Chapter 49
An Automatic Dynamics Generation Method for Reconfigurable Modular Robot

Wenbin Gao and Hongguang Wang

Abstract A method of automatic dynamics generation for modular robots is presented on modular level. The robot's link parameters are got from the modules' parameters and the Assemble Incidence Matrix (AIM) describing the module types, assembled orientations and sequence of a given robot configuration. Adjoint matrices are adopted to describe the forward mapping of velocities, accelerations from frames on the $(i-1)$ link to that of the ith link, as well as the dual adjoint matrices are taken to describe the backward mapping of moments and forces between frames on the i and the $(i-1)$th links. A mathematically consistent recursive approach for dynamics of modular robot is got from the Newton–Euler formula in Lie Group form.

Keywords Reconfigurable modular robot · Dynamics · Lie group

49.1 Introduction

Reconfigurable Modular Robot System (RMMS) is comprised by a serious of standardized modules with different dimensions and certain assembling styles, such as link modules, joint modules and gripper modules. Joint modules are all-in-one

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components which can generate rotary or translational motions and communicate with the upper controller. Manipulators with different DOFs or configurations can be rapidly constructed by choosing proper modules and then connecting them together. The closed-form dynamics model is widely applied in robot design, calibration and motion optimization. However, it's impractical to derive the dynamics manually for every configuration of modular robot, because the number of possible robot geometries is very huge. So this method that can generate the dynamics model of a given configurations automatically is important in module robot's researches and applications [1–4].

The researches of dynamics of modular robots mostly are based on the Lagrangian and Newton–Euler formulation. Wang [5] and [6] proposed the dynamics constructing method for modular robots based on Lagrangian formula. In both of their works, the given configurations are regarded as normal manipulators, which cannot be adopted to automatically generate the dynamics. Fei [7] and [8, 9] gave the recursive methods based on Newton–Euler formula, both of their methods can automatically generate the dynamics. Chen's method which is expressed in the Lie Group form is a more compact approach.

In this paper, it introduces a Modular Reconfigurable Robot Experiment System (MRRES) and presents a modular level method for automatic dynamics generation of modular robots based on Newton–Euler formulation in the form of Lie Group and Lie Algebra. The links' dynamics parameters of a given modular robot are obtained automatically from the information of the AIM and module parameters. Compare with Chen's method, our method is a complete modular level approach.

49.2 MRRES and AIM

The MRRES is developed by the State Key Laboratory of Robotics in Shenyang Institute of Automation, China. It takes a distributed control system with an Industrial Personal Computer as the main controller and DSPs in the joint modules. The data communication is based on CAN bus. Figure 49.1 shows the module library and a 6-DOF configuration.

As shown in Fig. 49.2a, both the revolute and prismatic joints are 1-DOF modules. In a joint module, the input part connecting to the lower part of a given manipulator is named as part I, and the output part connecting to the gripper direction is named as part II. Both part I and II have their own input ports and output ports. The Cartesian coordinates are set on the input port (subscript 0) and output port (subscript 1) respectively to describe the position and orientation of the output port relative to input port. The output frame of part I and the input frame of part II are superposed when the joint is at zero position. Their z-axes are collinear with the joint axis and their origins are set on the midpoint of the joint axis. As shown in Fig. 49.2b–d, the Cartesian coordinate is set on each link modules, based modules and gripper are alike, to describe the position and orientation of the output port relative to input port.

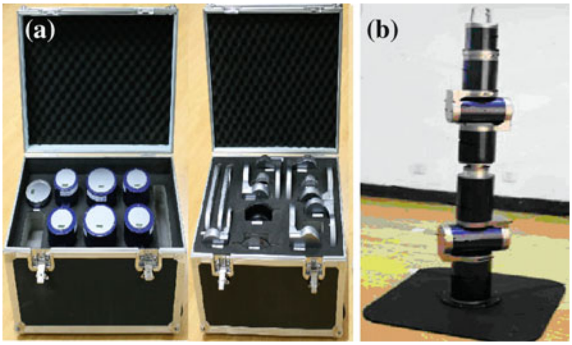

Fig. 49.1 Module library and a 6-DOF configuration. **a** Module library. **b** A 6-DOF configuration

