Study on Linear Vibration Model of Shield TBM Cutterhead Driving System

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In this paper, a general linear time-varying multiple-axis (LTVMA) vibration model of shield tunnel boring machine (TBM) cutterhead driving system is established. The corresponding multiple inputs and multiple outputs (MIMO) state-space model is also presented. The linear vibration model is analysed, and the vibration-torque transfer function matrix and the vibration-torque static gain matrix are obtained. The linear vibration model is simulated, and the physical parameters’ effects on the vibration response are investigated. A preliminary approach is proposed to reduce vibration by increasing motor rotor inertia and viscous damped. The LTVMA vibration model provides a solid foundation for fault detection and diagnosis (FDD), as long as the health monitoring of cutterhead driving system.

NOMENCLATURE

\( n \): The number of cutterhead driving motors;
\( q \): the reducer speed reduction ratio
\( J_{i,d,i} \): i-th induction motor rotor inertia after equivalent coupling-i
\( b_{i,d,i} \): i-th induction motor rotor viscous damped after equivalent coupling-i
\( J_{i,r,i} \): Inertia of the i-th coupling between motor-i and reducer-i
\( b_{i,r,i} \): Viscous damped of the i-th coupling between motor-i and reducer-i
\( J_{i,f,i} \): i-th induction motor rotor inertia
\( J_{m} \): Large gear inertia
\( b_{m} \): Large gear viscous damped
\( r_{m} \): large gear radius
\( \theta_{p,i} \): Angular displacement of i-th active pinion
\( \theta_{l} \): angular displacement of i-th motor rotor
\( \omega_{m} \): Large gear angular speed
\( J_{i,c,i} \): i-th pinion inertia after equivalent coupling-i
\( b_{i,c,i} \): i-th pinion viscous damped after equivalent coupling-i
\( J_{i,w,i} \): Inertia of the i-th coupling between reduce-i and pinion-i
\( b_{i,w,i} \): Viscous damped of the i-th coupling between reducer-i and pinion-i
\( J_{p,i} \): i-th pinion inertia
\( b_{p,i} \): i-th pinion viscous damped
\( T_{e,i} \): i-th induction motor electrical magnetic torque
\( \omega_{p,i} \): Angular speed of pinion-i
\( \omega_{i} \): angular speed of induction motor-i

\( F_{1} \): Elastic mesh force of the pinion-i and large gear
\( r_{i} \): radius of pinion-i
\( M_{c,i} \): Elastic mesh torque of the pinion-i and large gear
\( p_{i} \): Relative position of the pinion-i and large gear
\( k_{i} \): Mesh stiffness of the pinion-i and large gear
\( c_{i} \): Mesh damped of the pinion-i and large gear
\( m_{p,i} \): Mass of the pinion-i
\( k_{y,i} \): support stiffness of pinion-i
\( c_{y,i} \): support damped of pinion-i
\( m_{g} \): Mass of the large gear
\( k_{y,m} \): support stiffness of large gear
\( c_{y,m} \): support damped of large gear
\( y_{i} \): The contact direction (line-of-action direction) vibration displacement of i-th pinion
\( y_{m} \): The large gear’s contact vibration displacement along i-th pinion’s line-of-action direction
\( T_{L} \): Shield TBM cutterhead’s load torque
\( k_{r,i}, c_{r,i} \): elastic mesh force coefficient of pinion-i
\( k_{t,i}, c_{t,i} \): elastic mesh torque coefficient of pinion-i
\( i_{m,i} \): gear transmission ratio
\( T_{1} \): The resistant torque of soil and rocks
\( T_{2} \): The friction torque of soil and rocks which chafe with the front of the cutterhead
\( T_{3} \): The friction torque of soil and rocks which chafe with the back of the cutterhead
\( T_{4} \): The friction torque of soil and rocks which chafe with bulkhead
\( T_{5} \): The stirring torque of soil and rocks by cutterhead stirring rod
\( T_{6} \): The friction torque of cutterhead bearings and sealed chamber
1. INTRODUCTION

The shield TBM is a large-scale, underground machine used to excavate tunnels or subways. Modern shield TBM integrates the information and communication, network control, intelligent sensing, laser measuring guide, infrared detection, and other advanced science and technology with cutting geological, transporting rock and soil, support excavation section, orientation correction, tunnel segments assembly, and other functions. Therefore, the shield TBM excavates tunnels with high safety and reliability, low manpower, minor environmental damages, and rapid speed. The cutterhead driving system is a core system of shield TBM, and it drives the cutterhead to cut rock and soil (the cutterhead has a thrust system). The components of a shield TBM are shown in Fig. 1. To adapt to geological conditions, the shield TBM has different cutterhead forms: spokes form, panel form, and hybrid form. The shield TBM cutterhead includes two driven modes: hydraulic-driven and motor-driven. In this paper, the motor-driven mode’s shield TBM cutterhead is studied. The profile and transmission structure of the shield TBM cutterhead is shown in Fig. 2.20–22 Multiple gear structure synthesizes the motors’ electrical magnetic torque (EMT). Overall, the principle structure of the cutterhead driving system is shown in Fig. 3.20–22 The cutterhead and the central large gear have identical bearing, central axis, and rotation speed.

