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Stabilization for Networked Control Systems with Variable Sampling Intervals ⋆

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Abstract

In this paper, the problem of stabilization for networked control systems (NCSs) with variable sampling intervals is investigated. Taking the network induced delays and the variable sampling intervals into consideration in the same time, the NCS model is presented. By exploiting a novel Lyapunov-Krasovskii functional, using the Leibniz-Newton formula and the free-weighting matrix method, the sufficient conditions of the stability analysis and stabilization controller design for such systems are respectively given. Numerical examples demonstrate the effectiveness of the proposed methods.

Keywords: Networked Control Systems(NCSs); Sampled-data Control; Stabilization

1 Introduction

With the rapid development of computer and communication technology, networks play important roles in many industrial control applications. NCSs receive more and more attention and become more and more popular in many practical applications in recent years. Stability analysis, stabilization, and H_∞ control of network-based feedback systems have been receiving increasing attention [1–4].

In networked control systems, the digital control, digital filtering, and signal processing are widely used, which make the closed-loop systems hybrid. This kind of the systems is usually referred as the sampled-data control systems which simultaneously contain continuous-time and discrete-time signals. NCSs have been studied extensively by many researchers using sampled-data control theory over the past decades [5–7]. These works are generally appropriate for fixed sampling interval. In networked control systems with a limited bandwidth, the strategy of variable

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sampling interval could achieve better performance than the one of fixed sampling interval. Recent interest in sampled-data systems with uncertain sampling interval is mainly from networked control systems [8]. Recently, some research focus on the sampled-data control problems of linear systems [9–12]. To the best of the author's knowledge, the sampled-data control problems of NCSs with variable sampling have not been fully investigated and still remain challenging.

2 Problem Formulation

Consider the following system:

$$\dot{x}(t) = Ax(t) + Bu(t), \tag{1}$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^p$ is the control input. A, B are some constant matrices of appropriate dimensions.

Define the zero-order hold control action $u_c(t) = u_d(s_k)$, $s_k \le t < s_{k+1}$, where u_d is a discrete-time control signal and the time s_k is the sampling instant satisfying $0 = s_0 < s_1 < \cdots < s_k < \cdots$.

Modeling of continuous-time systems with digital control in the form of continuous-time systems with delayed control input was introduced by [9]. The digital control law may be represented as delayed control as follows:

$$\begin{aligned} u_c(t) = u_d(s_k) = u_d(t - (t - s_k)) = u_d(t - \tau(t)), \\ s_k \le t < s_{k+1}, \tau(t) = t - s_k, \end{aligned} \tag{2}$$

where u_d is a zero-order control signal and the time-varying delay $\tau(t) = t - s_k$ is piecewise linear with the derivative $\dot{\tau}(t) = 1$ for $t \ne s_k$. The sampling interval $T_k = s_{k+1} - s_k$ may vary but it is bounded. In this paper, it is assumed that T_k is time-varying and its lower bound and upper bound are known: $0 < T_{min} \le T_k \le T_{max}$ for all k.

In NCSs, the control signal from the sampler at s_k takes δ_k time units to reach the actuator, then $u(t) = u_c(t-\delta_k) = u_d(t-\delta_k-\tau(t-\delta_k))$. In the following, $d(t) = \tau(t-\delta_k)+\delta_k$, so, $s_k = t-d(t)$ for $s_k + \delta_k \le t \le s_{k+1} + \delta_{k+1}$. Note that the variable sampling interval and networked delay are integrated in a single input delay $d(t)$. Network communication delay δ_k is naturally assumed as

$$\delta_m \le \delta_k \le \delta_M,$$

where δ_m and δ_M denote the minimum communication delay and the maximum communication delay, respectively.

Our objective is to find a state-feedback controller of the form

$$u(t) = Kx(s_k),\ s_k + \delta_k \le t < s_{k+1} + \delta_{k+1}, \tag{3}$$

which stabilizes the system (1).

