Abstract—To improve the tracking accuracy of an underwater maneuvering target, according to its characteristics of low speed and weak maneuvering performance, an adaptive Kalman filter is given based on the online estimation of the process noise variance. As the main filter analyzes the target motion, the process noise variance of the main filter is estimated by an auxiliary filter for being adaptively adjusted according to the target maneuvering intensity to improve the target tracking accuracy for uniform motions, as well as improving response speed of the filter for maneuvering behavior of the target. Simulation results show that the proposed algorithm performs well, which, to a certain extent, effectively improves the tracking accuracy of an underwater maneuvering target.

Keywords—underwater target; maneuver; tracking; adaptive Kalman filter; active sonar

I. INTRODUCTION

Underwater maneuvering target tracking refers to estimating the state trajectories of a moving or movable underwater object via acoustic or non-acoustic sensors. Currently, the research on maneuvering target tracking focuses mainly on aerospace, which generally is divided into three categories. (1) Methods based on the target maneuvering detection [1], [2], [3], require rational detection criteria and thresholds which are designed according to characteristics of the residual in different application circumstance. When maneuver is detected to occur or be eliminated, the model used in the filter is converted or the parameters are adjusted. The time delay between the occurrence of the maneuver and its detection is unavoidable. (2) Methods based on the online identification of maneuvering acceleration and its statistical characteristics [4], [5], [6], which do not detect maneuver of targets, but need a priori assumption on the maneuvering characteristics of targets, perform well when targets maneuver but the performance declines as maneuver is eliminated. (3) Multiple model methods [7], [8], [9], which are a combination of the two methods mentioned above, to some extent, compensate for the disadvantages of them. These methods need extra effort such as predefining multiple sub-filters and updating the model transition probability, in addition to the large computational load imposed by using multiple sub-filters and low accuracy in tracking a constant velocity target. Unlike moving targets flying in the air, running on the ground or sliding on the surface of water, underwater targets with low speed and weak maneuvering performance, cannot make such a big maneuver. Generally, underwater moving targets are in a non-maneuvering motion, which is the straight and level motion at a constant velocity. Only under special circumstances, they maneuver with a small acceleration.

An adaptive filter algorithm based on the online estimation of the process noise variance is given. The algorithm uses an auxiliary filter to estimate the process noise variance of the main filter online and adjusts it adaptively according to the intensity of the target maneuver. The Kalman filter based on a constant velocity (CV) model effectively improves the precision of estimation for a non-maneuvering motion, while the process noise variance is increased adaptively by the algorithm to improve the tracking performance when the target maneuvers.

The rest of this paper is organized as follows. Section II describes the process and measurement model of the target tracking system. Then the structure and principle of the algorithm proposed are given in detail in section III. Section IV shows simulation results and relevant analysis. Finally, conclusions are drawn in section V.

II. SYSTEM MODELING

In the underwater maneuvering target tracking system, a target moving with nearly constant velocity is represented by a state vector with position and velocity as elements. The observations made can be assumed as a linear combination of the state vector corrupted by additive measurement noise due to the wave action and other physical characteristics of complex underwater environment.

The velocity of the target at the discrete time $t_{k+1}$ can be written as

$$v_{k+1} = v_k + T a_k + \bar{v}_k$$

(1)

where $T$ is the sampling interval; $\bar{v}_k$ is noise sequences of the velocity; and $a_k$ is the target acceleration at the discrete time $t_k$. A similar equation for the position $s$ of the target at the discrete time $t_{k+1}$ is expressed as
\[ s_{k+1} = s_k + T v_k + \frac{1}{2} T^2 a_k + \tilde{s}_k \]  
\[ x_{k+1} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2} T^2 \\ T \end{bmatrix} a_k + w_k \]  
where \( \tilde{s}_k \) is noise sequences of the position. So the process equation of the target is represented as:

\[ x_k = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x_k + \begin{bmatrix} \frac{1}{2} T^2 \\ T \end{bmatrix} a_k + w_k \]  
where \( x_k = [s_k \ v_k]^T \) is the process vector of the target; \( w_k = [\tilde{s}_k \ \tilde{v}_k]^T \) represents the trajectory perturbations due to uncertainty in the target state.