The development of the shield TBM cutterhead driving system mainly goes through the human-driven, mechanical-driven, hydraulic-driven, and motor-driven stages. For the motor-driven mode’s cutterhead driving system, it needs to solve the cutterhead speed tracking (CST) problems, multiple motors torque synchronization (MMTS) problems, and FDD problems of the cutterhead driving system. The LTVMA vibration model provides a theoretical foundation for FDD and health monitoring. This paper is arranged as follows: in Section 2, a general LTVMA vibration model of the cutterhead driving system is presented, and MIMO state-space is also presented. In Section 3, the linear vibration model is analysed, and the vibration-torque transfer function matrix and vibration-torque static gain matrix are obtained. In Section 4, the LTVMA vibration model is simulated, and a preliminary approach is presented to reduce the cutterhead driving system’s vibration. In Section 5, the study contents and results are briefly reviewed, and some conclusions are made.

2. GENERAL LTVMA VIBRATION MODEL OF CUTTERHEAD DRIVING SYSTEM

2.1. Preliminary and Preparation

To establish the linear vibration model of cutterhead driving system, the gear backlash and transmission error is ignored, and the spring-damped is usually used to depict the gear mesh process. Figure 4 shows the linear vibration dynamic model of the gear mesh process, where \( k \) and \( c \) are the mesh stiffness and damped, respectively, \( k_{y,1} \) and \( k_{y,m} \) are the pinion-1 support stiffness and large gear support stiffness, respectively, and \( c_{y,1} \) and \( c_{y,m} \) are the pinion-1 support damped and large gear support damped respectively. Mesh stiffness \( k \) and mesh damped \( c \) are the physical parameters that are affected by mesh point. In addition, the gear mesh point will change along with gear mesh action line. Therefore, mesh stiffness \( k \) and mesh damped \( c \) are time-varying physical parameters. The linear vibration model of the shield TBM cutterhead driving system is shown in Fig. 5.

Based on the gear mesh dynamic,1–6 the relative position deviation function, elastic mesh force, and elastic mesh torque...
Figure 3. Overall principle structure of shield TBM cutterhead driving system (motor-driven mode).

Figure 4. Linear vibration dynamic model.

Figure 5. Linear vibration dynamic model of cutterhead driving system.

can be obtained where \(y_i(t)\) is the contact direction (line-of-action) vibration displacement of \(i\)-th pinion, and \(y_m(t)\) is the large gear’s contact vibration displacement along contact direction of \(i\)-th pinion.

\[
p_i = r_i \cdot \theta_{p,i} - r_m \cdot \theta_m + (y_i(t) - y_m(t)) \quad (1)
\]

\[
F_i = k_i p_i + c_i \dot{p}_i \quad (2)
\]

\[
M_{c,i} = F_i \cdot r_i = k_i p_i r_i + c_i \dot{p}_i r_i \quad (3)
\]

Then, the elastic mesh force \(F_i\) and mesh torque \(M_{c,i}\) could be expressed as

\[
F_i = k_i \left( r_i \cdot \theta_{p,i} - r_m \cdot \theta_m + (y_i(t) - y_m(t)) \right) + c_i \left( r_i \cdot \dot{\theta}_{p,i} - r_m \cdot \dot{\theta}_m + (\dot{y}_i(t) - \dot{y}_m(t)) \right) \quad (4)
\]

\[
p_i = r_i \cdot \theta_{p,i} - r_m \cdot \theta_m + (y_i(t) - y_m(t)) \quad \Leftrightarrow \quad \frac{p_i}{r_i} = \theta_{p,i} - i_{m,i} \cdot \theta_m + \frac{(y_i(t) - y_m(t))}{r_i} \quad (5)
\]

\[
M_{c,i} = k_i r_i \left( r_i \cdot \theta_{p,i} - r_m \cdot \theta_m + (y_i(t) - y_m(t)) \right) + c_i r_i \left( r_i \cdot \theta_{p,i} - r_m \cdot \theta_m + (\dot{y}_i(t) - \dot{y}_m(t)) \right) \quad (6)
\]

Therefore, the elastic mesh force and torque of \(i\)-th pinion can be equivalently expressed as

\[
F_i = k_{f,i} \left( \theta_{p,i} - i_{m,i} \cdot \theta_m + \frac{y_i(t) - y_m(t)}{r_i} \right) + c_{f,i} \left( \dot{\theta}_{p,i} - i_{m,i} \cdot \dot{\theta}_m + \frac{\dot{y}_i(t) - \dot{y}_m(t)}{r_i} \right) \quad (7)
\]

and

\[
M_{c,i} = k_{t,i} \left( \theta_{p,i} - i_{m,i} \cdot \theta_m + \frac{y_i(t) - y_m(t)}{r_i} \right) + c_{t,i} \left( \dot{\theta}_{p,i} - i_{m,i} \cdot \dot{\theta}_m + \frac{\dot{y}_i(t) - \dot{y}_m(t)}{r_i} \right) \quad (8)
\]

The physical parameters in Eqs. (7) and (8) are defined as

\[
k_{f,i} = k_i r_i \quad (i = 1, 2, ..., n); \quad (9)
\]

\[
k_{t,i} = k_i r_i^2 \quad (i = 1, 2, ..., n); \quad (10)
\]

\[
c_{f,i} = c_i r_i \quad (i = 1, 2, ..., n); \quad (11)
\]

\[
c_{t,i} = c_i r_i^2 \quad (i = 1, 2, ..., n); \quad (12)
\]

\[
i_{m,i} = \frac{r_m}{r_i} \quad (i = 1, 2, ..., n) \quad (13)
\]

where \(k_{f,i}\) and \(c_{f,i}\) are elastic mesh force coefficients, \(k_{t,i}\) and \(c_{t,i}\) are the mesh torque coefficients, and \(i_{m,i}\) is the gear
transmission ratio. The mesh stiffness $k_i$ and mesh damping $c_i$ are time-varying parameters, which can be expressed as the Fourier series

$$k_i(t) = k_{i,0} + \sum_{j=1}^{\infty} k_{i,j} \cos \left( j2\pi f_{z,i} t + \phi_{i,j} \right);$$

$$c_i(t) = c_{i,0} + \sum_{j=1}^{\infty} c_{i,j} \cos \left( j2\pi f_{z,i} t + \phi_{i,j} \right);$$

where $f_{z,i}$ is the mesh frequency, $k_{i,0}$ and $c_{i,0}$ are the mean mesh stiffness and mean mesh damping respectively, and $\phi_{i,j}$ and $\phi_{i,j}$ are the phase angle of mesh stiffness and mesh damped, respectively.