Substituting the latter controller into (1), and setting $t_k = s_k + \delta_k$ $(k = 0, 1, 2, \cdots .)$, we can obtain the closed-loop sampled-data networked control system:

$$\dot{x}(t) = Ax(t) + BKx(t - d(t)), t_k \le t < t_{k+1}. \tag{4}$$

It is assumed that the delay $d(t)$ is a bounded function that satisfies

$$0 \le h_1 \le d(t) \le h_2,\quad t_k \le t < t_{k+1}, \tag{5}$$

where the time-varying delay $d(t) = t - s_k$ is piecewise linear with the derivative $\dot{d}(t) = 1$ for $t \ne t_k$.

3 Main Results

3.1 Stability analysis

Assuming that the matrices A, B and the control gain K in the system (4) are known, we shall develop a condition under which the closed-loop system in (4) is asymptotically stable.

Theorem 1. For given scalars h_1, h_2 $(0 \le h_1 < h_2)$, $0 \le \alpha < 1$, and a matrix K, system (4) is asymptotically stable if there exist matrices $P = P^T > 0$, $Q_m = {Q_m}^T > 0$ $(m = 1, 2, 3)$, $Z_j = {Z_j}^T > 0$ $(j = 1, 2, 3)$, and N_i, T_i, M_i, E_i, L_i $(i = 1, 2, 3, 4, 5)$ such that

$$\Pi = \begin{bmatrix} \Pi_1 & \Pi_2 \\ * & \Pi_3 \end{bmatrix} < 0, \tag{6}$$

$$\Pi_1 = \begin{bmatrix} \Gamma_{11} + A^TUA & \Gamma_{12} + A^TUBK & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & \Gamma_{22} + (BK)^TUBK & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \Gamma_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix}, \Pi_2 = \begin{bmatrix} N_1 & T_1 & M_1 & E_1 & L_1 \\ N_2 & T_2 & M_2 & E_2 & L_2 \\ N_3 & T_3 & M_3 & E_3 & L_3 \\ N_4 & T_4 & M_4 & E_4 & L_4 \\ N_5 & T_5 & M_5 & E_5 & L_5 \end{bmatrix},$$

$$\begin{aligned}
\Gamma_{11} =& PA + A^TP + \sum_{i=1}^{3} Q_i + N_1 + N_1^T + L_1 + L_1^T, \\
\Gamma_{12} =& PBK + N_2^T - T_1 + M_1 - E_1 + L_2^T, \Gamma_{13} = E_1 + N_3^T + L_3^T + L_1, \\
\Gamma_{14} =& - M_1 + N_4^T + L_4^T, \Gamma_{15} = T_1 - N_1 + N_5^T + L_5^T, \\
\Gamma_{22} =& M_2 + M_2^T - T_2 - T_2^T - E_2 - E_2^T, \Gamma_{23} = E_2 + M_3^T - T_3^T - E_3^T + L_2, \\
\Gamma_{24} =& - M_2 + M_4^T - T_4^T - E_4^T, \Gamma_{25} = T_2 - N_2 + M_5^T - T_5^T - E_5^T, \\
\Gamma_{33} =& - Q_1 + E_3 + E_3^T + L_3 + L_3^T, \Gamma_{34} = -M_3 + E_4^T + L_4^T, \\
\Gamma_{35} =& T_3 - N_3 + E_5^T + L_5^T, \Gamma_{44} = -Q_2 - M_4 - M_4^T, \Gamma_{45} = T_4 - N_4 - M_5^T, \\
\Gamma_{55} =& - (1 - \alpha)Q_3 + T_5 - N_5 + T_5^T - N_5^T, U = h_2Z_1 + h_{12}Z_2 + h_2Z_3, h_{12} = h_2 - h_1. \\
\Pi_3 =& diag \begin{bmatrix} -\frac{1}{\alpha h_2}Z_1 & -\frac{1}{(1-\alpha)h_2}Z_1 & -\frac{1}{h_{12}}(Z_1 + Z_2) & -\frac{1}{h_{12}}Z_2 & -\frac{1}{h_2}Z_3 \end{bmatrix}.
\end{aligned}$$