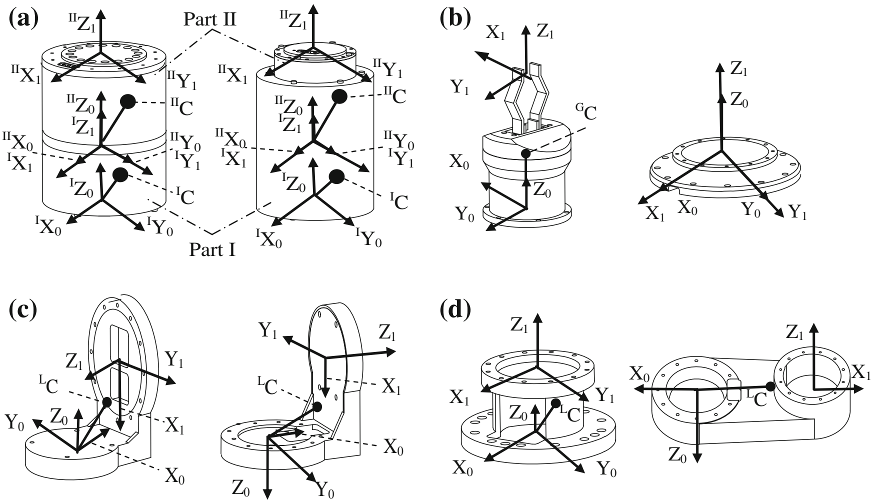

Fig. 49.2 Cartesian coordinates on modules. **a** Revolute joint and Prismatic joint. **b** Gripper and base modules. **c** A and B styles of right-angled links. **d** Linear link and parallel link

AIM is taken to represent module types, assembled orientations and sequence of a given configuration of modular robots. The AIM of the 6-DOF configuration shown in Fig. 49.1 is given in Table 49.1. The first row describes the joint modules types and connecting sequence from base to gripper. J_R, J_P, J_G are the representations of the revolute joint, prismatic joint and gripper respectively. The first column describes the possible types of the link modules. L_C, L_P, L_R are the representations of linear link, parallel link, A and B styles of right-angled links separately. The other part of Table 49.1 excluding the first row and first column above mentioned is the AIM. Each element of AIM contains three numbers. The first one represents the orientation of the two assembled modules and the 1, 2, 3, 4 mean that the input port of a module turning 0, 90, 180, 270° about the *z*-axis of output port of former module in a clockwise direction separately; The second and third one are the numbers of the given types of joint and link modules respectively.

Table 49.1 AIM of the configuration shown in Fig. 49.2

	J_R	J_R	J_P	J_R	J_R	J_R	J_G
B	(1,1,1)						
L_C	↓		(1,1,1)→	(1,3,1) ↓		(1,5,2)→	(2,1,2)
^AL_R	(2,2,1)→	(1,2,1) ↓	↑	(3,3,2)→	(1,4,2) ↓	↑	
^BL_R		(3,2,1)→	(1,1,1)		(3,4,2)→	(1,5,2)	
L_P							

Taking the third row and the fourth column of AIM for example, G_{34} (3, 3, 2) means that the number 2 of A style right-angled links turning 180° in a clockwise direction relative to the output port of the number 3 revolute joint.

49.3 Kinemics of Modular Robot

Based on the assembly information from AIM, the link's information of robot can be got from the parameters of link and joint modules (In the following, links are all mean the robot's links after being assembled). As shown in Fig. 49.3, the part II of joint module ($j-1$), link module ($j-1$) and part I of joint module j consist the ($i-1$)th link of the robot. And the ($i-1$)th joint is formed by joint module ($j-1$), the body's fixed frame of the ($i-1$)th link is superposed with input frame of part II of joint module ($j-1$).