As the measurement vector contains only the position element, the linear equations of observation system can be represented as:

\[ z_k = [1 \ 0] x_k + v_k \]  
where \( z_k \) is the observation data; \( v_k \) is the measurement noise, which represents the inability of the tracking device to precisely measure the position of the target due to unavoidable errors in the measurement system. \( v_k \) and \( w_k \) are both random Gaussian white noise with variance \( Q \) and \( R \).

In an active sonar observation system, the three dimensional position of underwater targets is determined by the azimuth, elevation angle, range, Doppler shift, radial velocity and so on. The low accuracy of Doppler shift measured usually causes great positioning error. For ease of illustration, the active sonar and the underwater maneuvering target are assumed in the same depth. That means the target motion is assumed in the horizontal \( xy \) plane and the target motion state is estimated only through the azimuth and range with respect to the sonar. \( z_k \) is the observation data of positional coordinates along \( x \) or \( y \) in the Cartesian coordinate converted from the spherical coordinate[10].

The acceleration \( a_k \) can be assumed to be zero without disturbing the generality of the system for a target moving with a constant velocity, because of its weak underwater maneuvering performance.

Finally, the model of the system is represented as:

\[ x_{k+1} = F x_k + G w_k \]
\[ z_k = H x_k + v_k \]  
where \( F = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}, G = \begin{bmatrix} \frac{1}{2} T^2 \\ T \end{bmatrix}, H = [1 \ 0] \).

### III. ADAPTIVE KALMAN FILTER

The adaptive Kalman filter consists of a main filter and an auxiliary filter, as shown in Fig. 1.

The main filter and auxiliary filter are both standard Kalman filters. At each time step, the main filter receives an updated process noise variance estimated by the auxiliary filter to analyze the motion of the target, while the auxiliary filter estimates the process noise variance of the main filter according to the measurement residual and innovation covariance from the main filter. The main filter is able to work independently as an integrated Kalman filter, so when the target travels at a constant velocity, the auxiliary filter can be turned off to reduce the system computational burden.

The model of the process noise variance of the main filter is:

\[ Q_{k+1} = f(Q_k) + w_{Q_k}, w_{Q_k} \sim N(0, Q_o) \]  
\[ z_{Q_k} = g(Q_k) + v_{Q_k}, v_{Q_k} \sim N(0, R_o) \]  
where \( Q_k \) and \( z_{Q_k} \) are the value and observation of the process noise variance of the main filter, respectively, at the discrete time \( t_k \); \( w_{Q_k} \) and \( v_{Q_k} \) are process and measurement noise sequences, respectively; \( f(\cdot) \) and \( g(\cdot) \) are some transition functions.

Generally, the change rule of the process noise variance of the main filter is unknown, so it is assumed as irrelevant.
random drift. Equation (7) can be expressed as

$$Q_{k+1} = Q_k + w_{k}$$

(9)

to estimate the process noise variance of the main filter. The innovation variance of the main filter is chosen as the measurement prediction of the auxiliary filter,

$$\hat{z}_{k|k-1} = S_k$$

$$= H P_{k|k-1} H^T + R$$

$$= H F P_{k+1|k-1}(HF)^T + HG Q_{k+1} G^T H^T + R$$

(10)

Because of $Q_e \in R$, (10) can be expressed as

$$\hat{z}_{k|k-1} = HG(HG)^T Q_e + H F P_{k+1|k-1}(HF)^T + R.$$  

(11)

The measurement of the auxiliary filter is

$$z_{k|k} = \frac{1}{N} \sum_{i=1}^{N-e_k} e_k^T e_k$$

(12)

where $N$ and $e_k$ are the length of the slide window and residual of the main filter, respectively. $e_k$ is represented as

$$e_k = z_k - \hat{z}_{k|k-1}.$$  

(13)

In conclusion, the algorithm of the auxiliary filter is as follows:

Step 1: Initialization

$$\bar{Q}_o = E[Q_o]$$

$$P_{o|0} = E[(Q_o - \bar{Q}_o)(Q_o - \bar{Q}_o)^T]$$

(14)

Step 2: One step prediction

$$\hat{Q}_{k+1|k} = \bar{Q}_{k+1}$$

$$P_{o|k} = P_{o|k} + Q_o$$

$$\hat{z}_{k|k} = S_k$$

(15)

Step 3: Kalman gain

$$K_{o} = P_{o|k} \cdot H_{o}^T (H_{o} \cdot P_{o|k} \cdot H_{o}^T)^{-1}$$

(16)

Step 4: Update state estimation and its covariance

$$P_{o|k+1} = (I - K_{o} \cdot H_{o}^T)P_{o|k}$$

$$\hat{Q}_o = \hat{Q}_{k+1} + K_{o} (z_{k|k} - \hat{z}_{k|k})$$

(17)

where $H_{o} = HG(HG)^T$.