Proof. Consider the following Lyapunov-Krasovskii functional

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) + V_7(t), \tag{7}$$

where

$$\begin{aligned}
V_1(t) =& x^T(t)Px(t),\ V_2(t) = \int_{t-h_1}^{t} x(s)^TQ_1x(s)ds,\ V_3(t) = \int_{t-h_2}^{t} x(s)^TQ_2x(s)ds, \\
V_4(t) =& \int_{t-\alpha d(t)}^{t} x(s)^TQ_3x(s)ds, V_5(t) = \int_{-h_2}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z_1\dot{x}(s)dsd\beta, \\
V_6(t) =& \int_{-h_2}^{-h_1}\int_{t+\beta}^{t} \dot{x}^T(s)Z_2\dot{x}(s)dsd\beta, V_7(t) = \int_{-h_2}^{0}\int_{t+\beta}^{t} \dot{x}^T(s)Z_3\dot{x}(s)dsd\beta,
\end{aligned}$$

$P = P^T > 0$, $Q_m = {Q_m}^T > 0$ $(m = 1, 2, 3)$, $Z_j = {Z_j}^T > 0$ (j=1, 2, 3).

1). For $t_k < t < t_{k+1}$, calculating the derivative of $V(t)$ with respect to t along the solutions of the system (4) and using Leibniz-Newton formula, it yields that

$$\begin{aligned}
&\dot{V}(t) = 2x^T(t)P\dot{x}(t) + \sum_{i=1}^{3} x^T(t)Q_i x(t) - x^T(t-h_1)Q_1 x(t-h_1) - x^T(t-h_2)Q_2 x(t-h_2)\\
&-(1-\alpha)x^T(t-\alpha d(t))Q_3 x(t-\alpha d(t)) + \dot{x}^T(t)(h_2 Z_1 + h_{12}Z_2 + h_2 Z_3)\dot{x}(t) - \int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds\\
&-\int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-h_2}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds - \int_{t-d(t)}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-h_2}^{t} \dot{x}^T(s)Z_3\dot{x}(s)ds\\
&\le 2x^T(t)P(Ax(t) + BKx(t-d(t)) + \sum_{i=1}^{3} x^T(t)Q_i x(t) - x^T(t-h_1)Q_1 x(t-h_1)\\
&-x^T(t-h_2)Q_2 x(t-h_2) - (1-\alpha)x^T(t-\alpha d(t))Q_3 x(t-\alpha d(t))\\
&+\dot{x}^T(t)(h_2 Z_1 + (h_2-h_1)Z_2 + h_2 Z_3)\dot{x}(t) - \int_{t-\alpha d(t)}^{t} \dot{x}^T(s)Z_1\dot{x}(s)ds - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}^T(s)Z_1\dot{x}(s)ds\\
&-\int_{t-h_2}^{t-d(t)} \dot{x}^T(s)(Z_1+Z_2)\dot{x}(s)ds - \int_{t-d(t)}^{t-h_1} \dot{x}^T(s)Z_2\dot{x}(s)ds - \int_{t-h_2}^{t} \dot{x}^T(s)Z_3\dot{x}(s)ds\\
&+2\zeta^T(t)N[x(t) - x(t-\alpha d(t)) - \int_{t-\alpha d(t)}^{t} \dot{x}(s)ds] + 2\zeta^T(t)T[x(t-\alpha d(t)) - x(t-d(t)) - \int_{t-d(t)}^{t-\alpha d(t)} \dot{x}(s)ds]\\
&+2\zeta^T(t)M[x(t-d(t)) - x(t-h_2) - \int_{t-h_2}^{t-d(t)} \dot{x}(s)ds]\\
&+2\zeta^T(t)E[x(t-h_1) - x(t-d(t)) - \int_{t-d(t)}^{t-h_1} \dot{x}(s)ds] + 2\zeta^T(t)L[x(t) - x(t-h_2) - \int_{t-h_2}^{t} \dot{x}(s)ds]\\
&\le \zeta^T(t)(\Gamma_1 + \alpha h_2 N Z_1^{-1} N^T + (1-\alpha)h_2 T Z_1^{-1} T^T + h_{12} M (Z_1+Z_2)^{-1} M^T\\
&+h_{12} E Z_2^{-1} E^T + h_2 L {Z_3}^{-1} L^T + \bar{A}^T U \bar{A})\zeta(t) - \int_{t-\alpha d(t)}^{t} \mathcal{H}_1 Z_1^{-1} {\mathcal{H}_1}^T ds - \int_{t-d(t)}^{t-\alpha d(t)} \mathcal{H}_2 Z_1^{-1} {\mathcal{H}_2}^T ds\\
&-\int_{t-h_2}^{t-d(t)} \mathcal{H}_3 (Z_1+Z_2)^{-1} {\mathcal{H}_3}^T ds - \int_{t-d(t)}^{t-h_1} \mathcal{H}_4 Z_2^{-1} {\mathcal{H}_4}^T ds - \int_{t-h_2}^{t} \mathcal{H}_5 Z_3^{-1} {\mathcal{H}_5}^T ds,
\end{aligned}$$

