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TOOL ORIENTATION PLANNING FOR FIVE-AXIS CNC MACHINING OF OPEN FREE-FORM SURFACES*

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Abstract For the geometry characteristics of open free-form surfaces, it is hard to consider global interference during the planning of feasible domains. Therefore, the optimal kinematic orientation of tool axis will no longer be confined to the boundary of feasible domains. In this paper, according to the principle demanding that the tool should be fitted to a surface as close as possible and relevant processing parameters, a feasible domain of tool orientation for each cutter contact is planned in the local feed coordinates system. Then, these feasible domains of the tool orientation are transformed into the same coordinates system of the machine tool by the inverse kinematics transformation. The linear equations based feasible domain method and Rosen gradient projection algorithm are used to improve the optimization process in precision and efficiency of the algorithm. It constructs the variation of tool orientation optimization model and ensures the smoothness of tool orientation globally. Simulation and analysis of examples show that the proposed method has good kinematics performance and greatly improves the efficiency.

Keywords Feasible domains, five-axis CNC machining, open free-form surface, planning tool orientation.

1 Introduction

The five-axis CNC machine tool is originated from traditional three-axis ones by adding two rotary axis. It can make the tool position and orientation with respect to workpieces arbitrarily

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controllable, which improves the accuracy, quality, and efficiency of free-form surface machining and is conducive to the integration of manufacturing systems[1, 2]. However, the controllability of tool orientation in a certain range makes tool path planning more complex. The tool orientation directly affects machining efficiency, since the speed, acceleration of rotation axis are limited in a certain range. Therefore, the kinematics performance of tool orientation is of great significance to the machining quality and stability.

In recent years, many domestic and foreign scholars focused on the tool orientation planning problems. Its basic pipeline is first tool path planning[3], then tool orientation planning (i.e., modify the tool orientation at each cutter contact point to satisfy some criterion). Through quaternion interpolation, Ho, et al.[4] reduced the variation of adjacent tool orientation in order to decrease the nonlinear error in cutting process, finally obtaining better surface quality; Beudaert, et al.[5] improved the local curvature change of feed rate and reduced the global feed rate change by smoothing the motion of each axis for a machine tool; Jun, et al.[6] introduced configuration-space search method into the smoothing of five-axis tool orientation. In recent ten years many scholars have done a lot of research work on tool orientation. Wang and Tang[7] determined the tool orientation accessible domain through the calculation of tool orientation accessible range under angular velocity limits and the non-interference range of intersection at each cutter contact points; Based on multi-axis machine tool stiffness model, Yan, et al.[8] planned tool orientation in the accessible spaces to optimize the multi-axis machine tool stiffness performance parameters; Li and Zhang[9] developed a tool-path generation method targeting at both posture smoothing and machining efficiency; The feasible domain of the tool orientation in Reference [10] optimize the tool orientation under multiple constraints to satisfy geometric characteristics and kinematic performance of machine tool; Bi, et al.[11] proposed an analogy of the spring mechanics overall grid optimization model to smooth tool orientation between adjacent rows. Although the feasible domain and their boundaries can be quickly obtained by edge detection techniques in computer vision, the boundary and domain are node data making precision of the following optimization be limited to the density of these nodes. Meanwhile, feasible domain, in a node sense, of the optimization algorithm costs a lot of computing resources in selecting nodes, which seriously affects the efficiency of tool orientation planning algorithms.

For the above reasons, this paper proposes a feasible domain constrained by linear boundary equations and the evaluation model. Simulation and analysis show that the tool orientation sequences have good kinematical performance and efficiency.

2 Estimation of Feasible Domain and Boundary Equations

For five-axis CNC machining, tool orientation can be described in three different coordinates systems. As shown in Figure 1, the tool feed vector f, the surface normal vector n at some cutter contact point. Through the right-hand coordinates system principles, a feed coordinates system $Ofcn$ at the cutter contact point can be established; As shown in Figure 2, tool lead angle α, the tool go around feed direction c rotation angle between tool orientation and normal plane; tool tilt angle β, the tool go around feed direction f rotation angle.

