Wavelength-sensitive-function controlled reflectance reconstruction

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Spectral reflectance is defined as the “fingerprint” of an object and is illumination invariant. It has many applications in color reproduction, imaging, computer vision, and computer graphics. In previous reflectance reconstruction methods, spectral reflectance has been treated equally over the whole wavelength. However, human eyes or sensors in an imaging device usually have different sensitivities over different wavelengths. We propose a novel method to reconstruct reflectance, considering a wavelength-sensitive function (WSF) that is constructed from sensor-sensitive functions (or color matching functions). Our main idea is to achieve more accurate reconstruction at wavelengths where sensors have high sensitivities. This more accurate reconstruction can achieve better imaging or color reproduction performance. In our method, we generate a matrix through the Hadamard product of the reflectance matrix and the WSF matrix. We then obtain reconstructed reflectance by applying the singular value decomposition on the generated matrix. The experimental results show that our method can reduce 47% mean-square error and 55% Lab error compared with the classical principal component analysis method. © 2013 Optical Society of America

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Spectral reflectance is a significant physical property of materials and is robust to illumination variation. It is useful in many applications. For example, in computer graphics, the increased dimensionality from tristimulus values to full spectral reflectance allows more accurate illumination computation [1,2]. In image relighting, the reflectance spectra can be used to estimate the color appearance of a scene under various illuminants [3,4]. In scene classification, spectral reflectance has been successfully applied as a classification and recognition descriptor that is robust to changes in illumination and geometry [5]. To summarize, more accurate reflectance reconstruction will promote the progress of these research topics.

A set of reflectance vectors \( r_i \in \mathbb{R}^n \) \((i = 1 \cdots m)\) can be represented by \( r_i = \sum_{j=1}^n a_j b_j, \) \( m \geq n, \) where \( b_j \) are known as basis function vectors. Spectral reflectance can be approximated by using only a few basis function vectors \([6,7]\), i.e., \( \hat{r}_i \approx \sum_{j=1}^d a_j b_j, \) \( d \ll n. \) In practice, the number of basis functions \( d \) usually corresponds to the channel number of an imaging device. The existing methods use either a multispectral camera system \((d > 3)\) [8] or a tricolor camera system \((d = 3)\) [2] for reflectance reconstruction.

Let \( R = [r_1, r_2, \ldots, r_m]^T \) and \( \hat{R} = [\hat{r}_1, \hat{r}_2, \ldots, \hat{r}_m]^T \) be two matrices that contain original and reconstructed reflectance vectors, respectively. The basis functions can be chosen by minimizing the following error function:

\[
\min_{b_j} \| R - \hat{R} \|_2^2 \quad j = 1, 2 \cdots d. \tag{1}
\]

The solution of Eq. (1) can be obtained by classical methods, such as singular value decomposition (SVD) or principal component analysis (PCA) [2,10–12]. We first define a centered matrix as \( X = [r_1 - \bar{r}, r_2 - \bar{r}, \ldots, r_m - \bar{r}]^T, \) where \( \bar{r} \) denotes the mean reflectance vector, i.e., \( \bar{r} = \frac{1}{m} \sum_{i=1}^m r_i. \) The SVD expression of \( X \) can be written as \( X = \sum_{j=1}^n \sigma_j u_j v_j^T, \) where \( \sigma_j, u_j, \) and \( v_j \) denote the singular values, \( m \)-dimensional left singular vectors, and \( n \)-dimensional right singular vectors of \( X, \) respectively.

Here \( \otimes \) denotes the tensor product of vectors. Similarly, the reconstructed spectral reflectance matrix by the PCA (or SVD) method can be obtained by

\[
\hat{R}_0 = \sum_{j=1}^d \sigma_j u_j v_j^T + h \otimes \bar{r}. \tag{2}
\]

Figure 1 shows one example of the original and reconstructed reflectance obtained by the PCA method. We note that the entire spectral reflectance receives equal treatments along different wavelengths in the PCA-based reconstruction process. Therefore, the error distribution between the original and reconstructed reflectance tends to be a uniform distribution as illustrated on the upper right-hand plot of Fig. 2. However, the sensor response in each channel usually has different sensitivities for

![Fig. 1. One example of reflectance reconstructed by the PCA method.](image-url)
In our experiment, we take XYZ color matching functions (400–700 nm) to test our method, since XYZ color space is the standard space in color, imaging, and related sciences. Figure 3 shows the XYZ color matching functions (left) and the WSF (right) generated by adding the three matching functions.