where

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \Gamma_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix}, N = \begin{bmatrix} N_1 \\ N_2 \\ N_3 \\ N_4 \\ N_5 \end{bmatrix}, T = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \end{bmatrix}, M = \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ M_4 \\ M_5 \end{bmatrix}, E = \begin{bmatrix} E_1 \\ E_2 \\ E_3 \\ E_4 \\ E_5 \end{bmatrix},$$

$$\begin{aligned}
L &= \begin{bmatrix} L_1^T & L_2^T & L_3^T & L_4^T & L_5^T \end{bmatrix}^T, \mathcal{H}_1 = \zeta^T(t)N + \dot{x}^T(s)Z_1, \mathcal{H}_2 = \zeta^T(t)T + \dot{x}^T(s)Z_1,\\
\mathcal{H}_3 &= \zeta^T(t)M + \dot{x}^T(s)(Z_1+Z_2), \mathcal{H}_4 = \zeta^T(t)E + \dot{x}^T(s)Z_2, \mathcal{H}_5 = \zeta^T(t)L + \dot{x}^T(s)Z_3,\\
\bar{A} &= \begin{bmatrix} A & BK & 0 & 0 & 0 & 0 \end{bmatrix},\\
\zeta(t) &= \begin{bmatrix} x^T(t) & x^T(t-d(t)) & x^T(t-h_1) & x^T(t-h_2) & x^T(t-\alpha d(t)) \end{bmatrix}^T.
\end{aligned}$$

By the Schur complements, combining (6), we obtain $\dot{V}(t) < 0$ for all $t_k < t < t_{k+1}$.

2). It is noting that $d(t_k) = t_k - s_k, \forall k \in N$, $d(t_k^-) = t_k - s_{k-1}, \forall k \in N$.

The value of x before and after t_k pionts remains unchanged (since x(t) is continuous). Then, we have $V_i(t_k^-) = V_i(t_k)$ $(i = 1, 2, 3, 5, 6, 7)$ in Lyapunov-Krasovskii functional (7). Moreover, for $V_4(t)$, there exists $V_4(t_k^-) \geq V_4(t_k)$. Thus, $V(t_k^-) \geq V(t_k)$ for $k = 0, 1, 2, 3, \cdots$. For $t \in \left[\ t_k,\ \ t_{k+1}\ \right)$, we have $V(t) - V(t_k) \leq 0$. Since $\lim_{k\to\infty} t_k = \infty$, we have $\bigcup_{k=0}^{\infty} \left[\ t_k\ \ t_{k+1}\ \right) = \left[\ t_0,\ \ \infty\ \right)$. It follows that

$$V(t) - V(t_0) \leq 0. \tag{8}$$

This completes the proof.