### 2.2. Linear Dynamic Vibration Model

Without losing generality, it was assumed that the cutterhead is driven by $n$ motors when establishing the general LTVMMA vibration model. If the coupling mass is far less than the motor rotor mass, then the coupling inertia can be ignored. In fact, motor rotor mass cannot be far greater than coupling mass, and therefore coupling inertia cannot be ignored. According to the torque balance principle, torque balance equations of the first induction motor will yield

$$T_{e,1} = J_{r,1} \ddot{\theta}_1 + b_{r,1} \dot{\theta}_1 + T_{o,1};$$

$$T_{o,1} = J_{z,1} \ddot{\theta}_1 + b_{z,1} \dot{\theta}_1 + M_{1,1}.$$  

For the second induction motor, the corresponding torque balance equations are obtained as

$$T_{e,2} = J_{r,2} \ddot{\theta}_2 + b_{r,2} \dot{\theta}_2 + T_{o,2};$$

$$T_{o,2} = J_{z,2} \ddot{\theta}_2 + b_{z,2} \dot{\theta}_2 + M_{1,2}.$$  

Likewise, the torque balance equations of the $n$-th induction motor will yield

$$T_{e,n} = J_{r,n} \ddot{\theta}_n + b_{r,n} \dot{\theta}_n + T_{o,n};$$

$$T_{o,n} = J_{z,n} \ddot{\theta}_n + b_{z,n} \dot{\theta}_n + M_{1,n};$$

where $T_{o,i}$ is the output torque of the $i$-th motor and $M_{1,i}$ is the input torque of the $i$-th reducer ($i = 1, 2, ..., n$). The Equations (16), (19), and (21) are then substituted into Eqs. (16), (18), and (20), respectively. The following equations will be obtained:

$$T_{e,1} = (J_{r,1} + J_{z,1}) \ddot{\theta}_1 + (b_{r,1} + b_{z,1}) \dot{\theta}_1 + M_{1,1};$$

$$T_{e,2} = (J_{r,2} + J_{z,2}) \ddot{\theta}_2 + (b_{r,2} + b_{z,2}) \dot{\theta}_2 + M_{1,2};$$

$$T_{e,n} = (J_{r,n} + J_{z,n}) \ddot{\theta}_n + (b_{r,n} + b_{z,n}) \dot{\theta}_n + M_{1,n};$$

Then, Eqs. (22)–(24) can be rewritten as

$$T_{e,i} = J_{d,i} \ddot{\theta}_i + b_{d,i} \dot{\theta}_i + M_{1,i}, \ (i = 1, 2, ..., n);$$

where the physical parameters are defined as $J_{d,i} = J_{r,i} + J_{z,i}$, $b_{d,i} = b_{r,i} + b_{z,i} (i = 1, 2, ..., n)$. The input and output relationships of the $i$-th reducer can be described by the equation

$$\theta = q\dot{\phi}_{p,i}, M_{2,i} = qM_{1,i}, \ (i = 1, 2, ..., n);$$

where $M_{2,i}$ is the output torque of the $i$-th reducer. The torque balance equation of the reduce-$i$ or pinion-$i$ will yield

$$M_{2,i} = J_{w,i} \ddot{\theta}_{p,i} + b_{w,i} \dot{\theta}_{p,i} + T_{p,i}, \ (i = 1, 2, ..., n);$$

$$T_{p,i} = J_{p,i} \ddot{\theta}_{p,i} + b_{p,i} \dot{\theta}_{p,i} + M_{c,i}, \ (i = 1, 2, ..., n);$$

where $T_{p,i}$ is the input torque of pinion-$i$, and $M_{c,i}$ is the elastic mesh torque of pinion-$i$ or gear pair-$i$. Substituting Eq. (28) into the Eq. (27), then the torque balance equation can be written as

$$M_{2,i} = (J_{p,i} + J_{w,i}) \ddot{\theta}_{p,i} + (b_{p,i} + b_{w,i}) \dot{\theta}_{p,i} + M_{c,i}, \ (i = 1, 2, ..., n).$$