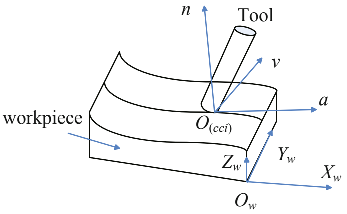


Figure 1 Feed coordinate and workpiece coordinate

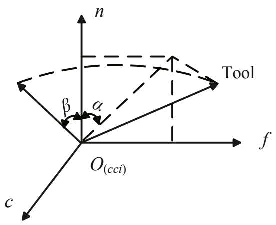


Figure 2 Lead angle and tilt angle representation

Marine propeller blades, aircraft cockpit surfaces and other parts often have open surfaces. Open geometrical features of these machined surface make the planning of feasible domain for tool orientation free from the need to consider global interference. Generally, surfaces fill into two categories, convex curved surfaces, and concave curved surfaces. When the machined surface is convex, tool angle α can vary from 0° to 90° and the effective radius of curvature varies from the infinite to the tool radius. While, when the machined surface is a concave surface, local-gouging will occur between the tool cutting ellipse and machined surface (as shown in Figure 3(a), $\alpha = 2°$). Based on the differential geometry theory, the tool cutting elliptical curvature is

$$K_c = \frac{\sin\alpha}{R}, \tag{1}$$

where K_c is the curvature of tool cutting ellipse, α is the tool lead angle, and R is the tool radius.

In order to avoid local-gouging phenomenon in the machining, the effective cutting curvature should be no less than the machined surface curvature, that is, α must satisfy the following equation:

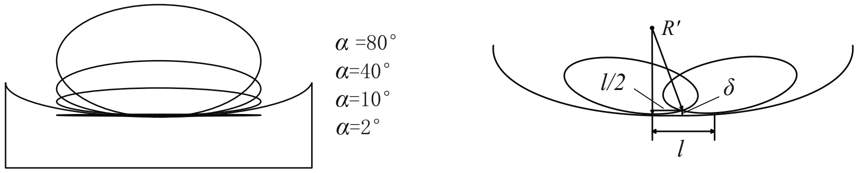
$$\frac{\sin\alpha}{R} \geq K_i \Rightarrow \alpha \geq \sin^{-1}(RK_i), \tag{2}$$

where K_i is the surface normal curvature along side direction.

In five-axis CNC milling, scallop height δ is determined by the side step l and surface curvature K_c. As illustrated in Figure 3(b), the scallop is formed by the boundaries of two adjacent cutter-end faces and the surface cross-section. In order to make scallop height δ less than the permitted range, α must satisfy

$$\alpha \leq \sin^{-1}\left(\frac{R'}{l^2/8\delta + \delta/2}\right), \tag{3}$$

where R' is the effective cutting ellipse radius.



(a) Effective cutting elliptical for Concave surface

(b) Description for scallop-height approximation

Figure 3 The relationship between tool section and the machined surface

Tool orientation domain is shown in Figure 4(a) without considering the global interference and $0 < \alpha < \pi/2$, $-\pi/2 < \beta < \pi/2$. Tool orientation domain is shown in Figure 4(b) and Figure 4(c) considering the machined surface convexity and tool section as well as surface fitting.

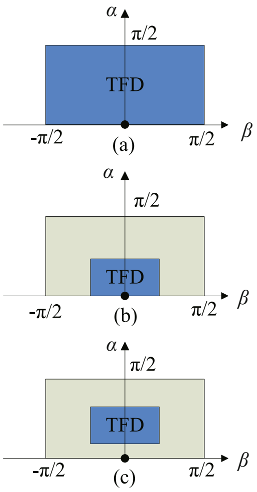


Figure 4 Tool orientation feasible domain (TFD)

Tool orientation is often described in feed coordinates system, workpiece coordinates system, and machine coordinates system, as shown in Figure 5. It adopts table rotation around A-tool spindle rotation around B type of machine tool to explain the calculating of the boundary of the feasible region in tool axis orientation planning. In the feed coordinates system, the tool orientation domain at each cutter contact point, calculated according to differential geometry information of the machined surface and preset machining parameters, must be transformed into the same machine coordinates system. Linear constraint equations of formula 4 are used to bound the boundary of the tool orientation feasible domain, as shown in Figure 6.

$$\begin{bmatrix} a_{i1} & b_{i1} \\ a_{i2} & b_{i2} \\ a_{i3} & b_{i3} \\ a_{i4} & b_{i4} \end{bmatrix} \times \begin{bmatrix} A_i \\ B_i \end{bmatrix} \leq \begin{bmatrix} c_{i1} \\ c_{i2} \\ c_{i3} \\ c_{i4} \end{bmatrix}, \quad \text{i.e.,} \quad C_i x_i \leq D_i, \tag{4}$$

where $|a_{i,n}, b_{i,n}|, |c_{i,n}|, n \in [1, 4]$ are the coefficient matrix of the i-th cutter contact at the linear boundary equations.