4 Stabilization Controller Design

This section is devoted to solve the problem of stabilization controller design for NCSs with variable sampling. The following theorem presents the conditions for the desired controller design.

Theorem 4. For given scalars h_1, h_2 $(0 \leq h_1 < h_2)$, and $0 \leq \alpha < 1$, the sampled-data closed-loop system (4) is asymptotically stable if there exist matrices $X = X^T > 0$, $\tilde{Q}_m = \tilde{Q}_m^T > 0$ $(m = 1, 2, 3)$, $\tilde{Z}_j = \tilde{Z}_j^T > 0$ (j=1, 2, 3), and $\tilde{N}_i$, $\tilde{T}_i$, $\tilde{M}_i$, $\tilde{E}_i$, $\tilde{L}_i$ $(i = 1, 2, 3, 4, 5)$ such that

$$\tilde{\Gamma} = \begin{bmatrix} \tilde{\Gamma}_1 & \tilde{\Gamma}_2 \\ * & \tilde{\Gamma}_3 \end{bmatrix} < 0, \tag{9}$$

where

$$\tilde{\Gamma}_1 = \begin{bmatrix} \tilde{\Gamma}_{11} & \tilde{\Gamma}_{12} & \tilde{\Gamma}_{13} & \tilde{\Gamma}_{14} & \tilde{\Gamma}_{15} \\ * & \tilde{\Gamma}_{22} & \tilde{\Gamma}_{23} & \tilde{\Gamma}_{24} & \tilde{\Gamma}_{25} \\ * & * & \tilde{\Gamma}_{33} & \tilde{\Gamma}_{34} & \tilde{\Gamma}_{35} \\ * & * & * & \tilde{\Gamma}_{44} & \tilde{\Gamma}_{45} \\ * & * & * & * & \tilde{\Gamma}_{55} \end{bmatrix}, \tilde{\Gamma}_2 = \begin{bmatrix} \tilde{N}_1 & \tilde{T}_1 & \tilde{M}_1 & \tilde{E}_1 & \tilde{L}_1 & XA^T & XA^T & XA^T \\ \tilde{N}_2 & \tilde{T}_2 & \tilde{M}_2 & \tilde{E}_2 & \tilde{L}_2 & Y^TB^T & Y^TB^T & Y^TB^T \\ \tilde{N}_3 & \tilde{T}_3 & \tilde{M}_3 & \tilde{E}_3 & \tilde{L}_3 & 0 & 0 & 0 \\ \tilde{N}_4 & \tilde{T}_4 & \tilde{M}_4 & \tilde{E}_4 & \tilde{L}_4 & 0 & 0 & 0 \\ \tilde{N}_5 & \tilde{T}_5 & \tilde{M}_5 & \tilde{E}_5 & \tilde{L}_5 & 0 & 0 & 0 \end{bmatrix},$$

$$\tilde{\Gamma}_{11} = AX + XA^T + \sum_{i=1}^{3} \tilde{Q}_i + \tilde{N}_1 + \tilde{N}_1^T + \tilde{L}_1 + \tilde{L}_1^T, \tilde{\Gamma}_{12} = BY + \tilde{N}_2^T - \tilde{T}_1 + \tilde{M}_1 - \tilde{E}_1 + \tilde{L}_2^T,$$

$$\tilde{\Gamma}_{13} = \tilde{E}_1 + \tilde{N}_3^T + L_3^T + \tilde{L}_1, \tilde{\Gamma}_{14} = -\tilde{M}_1 + \tilde{N}_4^T + \tilde{L}_4^T,$$

$$\tilde{\Gamma}_{15} = \tilde{T}_1 - \tilde{N}_1 + \tilde{N}_5^T + L_5^T, \tilde{\Gamma}_{22} = \tilde{M}_2 + \tilde{M}_2^T - \tilde{T}_2 - \tilde{T}_2^T - \tilde{E}_2 - \tilde{E}_2^T,$$

