.7 -671.7 10.6 682.3]^T)</td>
</tr>
<tr>
<td></td>
<td>Experimental ([-687.6 -26.1 673.2 -653.4 -5.1 705.9]^T)</td>
</tr>
</tbody>
</table>

The link forces are numerically computed firstly according to the static measuring model Eq.(26), when the sensor is in the three different poses. Then they are obtained by actual measurement in the same cases. By comparing the computational and the Experimental results (shown in TABLE I) the static model and the gravity compensation can be verified preliminarily.

In order to verify the theoretical model further, the other experiment is made subsequently. In this one, the second pose is selected and the resultant force is measured when loads in different directions \((Fx, Fy, Fz, Mx, My \text{ and } Me)\) are applied to the sensor. In each direction, the load is divided into six grades. The errors between the measured output value and the actual load value are shown in Fig. 4.
The results show that there are some discrepancies between the experimental link forces and the computational link forces (in the former experiment) or between the measured values and the actual values (in the latter experiment). The reason resulting in these discrepancies mainly includes two parts: one is the errors of the mechanism physical parameters, and the other is the friction in the joints.

VI. CONCLUSIONS

In this paper, a force/torque sensor has been developed based on the Stewart platform. Some necessary preliminary kinematics which will be used in the following parts of this paper are presented firstly. Then the static measuring model of the sensor, in which the gravity of the links is taken into account, has been established. To be used as a force/torque sensor, the influence of the deadweight of the mechanism must be eliminated. The deadweights of the links have been involved in the static modeling. And the weight of top platform of the sensor is compensated, subsequently. The feature of the measuring model is its generality, which make the sensor be used even in the case that the pose of it is time-variant. And the only thing the user need do is to input the pose matrix. Some experiments are carried out for verifying the theoretical studies, and the results show that the static model and the deadweight compensation are valid.

REFERENCES