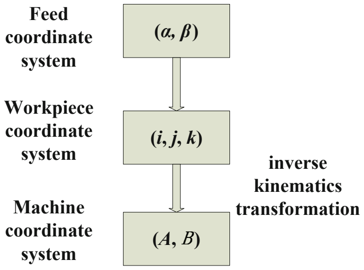


Figure 5 Tool orientation described in the three coordinate system

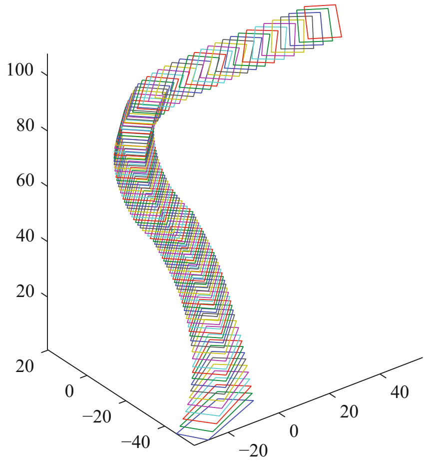


Figure 6 The global tool axis orientation feasible domain

3 Optimization Model

In machine coordinates system, tool orientation can be expressed as the feasible set $\{(A_i, B_i)\}$ and a space curve L can be formed along feed direction at each posture point (A_i, B_i), as (see Figure 7)

$$L = f(s) = (A(s), B(s), s). \tag{5}$$

Therefore, the tool orientation optimization problem turns out to be a problem smoothing a space curve L. The first order partial derivative of the function L with respect to A, B donate the changes in the angle of axis A and B, respectively, i.e., angular velocities of rotation axis. And the second order partial derivative of the function L with respect to A, B in the changes of angular velocity, i.e., angular accelerations of rotation axis.

The minimization of the global angle variation is computed in the form as

$$\min\sum_{i=1}^{n-1}\left[\left|\frac{A_{i+1}-A_i}{\Delta s}\right|+\left|\frac{B_{i+1}-B_i}{\Delta s}\right|\right],\ (A_i,B_i)\in TFD. \tag{6}$$

The minimization of the global angular velocity variation is computed in the form as

$$\min\sum_{i=1}^{n-2}\left[\left|\frac{A_{i+2}+A_i-2A_{i+1}}{\Delta^2 s}\right|+\left|\frac{B_{i+2}+B_i-2B_{i+1}}{\Delta^2 s}\right|\right],\ (A_i,B_i)\in TFD. \tag{7}$$

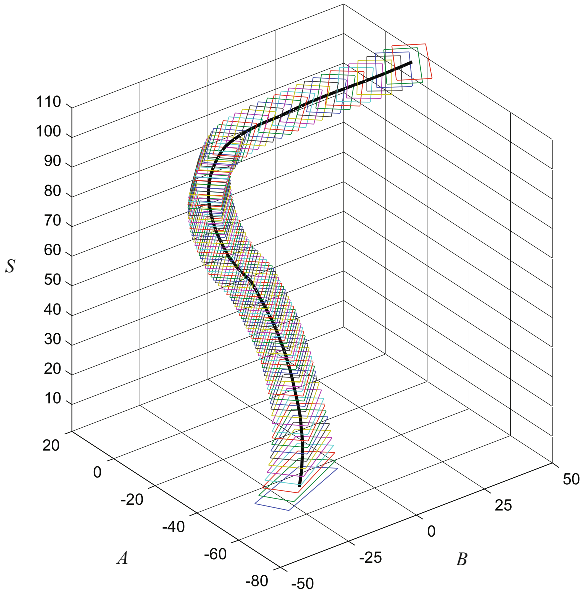


Figure 7 The global tool axis orientation sequence

4 Model Solution

With the extreme optimization model, the objective function of the steepest descent direction is the negative gradient direction[12]. However, since the constraints of tool orientation domain (see Equation (4)), it is possible that infeasible points may be touched along the steepest descent direction. Therefore, this paper applies the Rosen gradient projection method[13] to solve the optimization models, which can improve the objective function value and, meanwhile, can be maintained in the feasible domain of tool orientation. Specifically, there are mainly two cases here: 1) When an iterative point is inside the feasible domain, the negative gradient direction is set to be its descent direction; 2) When the point is on the boundary, the projection of the negative gradient direction to the boundary is chosen. The procedure is showed in Figure 8.

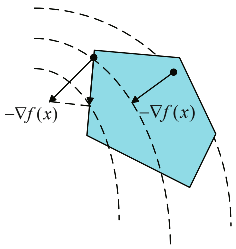


Figure 8 The gradient projection method of Rosen

The Rosen gradient projection method mathematical model is given as:

$$\begin{cases} \min f(x) \\ ax \geq b. \end{cases} \tag{8}$$

The object function $f(x)$ is the tool orientation planning model (Equation (6) or Equation (7)); Where $ax \geq b$ is the constraint function for extreme value model; At each cutter contact point, the linear boundary matrix $C_i x_i \leq D_i$ should be transformed to constraint function with Equation (8).

5 Examples and Analysis

In this paper, an example based on 3D ACIS Modeler is chosen to plan tool path and calculate tool orientation domain. Through the Rosen gradient projection method, a smooth tool orientation sequence can be obtained.