$$\tilde{\Gamma}_{23} = \tilde{E}_2 + \tilde{M}_3^T - \tilde{T}_3^T - \tilde{E}_3^T + \tilde{L}_2, \tilde{\Gamma}_{24} = -\tilde{M}_2 + \tilde{M}_4^T - \tilde{T}_4^T - \tilde{E}_4^T,$$

$$\tilde{\Gamma}_{25} = \tilde{T}_2 - \tilde{N}_2 + \tilde{M}_5^T - \tilde{T}_5^T - \tilde{E}_5^T, \tilde{\Gamma}_{33} = -\tilde{Q}_1 + \tilde{E}_3 + \tilde{E}_3^T + \tilde{L}_3 + \tilde{L}_3^T,$$

$$\tilde{\Gamma}_{34} = -\tilde{M}_3 + \tilde{E}_4^T + \tilde{L}_4^T, \tilde{\Gamma}_{35} = \tilde{T}_3 - \tilde{N}_3 + \tilde{E}_5^T + \tilde{L}_5^T,$$

$$\tilde{\Gamma}_{44} = -\tilde{Q}_2 - \tilde{M}_4 - \tilde{M}_4^T, \tilde{\Gamma}_{45} = \tilde{T}_4 - \tilde{N}_4 - \tilde{M}_5^T, \tilde{\Gamma}_{55} = -(1-\alpha)\tilde{Q}_3 + \tilde{T}_5 - \tilde{N}_5 + \tilde{T}_5^T - \tilde{N}_5^T,$$

$$\tilde{\Gamma}_3 = \begin{bmatrix} -\frac{1}{\alpha h_2}\tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\frac{1}{(1-\alpha)h_2}\tilde{Z}_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{h_{12}}(\tilde{Z}_1+\tilde{Z}_2) & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{h_{12}}\tilde{Z}_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{h_2}\tilde{Z}_3 & 0 & 0 & 0 \\ * & * & * & * & * & \frac{1}{h_2}\tilde{Z}_1{}' & 0 & 0 \\ * & * & * & * & * & * & \frac{1}{h_{12}}\tilde{Z}_2{}' & 0 \\ * & * & * & * & * & * & * & \frac{1}{h_2}\tilde{Z}_3{}' \end{bmatrix},$$

$\tilde{Z}_m{}' = \tilde{Z}_m - 2X (m = 1, 2, 3)$

In this case, state-feedback gain in (3) is given by

$$K = YX^{-1}. \tag{10}$$

Proof. By the Schur complements, (6) is equivalent to

$$\Gamma' = \begin{bmatrix} \Gamma_1 & \Gamma'_2 \\ * & \Gamma'_3 \end{bmatrix} < 0,$$

$$\Gamma_1 = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} & \Gamma_{13} & \Gamma_{14} & \Gamma_{15} \\ * & \Gamma_{22} & \Gamma_{23} & \Gamma_{24} & \Gamma_{25} \\ * & * & \Gamma_{33} & \Gamma_{34} & \Gamma_{35} \\ * & * & * & \Gamma_{44} & \Gamma_{45} \\ * & * & * & * & \Gamma_{55} \end{bmatrix}, \Gamma'_2 = \begin{bmatrix} N_1 & T_1 & M_1 & E_1 & L_1 & A^T & A^T & A^T \\ N_2 & T_2 & M_2 & E_2 & L_2 & K^TB^T & K^TB^T & K^TB^T \\ N_3 & T_3 & M_3 & E_3 & L_3 & 0 & 0 & 0 \\ N_4 & T_4 & M_4 & E_4 & L_4 & 0 & 0 & 0 \\ N_5 & T_5 & M_5 & E_5 & L_5 & 0 & 0 & 0 \end{bmatrix},$$

$$\Gamma'_3 = \begin{bmatrix} -\frac{1}{\alpha h_2}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & -\frac{1}{(1-\alpha)h_2}Z_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\frac{1}{h_{12}}(Z_1+Z_2) & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -\frac{1}{h_{12}}Z_2 & 0 & 0 & 0 & 0 \\ * & * & * & * & -\frac{1}{h_2}Z_3 & 0 & 0 & 0 \\ * & * & * & * & * & -\frac{1}{h_2}Z_1{}^{-1} & 0 & 0 \\ * & * & * & * & * & * & -\frac{1}{h_{12}}Z_2{}^{-1} & 0 \\ * & * & * & * & * & & * & -\frac{1}{h_2}Z_3{}^{-1} \end{bmatrix}.$$