When all joints are at zero position, the link fixed coordinate transformation from the i link to the ($i-1$)th link is

$$\boldsymbol{M}_{i-1,i} = \boldsymbol{T}_{j-1,j} \quad = \boldsymbol{T}^{\mathrm{II}}_{j-1} {}_{J}\boldsymbol{T}^{\mathrm{L}}_{j-1} \boldsymbol{T}^{\mathrm{L}}_{j-1} {}_{\mathrm{L}}\boldsymbol{T}^{\mathrm{J}}_{j} {}_{I}\boldsymbol{T}^{\mathrm{II}}_{j} = \begin{bmatrix} \boldsymbol{R}_i & \boldsymbol{p}_i \\ 0 & 1 \end{bmatrix} \quad i,j = 1 \cdots n+1 \quad (49.1)$$

Where, $\boldsymbol{T}_{j-1,j} \in \mathrm{SE}(3)$is the homogeneous transformation matrix from the input port of part II of joint module j to that of joint module ($j-1$); $\boldsymbol{T}^{\mathrm{II}}_{j-1} \in \mathrm{SE}(3)$ is the homogeneous transformation matrix from the output frame to the input frame of part II of joint module ($j-1$); ${}_{\mathrm{J}}\boldsymbol{T}^{\mathrm{L}}_{j-1} \in \mathrm{SE}(3)$ is the homogeneous matrix describing the orientation of link module ($j-1$) relative to joint module ($j-1$); $\boldsymbol{T}^{\mathrm{L}}_{j-1} \in \mathrm{SE}(3)$ is the homogeneous transformation matrix from the output frame to the input frame of link module ($j-1$); ${}_{\mathrm{L}}\boldsymbol{T}^{\mathrm{J}}_{j} \in \mathrm{SE}(3)$ is the homogeneous matrix describing the orientation of joint module j relative to link module ($j-1$); ${}_{\mathrm{I}}\boldsymbol{T}^{\mathrm{II}}_{j} \in \mathrm{SE}(3)$ is the homogeneous transformation matrix from the input frame of part II of joint module j to the input frame of part I.

The kinematics of sub-assembly (Fig. 49.3) from frame i to frame ($i-1$) is as following [10, 11]

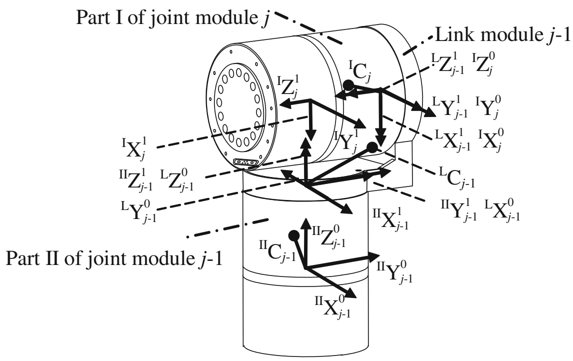

Fig. 49.3 Sub-assembly of a given configuration

$$f_{i-1,i} = \boldsymbol{M}_{i-1,i} e^{\boldsymbol{S}_i x_i} \tag{49.2}$$

where, $\boldsymbol{S}_i \in se(3)$ is the ith joint screw written in the body-fixed frame of the ith link and $x_i \in \boldsymbol{R}$ is the current position of the ith joint relative to a specified zero position. $\boldsymbol{S}_i$ can also be expressed in a 6-dimensional vector, $\boldsymbol{S}_i = [\boldsymbol{w}_i^{\mathrm{T}} \;\; \boldsymbol{u}_i^{\mathrm{T}}]^{\mathrm{T}} \in \boldsymbol{R}^{6\times1}$, termed twist coordinate. For revolute joint $\boldsymbol{S}_i = [\boldsymbol{w}_i^{\mathrm{T}}\, 0]^{\mathrm{T}}$, where $\boldsymbol{w}_i = [0\,0\,1]$ is the unit directional vector of the joint i expressed in frame i. For prismatic joint $\boldsymbol{S}_i = [0 \;\; \boldsymbol{u}_i]^{\mathrm{T}}$, where $\boldsymbol{u}_i = [0\,0\,1]$ is the unit directional vector of joint i relative to frame i.

According to the Eq. (49.2), the transformation of the frame fixed on the ith link to the inertial frame can be got from combining a series of sequential matrices, as follows

$$f_i = f_{0,1} f_{1,2} \cdots f_{i-1,i} \tag{49.3}$$

Then, the kinematics of a given modular robot's configuration can be modeled alike [11]

$$f_{0G} = f_{0,1}\, f_{1,2} \cdots f_{i-1,i}\, f_{i,G} \tag{49.4}$$

$f_{i,G}$ is the transformation matrix from the frame of gripper to that of the ith link.