In the experiment, a flat-end mill tool is chosen and its radius is 10mm. the tool path is planned by uniform regulations, with even step method of cutter contact point sequence, the adjacent row step l is set to 18mm, and scallop height δ is set to 0.1mm.

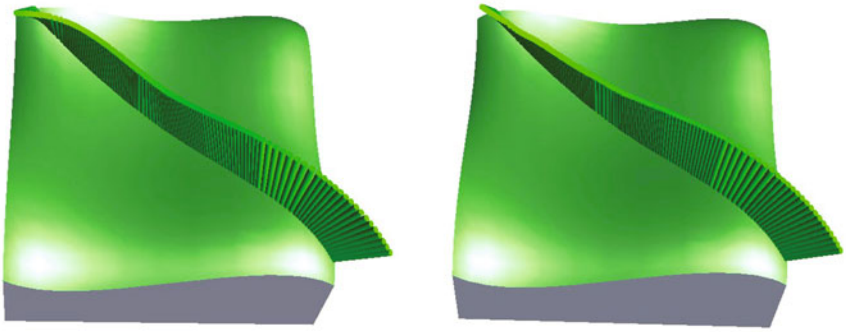

(a) Traditional method (b) Smoothed method

Figure 9 Tool orientation 3D simulation

In order to illustrate the global tool axis smoothing problem better, this paper adopts the traditional fixed tool lead angle and tool tilt angle method for planning a comparative example (as shown in Figure 9(a)). In Figure 9, there was no significant difference between the smoothing method and traditional method. But through the analysis of motion data for axis, we can find that the smoothing method has better kinematics performance than the traditional one. The smoothing method holds relatively stable angular velocity and angular acceleration, as in Figures 10 and 11.

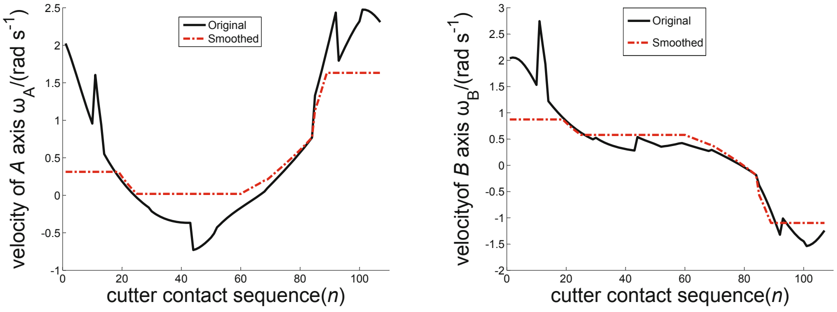


Figure 10 Angular velocity on axis A and B for original and smoothed

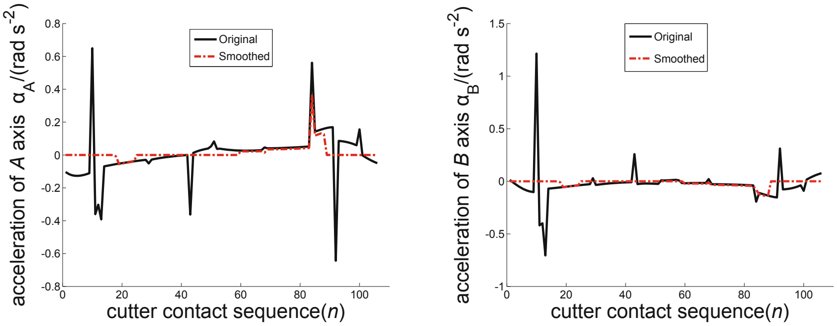


Figure 11 Angular acceleration on axis A and B for original and smoothed

Table 1 Different feasible domain method efficiency comparison

	Global tool axis orientation variation $(A_i^2 + B_i^2)$	Computing time (s)
Node feasible domain method	134.780	3294.21
Equation of constraint method method	120.902	13.501

In order to verify the computation efficiency of the proposed method the author used the method proposed in [14] to obtain the global optimization tool path with the Dijkstra algorithm. Comparison of the two methods about tool orientation variation and the computation time is show in Table 1. It is clear that the feasible domain method constructed with linear boundary constraint equations has higher calculation efficiency.

6 Conclusion

In this paper, a new approach is proposed to plan tool orientation for open free-form surfaces. The feasible domain is bounded by linear equations implying that the domain is a planar region instead of grid points. Thus, the planning of tool orientation won't be affected by the grid density, as opposed to conventional discrete feasible domains. The Rosen gradient projection method is then exploited to search for the optimal tool orientation. The proposed method can obtain the optimal solution for tool orientation planning. This method can be directly applied to surfaces without global interference.

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