Then, Eq. (29) can be rewritten as

$$M_{2,i} = J_{c,i} \ddot{\theta}_{p,i} + b_{c,i} \dot{\theta}_{p,i} + M_{c,i}, \ (i = 1, 2, ..., n);$$

where the system parameters are defined as $J_{c,i} = J_{p,i} + J_{w,i}$, and $b_{c,i} = b_{p,i} + b_{w,i} (i = 1, 2, ..., n)$. The elastic mesh force will excite contact direction (line-of-action) vibration displacement, because the mesh contact point is along with contact line direction. Thus, the pinions’ contact direction (line-of-action) vibration displacement in their own coordinates will be obtained as

$$m_{p,1} \dddot{y}_1(t) + c_{p,1} \ddot{y}_1(t) + k_{p,1} y_1(t) + F_1 = 0$$

$$m_{p,2} \dddot{y}_2(t) + c_{p,2} \ddot{y}_2(t) + k_{p,2} y_2(t) + F_2 = 0$$

$$\vdots$$

$$m_{p,n-1} \dddot{y}_{n-1}(t) + c_{p,n-1} \ddot{y}_{n-1}(t) + k_{p,n-1} y_{n-1}(t) + F_{n-1} = 0$$

$$m_{p,n} \dddot{y}_n(t) + c_{p,n} \ddot{y}_n(t) + k_{p,n} y_n(t) + F_n = 0$$

$$m_{p,1} \dddot{u}_1(t) + c_{u,1} \ddot{u}_1(t) + k_{u,1} u_1(t) + F_1 = 0, \ (i = 1, 2, ..., n);$$

$$m_{p,2} \dddot{u}_2(t) + c_{u,2} \ddot{u}_2(t) + k_{u,2} u_2(t) + F_2 = 0$$

$$\vdots$$

$$m_{p,n} \dddot{u}_n(t) + c_{u,n} \ddot{u}_n(t) + k_{u,n} u_n(t) + F_n = 0$$

$$m_{p,1} \dddot{y}'_1(t) + c_{u,1} \ddot{y}'_1(t) + k_{u,1} y'_1(t) = F_1, \ (i = 1, 2, ..., n);$$

$$m_{p,2} \dddot{y}'_2(t) + c_{u,2} \ddot{y}'_2(t) + k_{u,2} y'_2(t) = F_2$$

$$\vdots$$

$$m_{p,n} \dddot{y}'_n(t) + c_{u,n} \ddot{y}'_n(t) + k_{u,n} y'_n(t) = F_n \ (i = 1, 2, ..., n).$$

where \( m_g \) is the mass of large gear, and \( k_{y,m} \) and \( c_{y,m} \) are the large gear support stiffness and viscous damped. The elastic mesh torque of pinion-\( i \) or gear pair-\( i \) has been presented in Section 2.1, and the elastic mesh force \( F_i \) and mesh torque \( M_{c,i} \) of pinion-\( i \) will be obtained

\[
\begin{align*}
F_i &= k_{f,i} \left( \theta_{p,i} - i_{m,i} \theta_m + \frac{y_i(t) - y_m^i(t)}{r_i} \right) + c_{f,i} \left( \dot{\theta}_{p,i} - i_{m,i} \dot{\theta}_m + \frac{\dot{y}_i(t) - \dot{y}_m^i(t)}{r_i} \right) \\
M_{c,i} &= k_{t,i} \left( \theta_{p,i} - i_{m,i} \theta_m + \frac{y_i(t) - y_m^i(t)}{r_i} \right) + c_{t,i} \left( \dot{\theta}_{p,i} - i_{m,i} \dot{\theta}_m + \frac{\dot{y}_i(t) - \dot{y}_m^i(t)}{r_i} \right).
\end{align*}
\]

(35)

Then, the torque balance equations of the large gear will be obtained as

\[
\begin{align*}
J_m \ddot{\theta}_m + b_m \dot{\theta}_m + T_L &= M_m; \quad (36) \\
M_m &= i_{m,1} M_{c,1} + i_{m,2} M_{c,2} + \cdots + i_{m,n} M_{c,n} \\
&= \sum_{k=1}^{n} i_{m,k} M_{c,k}; \quad (37)
\end{align*}
\]

where \( T_L \) is total load torque of the shield TBM cutterhead. Cutterhead design parameters and geology conditions will directly affect the load torque, for instance, the cutterhead cutting amount and distribution, rocks and soil properties, boulders, and so on. The shield TBM cutterhead’s load torque is estimated and calculated, and the total load torque includes six parts of torque.\(^{17}\)

\[
T_L = T_1 + T_2 + T_3 + T_4 + T_5 + T_6 = \sum_{j=1}^{6} T_j.
\]

(38)

Detailed derivations and descriptions of load torque components and parameters can be found in previous work.\(^{17}\) Therefore, calculation formulas of load torque are directly presented here.

\[
\begin{align*}
T_1 &= \frac{D^2 v_s d_s}{8 N_r}; \\
T_2 &= \frac{\pi \gamma H_0 \mu T^3}{12}; \\
T_3 &= \frac{\pi \gamma H_0 \mu_1 (D_1 - D_2)^2}{12}; \\
T_4 &= \frac{\pi \gamma H_0 \mu_1 B (D_1 + D_2) \sin \alpha}{2 B}; \\
T_5 &= \gamma H_0 \mu_1 D_0 L_5 R_6, T_6 = \frac{(1.414 G_0 + K_{H_H}) \mu_2 D_0 g_0}{2}.
\end{align*}
\]

(39)-(44)

The general LTVMA dynamic vibration model of the shield TBM cutterhead driving system is obtained, and it can be depicted as in Eq. (45). The LTVMA dynamic vibration model can be combined and simplified, and LTVMA vibration model is equivalently expressed as in Eq. (46). Hence, the linear dynamic vibration model of shield TBM cutterhead driving system is finally expressed as in Eq. (47). The LTVMA vibration model can be transformed into the state-space dynamic vibration model via selecting suitable state variables, output variables, and control variables. It could select suitable state variables for motion equations (Eq. (48)—see the next page). The motion Eq. (48) is transformed into the MIMO state-space form via selecting the the state vector, output vector, and control vector as in Eqs. (49)–(51)—see the next page.