49.4 Dynamics of Modular Robot

49.4.1 Dynamics Parameters of Sub-Assembly

Frames, which are parallel to the input frames of link modules or each part of joint modules, are set on the centers of masses (Marked by C in Fig. 49.2) respectively. The vector from the origin of input frame of module to the center of mass is named as $\boldsymbol{r}$. And the homogeneous matrix of the frame on the center of mass relative to the input frame is

$$\boldsymbol{T}_j^{k\mathrm{C}} = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{r}^{k\mathrm{C}} \\ 0 & 1 \end{bmatrix} \quad k = \mathrm{I, II, L} \tag{49.5}$$

where, L means a link module and I and II mean the two parts of a joint module respectively.

Taking a sub-assembly for example (As shown in Fig. 49.3), the frame on the center of mass of link module ($j-1$) which is expressed in the ($i-1$)th link frame (superposed with input frame of part II of link module ($j-1$)) is

$$\boldsymbol{T}_{i-1}^{\mathrm{LC}} = \boldsymbol{T}_{j-1}^{\mathrm{II}}{}_{J}\boldsymbol{T}_{j-1}^{\mathrm{L}}\boldsymbol{T}_{j-1}^{\mathrm{LC}} = \begin{bmatrix} \boldsymbol{R}_{i-1}^{\mathrm{LC}} & \boldsymbol{r}_{i-1}^{\mathrm{LC}} \\ 0 & 1 \end{bmatrix} \tag{49.6}$$

$\boldsymbol{T}_{j-1}^{\mathrm{II}}$, ${}_{\mathrm{J}}\boldsymbol{T}_{j-1}^{\mathrm{L}}$ and $\boldsymbol{T}_{j}^{\mathrm{LC}}$ are defined in Eqs. (49.1) and (49.5).

The frame on the center of mass in part I of joint module j which is expressed in the ($i-1$)th link frame is

$$\boldsymbol{T}_{i-1}^{\mathrm{IC}} = \boldsymbol{T}_{j-1}^{\mathrm{II}}{}_{J}\boldsymbol{T}_{j-1}^{\mathrm{L}}\boldsymbol{T}_{j-1}^{\mathrm{L}}\boldsymbol{T}_{j}^{\mathrm{IC}} = \begin{bmatrix} \boldsymbol{R}_{i-1}^{\mathrm{IC}} & \boldsymbol{r}_{i-1}^{\mathrm{IC}} \\ 0 & 1 \end{bmatrix} \tag{49.7}$$

The definitions of $\boldsymbol{T}_{j-1}^{\mathrm{II}}$, ${}_{\mathrm{J}}\boldsymbol{T}_{j-1}^{\mathrm{L}}$, $\boldsymbol{T}_{j-1}^{\mathrm{L}}$ and $\boldsymbol{T}_{j}^{\mathrm{IC}}$ are in Eqs. (49.1) and (49.5).

The links' dynamics parameters are the basic informations for constructing the dynamics equations. Taking ($i-1$)th link for example, its total mass is

$$\mathrm{m}_{i-1} = \mathrm{m}_{j-1}^{\mathrm{II}} + \mathrm{m}_{j-1}^{\mathrm{L}} + \mathrm{m}_{j}^{\mathrm{I}} \tag{49.8}$$

$\mathrm{m}_{j-1}^{\mathrm{II}}$, $\mathrm{m}_{j-1}^{\mathrm{L}}$, and $\mathrm{m}_{j}^{\mathrm{I}}$ are the masses of part II of joint module ($j-1$), link module ($j-1$) and part I of joint module j respectively.