To the best of our knowledge, the reflectance spectral data introduced in [13] are the most complete data. They contain 1995 different surface reflectances, including the 24 Macbeth ColorChecker patches, 1269 Munsell chips, 120 Dupont paint chips, and 577 natural objects. Therefore, this reflectance dataset is employed to test our method.

By performing calculation on this reflectance dataset, we obtain that the SVDs of \( W^* \mathbf{R} \) and \( \mathbf{R} \) satisfy

\[
\left\| \sum_{j=d+1}^{n} \sigma_j \tilde{u}_j \tilde{v}_j^T \right\|_2^2 < \left\| W^* \sum_{j=d+1}^{n} \sigma_j u_j v_j^T \right\|_2^2.
\]  

In the following, we will prove that the reconstructed reflectance \( \hat{\mathbf{R}}_1 \) obtained by our method and \( \hat{\mathbf{R}}_0 \) obtained by the PCA method satisfy the following inequality:

\[
\left\| W^*(R - \hat{\mathbf{R}}_1) \right\|_2^2 < \left\| W^*(R - \hat{\mathbf{R}}_0) \right\|_2^2.
\]  

**Proof of inequality** (9).

\[
\left\| W^*(R - \hat{\mathbf{R}}_1) \right\|_2^2 = \left\| W^* \mathbf{R} - W^* \hat{\mathbf{R}}_1 \right\|_2^2
\]

\[
= \left\| \sum_{j=d+1}^{n} \sigma_j \tilde{u}_j \tilde{v}_j^T + W^* \mathbf{H} \otimes \tilde{r} \right\|_2^2
\]

\[
- W^* \left[ D^T \left( \sum_{j=1}^{d} \sigma_j \tilde{u}_j \tilde{v}_j^T + W^* \mathbf{H} \otimes \tilde{r} \right) \right] \right\|_2^2
\]

\[
\left\| \sum_{j=d+1}^{n} \sigma_j \tilde{u}_j \tilde{v}_j^T \right\|_2^2 < \left\| W^* \sum_{j=d+1}^{n} \sigma_j u_j v_j^T \right\|_2^2
\]

\[
\left\| W^*(R - \hat{\mathbf{R}}_0) \right\|_2^2.
\]

Inequality (9) shows that our reconstructed reflectance is more accurate than PCA-based reconstructed reflectance at wavelengths where sensor sensitivities are high.

The comparison of our results, the PCA-based results, and the original reflectance is plotted in Fig. 4. We also superimpose the WSF on the plot to illustrate the sensitivity peaks.
Figure 4 clearly shows that our reconstructed results are more accurate than the PCA-based reconstructed results near the wavelengths where WSFs are high. It also indicates that our results can achieve better imaging or color reproduction performance as a result of the small reconstruction errors at the sensitivity peaks. To evaluate the results quantitatively, we calculate the mean-square errors (MSEs) of $\|W(R - \hat{R}_0)\|_2^2$ and $\|W(R - \hat{R}_1)\|_2^2$ for the 1995 reflectance. To compare the performance of imaging or color reproduction for each reconstructed spectral reflectance, we also calculate the CIE XYZ tristimulus values under illuminant D65 and convert them to CIE Lab values. We then calculate the color difference between the actual and reconstructed Lab values. In Table 1, we statistically summarize these two results. Table 1 shows that our method achieves the overall smaller MSEs in terms of mean, minimum, and maximum errors. Our method also achieves more accurate color reproduction performance because of its smaller errors at the sensitivity peaks. For the statistically significant mean values, our method reduces MSE and Lab errors by 47% and 55%, respectively, when compared to the PCA-based method.

In this Letter, we propose a method to selectively control the WSF influence on the reflectance reconstruction process. Our reconstructed results are more accurate than the classical PCA-based results near the wavelengths corresponding to the peaks of the WSF. Consequently, our reconstructed results lead to more accurate color reproduction performance. These results further imply that our method will bring better performance in related applications, such as scene classification, imaging simulation, computer vision, and computer graphics.

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### References