\[
\begin{align*}
\dot{x}_\rho &= A_\rho x_\rho + B_\rho u_\rho \quad ; \quad (53) \\
y_\rho &= C_\rho x_\rho \quad ; \quad (54) \\
A_\rho &= \begin{pmatrix} A_{11}^\rho & A_{12}^\rho \\ A_{21}^\rho & A_{22}^\rho \end{pmatrix}; \\
B_\rho &= \begin{pmatrix} B_{11}^\rho & B_{12}^\rho \\ B_{21}^\rho & B_{22}^\rho \end{pmatrix}; \\
C_\rho &= \begin{pmatrix} C_{11}^\rho & C_{12}^\rho \end{pmatrix}.
\end{align*}
\]

(53)-(56)

where \( x_\rho \) is a state vector, \( y_\rho \) is an output vector, and \( u_\rho \) is a control vector, \( A_\rho \) is the state matrix, \( B_\rho \) is a control matrix, and \( C_\rho \) is an output matrix. The matrices \( A_\rho, B_\rho, \) and \( C_\rho \) are presented in Appendix 1. Likewise, it could select suitable state variables for vibration equations of linear dynamic vibration model, Eq. (47):

\[
\begin{align*}
F_i &= k_{f,i} \left( \theta_{p,i} - i_{m,i} \theta_m + \frac{y_i(t) - y_m^i(t)}{r_i} \right) + c_{f,i} \left( \dot{\theta}_{p,i} - i_{m,i} \dot{\theta}_m + \frac{\dot{y}_i(t) - \dot{y}_m^i(t)}{r_i} \right) \\
&+ m_{p,i} \ddot{y}_i(t) + c_{p,i} \dot{y}_i(t) + k_{y,i} y_i(t) + F_i = 0; \quad (57)
\end{align*}
\]

\[
m_y \ddot{y}_m(t) + c_y \dot{y}_m(t) + k_y y_m(t) = F_i, \quad (i = 1, 2, ..., n). \quad (58)
\]

The vibration Eq. (57) is transformed into the MIMO state-space form by selecting the following state vector, output vector, and control vector. See Eq. (50), where \( x_\delta \) is a state vector, \( y_\delta \) is an output vector, and \( u_\delta \) is a control vector; \( A_\delta \) is the state matrix, \( B_\delta \) is a control matrix, and \( C_\delta \) is an output matrix. The matrices \( A_\delta, B_\delta, \) and \( C_\delta \) are presented in Appendix 1. Then, the state Eqs. (53) can be equivalently expressed as

\[
\begin{align*}
\dot{x}_\rho &= A_\rho x_\rho + B_\rho u_\rho \Leftrightarrow \dot{x}_\rho &= A_\rho x_\rho + B_\rho^1 u_{p1} + B_\rho^2 u_{p2}. \quad (59)
\end{align*}
\]

It can be seen that the control variables \( u_{p1} \) and \( u_{p2} \) have relationships with the state variables \( x_\delta \) and \( x_\rho \), respectively; therefore, the state space equations are expressed as

\[
\begin{align*}
\dot{x}_\rho &= A_\rho x_\rho + B_\rho^1 u_{p1} + B_\rho^2 u_{p2} \\
&= A_\rho x_\rho + B_\rho^1 u_{p1} + B_\rho^2 \overline{A}_\rho \overline{x}_\delta \quad \Leftrightarrow \quad (60) \\
\dot{x}_\delta &= A_\delta x_\delta + B_\delta u_\delta = A_\delta x_\delta + B_\delta \overline{A}_\delta \overline{x}_\rho \\
&= \left( \begin{array}{c} A_\delta \\ B_\delta \overline{A}_\delta \end{array} \right)(x_\delta) + \left( \begin{array}{c} 0 \\ B_\delta \overline{A}_\delta \end{array} \right)u_{p1}. \quad (62)
\end{align*}
\]

(60)-(62)
\[
\begin{align*}
\{ T_{c,i} = J_{d,i} \ddot{\theta}_{i} + b_{d,i} \dot{\theta}_{i} + M_{1,i} \theta_{i} + q_{\theta,p,i} M_{2,i} & = q M_{1,i}, \\
M_{2,i} = J_{c,i} \ddot{\theta}_{p,i} + b_{c,i} \dot{\theta}_{p,i} + M_{1,i} \theta_{p,i} + m_{p,i} \ddot{y}_{i}(t) + c_{y,p,i} \dot{y}_{i}(t) + k_{y,p,i} y_{i}(t) + F_{i} = 0, \\
m_{y} \ddot{y}_{m}(t) + c_{y} \dot{y}_{m}(t) + k_{y,m} y_{m}(t) = F_{i}, & \quad (i = 1, 2, \ldots, n) \\
J_{m,\theta_{m}} + b_{m,\theta_{m}} + T_{L} = M_{m,\theta_{m}} = \sum_{k=1}^{n} i_{m,k} M_{c,k} t_{L} = \sum_{j=1}^{6} T_{j}, \\
F_{i} = k_{f,i} \left( \theta_{p,i} - i_{m,i} \theta_{m} + \frac{y_{i}(t) - y_{m}(t)}{r_{i}} \right) + c_{f,i} \left( \dot{\theta}_{p,i} - i_{m,i} \dot{\theta}_{m} + \frac{\dot{y}_{i}(t) - \dot{y}_{m}(t)}{r_{i}} \right), \\
M_{c,i} = k_{t,i} \left( \theta_{p,i} - i_{m,i} \theta_{m} + \frac{y_{i}(t) - y_{m}(t)}{r_{i}} \right) + c_{t,i} \left( \dot{\theta}_{p,i} - i_{m,i} \dot{\theta}_{m} + \frac{\dot{y}_{i}(t) - \dot{y}_{m}(t)}{r_{i}} \right).
\end{align*}
\]
3. THEORETICAL ANALYSIS OF THE LINEAR VIBRATION MODEL