The product of the mass of the ($i-1$)th link and the vector from the origin of the ($i-1$)th frame to that of the center of mass is

$$\mathrm{m}_{i-1}[\boldsymbol{r}_{i-1}] = \mathrm{m}_{j-1}^{\mathrm{II}}\left[\boldsymbol{r}_{i-1}^{\mathrm{IIC}}\right] + \mathrm{m}_{j-1}^{\mathrm{L}}\left[\boldsymbol{r}_{i-1}^{\mathrm{LC}}\right] + \mathrm{m}_{j}^{\mathrm{I}}\left[\boldsymbol{r}_{i}^{\mathrm{IC}}\right] \tag{49.9}$$

Taking $\boldsymbol{r}_{i-1}^{\mathrm{IC}}$ (As defined in Eq. (49.7)) which is the vector from the origin of the ($i-1$)th frame to the center of mass of the part I of joint module j for example. If $\boldsymbol{r}_{i-1}^{\mathrm{IC}} = [\, r_1 \quad r_2 \quad r_3 \,]$, $[\boldsymbol{r}_{i-1}^{\mathrm{IC}}]$ is its 3×3 real screw-symmetric form belonged to the Lie Algebra of so (49.3), as follows

$$\left[\boldsymbol{r}_{i-1}^{\mathrm{IC}}\right] = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix} \tag{49.10}$$

Other three dimensional vectors expressed in the brackets are alike.

The inertia of the ($i-1$)th link relative to the ($i-1$)th frame is [12]

$$\begin{aligned} \boldsymbol{I}_{i-1} &= \boldsymbol{I}_{i-1}^{\mathrm{C}} - \mathrm{m}_{i-1}[\boldsymbol{r}_{i-1}]^2 \\ &= \boldsymbol{I}_{j-1}^{\mathrm{II}} - \mathrm{m}_{j-1}^{\mathrm{II}}\left[\boldsymbol{r}_{i-1}^{IIC}\right]^2 + \boldsymbol{R}_{i-i}^{\mathrm{LC}}\boldsymbol{I}_{j-1}^{\mathrm{L}}\left(\boldsymbol{R}_{i-1}^{LC}\right)^{\mathrm{T}} - \mathrm{m}_{j-1}^{\mathrm{L}}\left[\boldsymbol{r}_{i-1}^{\mathrm{LC}}\right]^2 \\ &\quad + \boldsymbol{R}_{i-1}^{\mathrm{IC}}\boldsymbol{I}_{j}^{\mathrm{I}}\left(\boldsymbol{R}_{i-1}^{\mathrm{IC}}\right)^{\mathrm{T}} - \mathrm{m}_{j}^{\mathrm{I}}\left[\boldsymbol{r}_{i-1}^{\mathrm{IC}}\right]^2 \end{aligned} \tag{49.11}$$

$\boldsymbol{I}^{C}_{i-1}$ is the inertia relative to a frame on the $(i-1)$th link's center of mass which is parallel to the $(i-1)$th frame; $\boldsymbol{r}_{i-1}$ is the vector from the origin of the $(i-1)$th frame to its center of mass. $\boldsymbol{I}^{II}_{j-1}$, $\boldsymbol{I}^{L}_{j-1}$ and $\boldsymbol{I}^{I}_{j}$ are the inertias relative to the frames at the centers of mass which are parallel to the input frames of part II of joint module $(j-1)$, link module $(j-1)$ and part I of joint module j, respectively. $\boldsymbol{R}^{LC}_{i-1}$ and $\boldsymbol{R}^{IC}_{i-1}$(As defined in Eqs. (49.6) and (49.7)) stand for the orientation matrixes of the frames on the centers of mass of link module $(j-1)$ and part I of joint module j relative to the frame of the $(i-1)$th link, respectively.

49.4.2 Dynamics Based on Lie Group

This method of automatic generation of dynamics is got from Newton–Euler formula based on the Lie Group and Lie Algebra [13]. The parameters of robot's links have been got from the modules' parameters in Sects. 49.3 and 49.4.1. So the link parameters used in the recursive method for dynamics are all mean the combined parameters.