Define $\Lambda = diag\{P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, P^{-1}, P^{-1}, P^{-1}, P^{-1}, P^{-1}, I, I, I, I\}$. Pre- and post-multiplying both sides of (11) by Λ, and denoting $X = P^{-1}$, $\tilde{M}_i = XM_iX$, $\tilde{N}_i = XN_iX$, $\tilde{L}_i = XL_iX$, $\tilde{T}_i = XT_iX$, $\tilde{E}_i = XE_iX$ $(i = 1, 2, 3, 4, 5)$, $\tilde{Q}_j = XQ_jX$ $(j = 1, 2, 3)$, $\tilde{Z}_m = XZ_mX$ $(m = 1, 2, 3)$, $Y = KX$. It is noted that $Z_m^{-1} = X\tilde{Z}_m^{-1}X$ $(m = 1, 2, 3)$, the conditions for controller design are not LMI because of the nonlinear terms $X\tilde{Z}_m^{-1}X$ $(m = 1, 2)$. In order to solve this non-convex problem, the inequalities in following are needed.

$$(\tilde{Z}_m - X)\tilde{Z}_m^{-1}(\tilde{Z}_m - X) \geq 0 \ (m = 1, 2), \tag{11}$$

it is equivalent to $-X\tilde{Z}_m^{-1}X \leq \tilde{Z}_m - 2X$ $(m = 1, 2)$. Then, we can obtain (9). This completes the proof.

5 Numerical Examples

In the following, two numerical examples will be given to illustrate the advancement of our methods.

Example 1. Consider the following system:

$$\dot{x}(t) = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix} x(t) + \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix} x(t - d(t)) \tag{12}$$

Our purpose is to find the upper bound h_2 for given lower bound h_1, under which the system is asymptotically stable. The computational results of Theorem 1 in this paper, Theorem 2 in [13], and Theorem 1 in [12] for different cases are listed in Table 1. It is seen from Table 1 that the computational results obtained in this paper are better than those obtained in [13], and [12], showing the advantage of the stability condition proposed in this paper.

Table 1: Allowable upper bound of h_2 with given h_1

Methods	$h_1 = 3$	$h_1 = 3.5$	$h_1 = 4$	$h_1 = 4.5$	$h_1 = 5$
Sun et al. [13]	3.34	3.75	4.16	4.59	5.02
Fridman [12]	4.07	4.52	4.96	5.42	5.88
Theorem 1	4.15	4.65	5.15	5.65	6.15

Example 2. Consider the following system [14]:

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t) \tag{13}$$

In this subsection, choose the low bound of communication delays $\delta_m = 0.001s$, the upper bound of communication delays $\delta_M = 0.006s$, the low bound of sampling intervals is $T_m = 0.01s$. By using Theorem 2 we can achieve the upper bound of sampling intervals up to $T_M = 1.561s$ and the state-feedback controller is $K = [-0.0380 \ -0.3398]$, while the result in [14] is 0.014s.

6 Conclusions

In this paper, we use the input delay approach to investigate the stabilization problem of NCSs with variable sampling. Under the consideration of time-varying communication delay and variable sampling interval, NCSs model is presented. In addition, when network communication delay is assumed to be zero, the previous model is transformed to general sampled-data systems model studied by input delay approach. By employing a novel Lyapunov functional, using relax matrix variables to reduce conservativeness, a sufficient condition which can guarantee the closed-loop systems asymptotically stabilize is obtained. Finally, numerical examples are provided to validate the proposed control strategy.

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