The general LTVMA vibration model of the cutterhead driving system has been established in the previous section. At present, there is no good theoretical analysis method for the LTVMA vibration model seen in Eq. (47). In order to analyse the LTVMA dynamic vibration model in theory, the time-varying system parameter's mesh stiffness, mesh damped, and load torque are replaced by the mean mesh stiffness, mean mesh damped, and mean load torque. Therefore, the vibration model of the cutterhead driving system can be seen as the linear time-invariant dynamic vibration system, and the dynamic vibration model can be analysed by a transfer function analysis method or frequency domain analysis method. Then, the Laplace transform of the dynamic vibration model seen in Eq. (47) will be obtained.

\[
qT_{z,i}(s) = (J_is^2 + b_is) \theta_{p,i}(s) + (k_{t,i} + c_{t,i}) \left( \theta_{p,i}(s) - i_{m,i} * \theta_m(s) \right) + \frac{y_i(s) - y_m(s)}{r_i}; \quad (65)
\]

\[
(J_ms^2 + b_ms) \theta_m(s) + T_L(s) = \sum_{k=1}^{n} i_{m,k} \left\{ (k_{t,k} + c_{t,k}s) \left( \theta_{p,k}(s) \right) - i_{m,k} * \theta_m(s) + \frac{y_k(s) - y_m^k(s)}{r_k} \right\}; \quad (66)
\]

\[
F_i(s) = (k_{f,i} + c_{f,i}) \left( \theta_{p,i}(s) - i_{m,i} * \theta_m(s) \right) + \frac{y_i(s) - y_m(s)}{r_i}; \quad (67)
\]

\[
m_{p,i}s^2 y_i(s) + c_{y,i}s y_i(s) + k_{y,i} y_i(s) + F_i(s) = 0; \quad (68)
\]

\[
m_{y}s^2 y_m(s) + c_{y,m}s y_m(s) + k_{y,m} y_m(s) = F_i(s); \quad (69)
\]

Substituting Eq. (67) into vibration Eq. (68) and Eq. (69), the vibration Eq. (68) and Eq. (69) can be expressed as

\[
m_{p,i}s^2 y_i(s) + c_{y,i}s y_i(s) + k_{y,i} y_i(s) + \left( k_{f,i} + c_{f,i} \right) \left( \theta_{p,i}(s) - i_{m,i} * \theta_m(s) \right) + \frac{y_i(s) - y_m(s)}{r_i} = 0; \quad (70)
\]

\[
m_{p,i}s^2 y_i(s) + c_{y,i}s y_i(s) + k_{y,i} y_i(s) + \left( k_{f,i} + c_{f,i} \right) \left( \theta_{p,i}(s) - i_{m,i} * \theta_m(s) \right) + \frac{y_i(s) - y_m^i(s)}{r_i} = 0. \quad (71)
\]

Then, the vibration equation (Eqs. (70)–(71)) can be further expressed as

\[
r_i (m_{p,i}s^2 + c_{y,i}s + k_{y,i}) \frac{y_i(s) - y_m^i(s)}{r_i} + (k_{f,i} + c_{f,i}) \theta_{p,i}(s) - i_{m,i} * \theta_m(s) = 0 \quad (72)
\]

\[
r_i \left( m_{y}s^2 + c_{y,m}s + k_{y,m} \right) \frac{y_m(s) - y_m^i(s)}{r_i} - (k_{f,i} + c_{f,i}) \left( y_i(s) - y_m^i(s) \right) = r_i \left( k_{f,i} + c_{f,i} \right) \theta_{p,i}(s) - i_{m,i} * \theta_m(s) \quad (73)
\]

Similarly, the motion (Eqs. (65)–(66)) can be equivalently expressed as

\[
qT_{z,i}(s) = (J_is^2 + b_is + k_{t,i} + c_{t,i}s) \theta_{p,i}(s) - i_{m,i} \left( k_{t,i} + c_{t,i}s \right) \theta_m(s) + \left( k_{t,i} + c_{t,i}s \right) \frac{y_i(s) - y_m^i(s)}{r_i} \Leftrightarrow (74)
\]

\[
qT_{z,i}(s) - r_i (J_is^2 + b_is + k_{t,i} + c_{t,i}s) \theta_{p,i}(s) - r_i \left( k_{t,i} + c_{t,i}s \right) \theta_m(s) + \left( k_{t,i} + c_{t,i}s \right) \frac{y_i(s) - y_m^i(s)}{r_i} \Leftrightarrow (75)
\]

and

\[
(J_ms^2 + b_ms) \theta_m(s) + T_L(s) = \sum_{k=1}^{n} i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \theta_{p,k}(s) - \sum_{k=1}^{n} i_{m,k}^2 \left( k_{t,k} + c_{t,k}s \right) \theta_m(s) + \sum_{k=1}^{n} i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \frac{y_k(s) - y_m^k(s)}{r_k} \Leftrightarrow (76)
\]

\[
T_L(s) + (J_ms^2 + b_ms) \theta_m(s) = \sum_{k=1}^{n} i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \theta_{p,k}(s) - \sum_{k=1}^{n} i_{m,k}^2 \left( k_{t,k} + c_{t,k}s \right) \theta_m(s) + \sum_{k=1}^{n} i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \frac{y_k(s) - y_m^k(s)}{r_k}. \quad (77)
\]