49.4.2.1 Forward Recursion for Velocities and Accelerations

Initialization:

$$\boldsymbol{V}_0 = 0, \quad \dot{\boldsymbol{V}}_0 = [0 \quad 0 \quad g]^{T} \tag{49.12}$$

Forward recursion: for i = 1 to n, do

$$\boldsymbol{V}_i = \mathrm{Ad}_{f^{-1}_{i-1,i}}(\boldsymbol{V}_{i-1}) + \boldsymbol{S}_i\dot{x}_i \tag{49.13}$$

$$\dot{\boldsymbol{V}}_i = \boldsymbol{S}_i\ddot{x}_i + \mathrm{Ad}_{f^{-1}_{i-1,i}}\left(\dot{\boldsymbol{V}}_{i-1}\right) + \mathrm{ad}_{\mathrm{Ad}_{f^{-1}_{i-1,i}}(\boldsymbol{V}_{i-1})}\boldsymbol{S}_i\dot{x}_i \tag{49.14}$$

- $\boldsymbol{V}_i = [\boldsymbol{\omega}_i^{T} \ \ \boldsymbol{v}_i^{T}]^{T}$ is the six-dimensional generalized velocity of the ith frame, which is expressed in the ith frame (The following three and six dimensional vectors are all expressed in the ith frame, except for special notes). And $\boldsymbol{v}_i$ represents the velocity of the ith frame, $\boldsymbol{\omega}_i$ represents the angular velocity of the ith frame. If $f_i = f_{0,1}f_{1,2}\cdots f_{i-1,i}$ is used to describe the location of the ith frame relative to the inertial reference frame, then $\boldsymbol{V}_i = f_i^{-1}\dot{f}_i$. $\dot{\boldsymbol{V}}_i = [\dot{\boldsymbol{\omega}}_i^{T} \ \ \dot{\boldsymbol{v}}_i^{T}]^{T}$is the six-dimensional generalized acceleration of the ith frame. $\dot{\boldsymbol{v}}_i$ represents acceleration of the ith frame; $\dot{\boldsymbol{\omega}}_i$ represents angular acceleration of the ith frame.
- $\mathrm{Ad}_{f^{-1}_{i-1,i}}$ which is used to transform $\boldsymbol{V}_{i-1}$ and $\dot{\boldsymbol{V}}_{i-1}$ from the $(i-1)$ frame to the ith frame is the adjoint matrix. $f_{i-1,i}$ is defined in Eq. (49.2), if $f_{i-1,i}$ is in the form as $f^{-1}_{i-1,i} = \begin{bmatrix} \boldsymbol{R} & \boldsymbol{p} \\ 0 & 1 \end{bmatrix}$. Then $\mathrm{Ad}_{f^{-1}_{i-1,i}} = \begin{bmatrix} \boldsymbol{R} & 0 \\ [\boldsymbol{p}]\boldsymbol{R} & \boldsymbol{R} \end{bmatrix}$.

- $\mathrm{ad}_{\mathrm{Ad}_{f_{i-1,i}^{-1}}(V_{i-1})} \boldsymbol{S}_i \dot{x}_i$ indicates $\boldsymbol{S}_i \dot{x}_i$'s linear mapping via Lie bracket. $\mathrm{Ad}_{f_{i-1,i}^{-1}}(\boldsymbol{V}_{i-1})$ and $\boldsymbol{S}_i \dot{x}_i$ are elements of Lie Algebra. If $\mathrm{Ad}_{f_{i-1,i}^{-1}}(V_{i-1}) = x = \begin{pmatrix} \omega_1^{\mathrm{T}} & v_1^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$ and $S_i \dot{x}_i = y = \begin{pmatrix} \omega_2^{\mathrm{T}} & v_2^{\mathrm{T}} \end{pmatrix}^{\mathrm{T}}$. Then, $\mathrm{ad}_x \boldsymbol{y} = (\boldsymbol{\omega}_1 \times \boldsymbol{\omega}_2, \boldsymbol{\omega}_1 \times \boldsymbol{v}_2 - \boldsymbol{\omega}_2 \times \boldsymbol{v}_1)$. The matrix representation is $\mathrm{ad}_{\boldsymbol{x}} \boldsymbol{y} = \begin{bmatrix} [\boldsymbol{\omega}_1] & 0 \\ [\boldsymbol{v}_1] & [\boldsymbol{\omega}_1] \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega}_2 \\ \boldsymbol{v}_2 \end{bmatrix}$.