IV. SIMULATIONS AND ANALYSIS

A long range target tracking application is simulated to examine the effects of the algorithm proposed on target state estimation. The target is assumed to be on the $x-y$ plane as discussed above, while an active sonar, as the sensor, is assumed fixed at the origin. The initial target location is at (1500,1000) m with initial velocity of 4 knots heading along -135° line to the x-axis. The target trajectory includes process noise (white noise acceleration) with a standard deviation 0.01 m/s^2 in each coordinate. The measurement variance is statistically obtained through actual trials, which, for the sake of convenience, is assumed as $R=100$ m^2. Tracking is performed using 163 measurements obtained with sampling interval of $T=1$ s. Tracks are initiated with two point differencing [11] to obtain the initial velocity estimate. The scenario is set as below: The target starts at a constant velocity of 4 knots from 1s to 50s. Then the target maneuvers for a 165° turn uniformly. It completes the turn at 83s and then takes an acceleration straight course from 4 to 9 knots until 113s. At last, it travels at a constant velocity of 9 knots from 114s to 163s.

Two filters, a conventional Interacting Multiple Model (IMM) algorithm and a standard Kalman filter based on the CV model are simulated to compare with the algorithm proposed.

The model set of the conventional IMM algorithm consists of a CV model and a constant acceleration (CA) model. Because the IMM algorithm has a powerful robustness for the selection of the transition probability matrix [12], which influences the results weakly between [0.8,0.95], the transition probability matrix is assumed to be $P = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$. The initial model probability for each sub-model is chosen as $u_o = [1, 0]$, as the underwater target usually travels at a constant velocity. The process noise variance of the two sub-model is set as $Q_{cv} = 0.01, Q_{ca} = 0.01$.

Considering the tracking accuracy of non-maneuvering motion and response speed of maneuvering motion, the standard Kalman filter based on the CV model has process noise variance $Q=0.05$, which makes it convenient and clarity to compare with algorithm proposed.

As the process noise variance of the adaptive Kalman filter is adjusted online according to the actual situation, the initial process noise variance is set as a small value, while other parameters are adjusted based on the actual simulation circumstances.

Figure 2 shows simulation results of the underwater maneuvering target tracking for the comparison among three kinds of algorithms. The figure shows the root mean square error (RMSE) curve in X-axis based on 100 Monte Carlo runs. The simulation curve shows that the adaptive Kalman filter has similar tracking accuracy with the standard Kalman filter when the target travels at a constant velocity. While the target maneuvers, because the process noise variance of the adaptive Kalman filter adaptively increases to improve the response speed, as shown in Fig. 4, the tracking performance is much better than the standard Kalman filter. Compared with the conventional IMM algorithm, the adaptive Kalman filter and the standard Kalman filter have better tracking accuracy than the IMM algorithm when the target travels at a constant velocity. However, the performance of the standard Kalman
filter is significantly worse than the IMM algorithm with a much larger tracking error when the target maneuvers. The adaptive Kalman filter and the IMM algorithm perform considerably but the algorithm proposed is a little worse than the latter.

V. CONCLUSIONS

An adaptive Kalman filter has been given for tracking an underwater maneuvering target and its feasibility has been examined. The algorithm brings in an auxiliary filter to estimate the process noise variance of the main filter online and adjust it adaptively. Simulation results show that the algorithm has a better tracking accuracy for non-maneuvering motion of the target, and improves the response speed when the target maneuvers by increasing the process noise variance adaptively to reduce the tracking error. Simulation results verify the effectiveness of the algorithm used in underwater maneuvering target tracking. In the premise of maintaining high tracking accuracy for the uniform motion, further improving the tracking performance of the filter when the target maneuvers is further research directions.

REFERENCES