Then, the motion equation could be further expressed as

\[
D_i(s) \theta_{p,i}(s) = qr_i T_{z,i}(s) + r_i i_{m,i} \left( k_{t,i} + c_{t,i}s \right) \theta_m(s) - (k_{t,i} + c_{t,i}s) \left( y_i(s) - y_m^i(s) \right) \Leftrightarrow (78)
\]
\[ T_L(s) + D_m(s)\theta_m(s) = \sum_{k=1}^{n} \left( i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \theta_p(k,s) + \frac{y_k(s) - y_{p,m}(s)}{r_k} \right) \]  
\[ D_l(s) = r_1 \left( J_1 s^2 + b_1 s + c_{t,1} + c_i \right) ; \]
\[ D_m(s) = \left( J_m s^2 + b_m s \right) + \sum_{k=1}^{n} \left( i_{m,k} \left( k_{t,k} + c_{t,k}s \right) \right) \]  

In order to simplify the analysis, it is assumed that constituted equipments of the cutterhead driving system have identical physical parameters with the formula \( J_{d,1} = J_d, J_{c,1} = J_c, b_{d,1} = b_d, b_{c,1} = b_c, r_1 = r, i_{m,1} = i_m, k_{t,1} = k_t, c_{t,1} = c_t, k_{f,1} = k_f, \) and \( c_{r,1} = c_r, m_{p,1} = m_p, k_{y,1} = k_y, c_{y,1} = c_y, (J_1 = q^2 J_d + J_c) = J, b_1 = q^2 b_d + b_c, \) then Eqs. (78)–(80) can be rewritten as

\[ D(s)\theta_{p,s}(s) = qr_1 T_{e,s}(s) + r_1 m (k_t + c_t) \theta_m(s) - (k_t + c_t) \left( y_m(s) - y_{m,s}(s) \right) ; \]  
\[ T_L(s) + D_m(s)\theta_m(s) = \sum_{k=1}^{n} \left( i_m (k_t + c_t) \theta_p(k,s) \right) \]  
\[ D(s) = r \left( J s^2 + b s + c_i \right) ; \]
\[ D_m(s) = \left( J_m s^2 + b_m s \right) + \sum_{k=1}^{n} \left( i_{m,k} (k_{t,k} + c_{t,k}s) \right) . \]

Likewise, the vibration Eqs. (72)–(73)) can be rewritten as

\[ r \left( m_p s^2 + c_g s + k_y \right) y_i(s) + (k_f + c_f s) (y_i(s) - y_{i,s}(s)) = -r (k_f + c_f s) (\theta_{p,s}(s) - i_m \ast \theta_m(s)) ; \]
\[ r \left( m_g s^2 + c_{g,m} s + k_{y,m} \right) y_m(s) - (k_f + c_f s) (y_m(s) - y_{m,s}(s)) = r (k_f + c_f s) (\theta_{p,s}(s) - i_m \ast \theta_m(s)) . \]

Substituting Eq. (82) into Eq. (83), Eq. (83) can be equivalently expressed in Eq. (88). Substituting Eqs. (82)–Eq. (88) into vibration Eq. (86), the vibration Eq. (86) can be equivalently expressed in Eq. (89). Likewise, substituting Eq. (82) and Eq. (88) into Eq. (87), the vibration Eq. (87) can be equivalently expressed as in Eq. (93), where the \( D_y(s), D_{y,m}(s), D_{r,s}(s), D_{2s}(s), D_{2r}(s), \) and \( D_{A,s}(s) \) are defined as in Eq. (97). Then, Eqs. (91) and Eq. (94) can be equivalently transformed into the transfer function matrix (see Eq. (100)).

Finally, the vibration transfer function matrix is acquired as

\[ Y(s) = \left\{ G_{vib}(s)^{-1} G_T(s) \right\} U(s) . \]

Thus, the vibration-torque static gain matrix of the shield TBM cutterhead driving system could be obtained as

\[ \begin{align*}
K_V &= G_{vib}(s)^{-1} G_T(s) |_{s=0}, \\
D_y(s)|_{s=0} &= -\frac{k_y}{k_f}, D_{y,m}(s)|_{s=0} = 0, D_A(s)|_{s=0} = \frac{q}{k_f}, \\
D_L(s)|_{s=0} &= -\frac{1}{nk_{1,1}m}, D_2(s)|_{s=0} = \frac{q}{nk_{1,1}m}, \\
D_3(s)|_{s=0} = 0, D_A(s)|_{s=0} = \frac{k_{y,m}}{k_f}.
\end{align*} \]  
\[ \begin{align*}
G_{11}(s)|_{s=0} &= \text{diag} \left( -\frac{k_y}{k_f}, -\frac{k_y}{k_f}, \ldots, -\frac{k_y}{k_f} \right), \\
G_{12}(s)|_{s=0} &= G_{vib}(s) |_{s=0} = 0, \\
G_{22}(s)|_{s=0} &= \text{diag} \left( \frac{k_{y,m}}{k_f}, \frac{k_{y,m}}{k_f}, \ldots, \frac{k_{y,m}}{k_f} \right), \\
G_{11}(s)|_{s=0} &= \left( \begin{array}{cccc}
\frac{k_{11}^v}{k_f} & \frac{k_{12}^v}{k_f} & \cdots & \frac{k_{1n}^v}{k_f} \\
\frac{k_{21}^v}{k_f} & \frac{k_{22}^v}{k_f} & \cdots & \frac{k_{2n}^v}{k_f} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{k_{n1}^v}{k_f} & \frac{k_{n2}^v}{k_f} & \cdots & \frac{k_{nn}^v}{k_f} \\
\end{array} \right).
\end{align*} \]  