49.4.2.2 Backward Recursion of Forces and Moments

The equation of motion for a rigid body by Newton–Euler method is expressed as following

$$\boldsymbol{F}_i^{\mathrm{I}} = \begin{bmatrix} \boldsymbol{m}_i^{\mathrm{I}} \\ \boldsymbol{f}_i^{\mathrm{I}} \end{bmatrix} = \boldsymbol{J}_i \dot{\boldsymbol{V}}_i - \mathrm{ad}_{\boldsymbol{V}_i}^{*}(\boldsymbol{J}_i \boldsymbol{V}_i) \tag{49.15}$$

- $\boldsymbol{F}_i^{\mathrm{I}}$ is the general inertial force of the ith link. m_i^{I} and f_i^{I} are inertial moment and inertial force respectively.
- $\boldsymbol{J}_i = \begin{bmatrix} \boldsymbol{I}_i^{\mathrm{C}} - m_i[\boldsymbol{r}_i]^2 & m_i[\boldsymbol{r}_i] \\ -m_i[\boldsymbol{r}_i] & m_i \cdot \boldsymbol{1} \end{bmatrix} \in \boldsymbol{R}^{6\times 6}$ is a symmetric positive definite matrix. $\boldsymbol{I}_i^{\mathrm{C}} - m_i[\boldsymbol{r}_i]^2$ and $m_i[\boldsymbol{r}_i]$ are defined in Eqs. (49.9) and (49.11).
- If $\mathrm{ad}_{V_i} = x = (\omega^{\mathrm{T}} \;\; v^{\mathrm{T}})^{\mathrm{T}}$, then $\mathrm{ad}_{\boldsymbol{V}_i}^{*} = \begin{bmatrix} -[\boldsymbol{\omega}_1] & -[\boldsymbol{v}_1] \\ 0 & -[\boldsymbol{\omega}_1] \end{bmatrix}$ is the dual operator of ad_{V_i}.

Backward recursion: for i = n to1 do

$$\boldsymbol{F}_i = \mathrm{Ad}_{f_{i,i+1}^{-1}}^{*}(\boldsymbol{F}_{i+1}) + \boldsymbol{J}_i \dot{\boldsymbol{V}}_i - \mathrm{ad}_{\boldsymbol{V}_i}^{*}(\boldsymbol{J}_i \boldsymbol{V}_i) \tag{49.16}$$

$$\tau_i = \boldsymbol{S}_i^{\mathrm{T}} \boldsymbol{F}_i \tag{49.17}$$

- $\boldsymbol{F}_i$ is the total generalized force transmitted from the ($i-1$) link to the ith link through joint i, the first three components are the moment vector.
- $\mathrm{Ad}_{f_{i,i+1}^{-1}}^{*} = \begin{bmatrix} \boldsymbol{R}^{\mathrm{T}} & \boldsymbol{R}^{\mathrm{T}}[\boldsymbol{p}]^{\mathrm{T}} \\ \boldsymbol{0} & \boldsymbol{R}^{\mathrm{T}} \end{bmatrix}$ is the dual operator of $\mathrm{Ad}_{f_{i,i+1}^{-1}}$ (Eq. (49.13)).
- τ_i is the ith actuator's torque or force.

49.5 Process of Automatic Generation for Dynamics

The process of the automatic generation of the dynamics for module robot is as following

(1) Giving the AIM of a modular robot's configuration;
(2) Getting the kinemics parameters of each link which is composed by parts of two joint modules and a link module;
(3) Getting the kinemics transformation between adjacent sub-assembles;
(4) Getting the dynamic parameters of each link;
(5) Generating dynamic equations by Lie Group and Lie Algebra formulation based on the Newton–Euler recursive method.

49.6 Conclusions

In this paper, a method for automatic generation of dynamics for modular robot is presented. The configuration information of a given manipulator about geometry and DOFs is described by a AIM. The robot's link parameters are got from the assembled modules' kinemics and dynamics parameters based on the assembling information from AIM. Adjoint matrix is adopted to describe the forward mapping of velocities and accelerations from frame of the $(i-1)$ link to that of ith. Meanwhile taking dual adjoint matrixes to describe the backward mapping from frame of the i to that of $(i-1)$th. And then a clear and elegant expression of closed-form dynamic equation is generated based on the recursive Newton–Euler algorithm in the form of Lie Groups and Lie Algebra. This automatic dynamics generation method can be applied to modular robot's configuration and motion optimization, calibration and optimal control and so on.

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