4. SIMULATION RESULTS OF THE LTVMA VIBRATION MODEL

In order to analyse the LTVMA vibration model, time-varying physical parameters such as mesh stiffness, mesh damped, and load torque are replaced by mean mesh stiffness, mean mesh damped, and mean load torque in the simulation. The corresponding simulation parameters are presented in Appendix 2, and the simulation results of the linear vibration model are shown in Figs. 6–7 (in the unload case), respectively. The vibration response results of the LTVMA dynamic model in Figs. 6–7 obviously demonstrate that the vibration speeds of pinions and the large gear appear as oscillation behaviours in the initial stage. The vibration response results also show that the vibration speeds of pinions and the large gear is not standard simple harmonic vibration. In addition, Figs. 6–7 show that the vibration speeds of pinions and the large gear appear as complex harmonic components. The maximum vibration speed appears in the initial stage and the maximum vibration speed attenuates slowly. The maximum vibration speed amplitude (MVSA) of the pinions and large gear is about 1.297 mm and 0.1082 mm, respectively. Pinions and large gear’s attenuation time of maximum vibration speed (ATMVS) is about 80.00 s, but the vibration setting time (VST) of pinions and large gear is much bigger than their ATMVS. The pinions and
\[
\left\{ \begin{array}{l}
\frac{r D(s) \overline{D}_m(s) - nr^2 i_m^2 (k_t + c_i s)^2}{r D(s) T_L(s) + \sum_{k=1}^{n} qr^2 i_m (k_t + c_i s) T_{e,k}(s) - \\
\sum_{k=1}^{n} i_m r (k_t + c_i s)^2 (y_k(s) - y_m^k(s)) + D(s) \times \sum_{k=1}^{n} i_m (k_t + c_i s) (y_k(s) - y_m^k(s))}
\end{array} \right. 
\]
\[
\theta_m(s) = \frac{1}{\Delta(s)} \times \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) = \left( r D(s) \overline{D}_m(s) - nr^2 i_m^2 (k_t + c_i s)^2 \right), \quad \Omega(s) = \left( r (k_t + c_i s)^2 - D(s) (k_t + c_i s) \right) i_m.
\]
\[
\frac{(m_i g^2 + c_y s + k_y)}{(k_f + c_f s)} y_m(s) - \frac{1}{r} (y_i(s) - y_m^i(s)) = \frac{1}{D(s)} \left\{ \begin{array}{l}
qr T_{e,i}(s) + ri_m (k_t + c_i s) \theta_m(s) - (k_t + c_i s) (y_i(s) - y_m^i(s)) \right\} - i_m * \theta_m(s)
\end{array} \right. 
\]
\[
\frac{1}{D(s)} \left\{ qr T_{e,i}(s) + ri_m (k_t + c_i s) \theta_m(s) - (k_t + c_i s) (y_i(s) - y_m^i(s)) \right\} - i_m * \theta_m(s)
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[
\Delta(s) \left\{ -r D(s) T_L(s) + qr^2 i_m (k_t + c_i s) \sum_{k=1}^{n} T_{e,k}(s) - \Omega(s) \sum_{k=1}^{n} (y_k(s) - y_m^k(s)) \right\}
\]
\[ D_y(s) = -\left(\frac{m_y s^2 + c_y s + k_y}{k_f + c_f s}\right) - \frac{1}{r} \left(\frac{k_t + c_t s}{D(s)}\right), \quad D_{y,m}(s) = \frac{1}{r} \left(\frac{k_t + c_t s}{D(s)}\right), \quad D_\lambda(s) = \frac{q r}{D(s)}; \]

\[ D_L(s) = \frac{r^2 i_m (k_t + c_t s) - r i_m D(s)}{-\Delta(s)}, \quad D_\Sigma(s) = \frac{r i_m (k_t + c_t s) - i_m D(s)}{D(s)} \times \frac{q r^2 i_m (k_t + c_t s)}{\Delta(s)}; \]

\[ D_\Omega(s) = \frac{r i_m (k_t + c_t s) - i_m D(s)}{D(s)} \times \frac{-\Omega(s)}{\Delta(s)}, \quad D_\lambda(s) = \frac{m_y s^2 + c_y y_m s + k_y y_m}{(k_f + c_f s)} + \frac{1}{r} \left(\frac{k_t + c_t s}{D(s)}\right); \]

\[
\begin{align*}
D_y(s)y_i(s) + D_{y,m}(s)y_m(s) &= D_\lambda(s)T_{e,i}(s) + D_L(s)T_L(s) + D_\Sigma(s) \sum_{k=1}^{n} T_{e,k}(s) + D_\Omega(s) \sum_{k=1}^{n} (y_k(s) - y^k_m(s)) \\
D_\lambda(s)y_m(s) - D_{y,m}(s)y_i(s) &= D_\lambda(s)T_{e,i}(s) + D_L(s)T_L(s) + D_\Sigma(s) \sum_{k=1}^{n} T_{e,k}(s) + D_\Omega(s) \sum_{k=1}^{n} (y_k(s) - y^k_m(s))
\end{align*}
\]

\[ \Rightarrow G_{y,b}(s)Y(s) = G_T(s)U(s), \quad Y(s) = (y_1(s)y_2(s) \cdots y_n(s)y_1^2(s) \cdots y_m^2(s)) \in \mathbb{R}^{2n} \]

\[ U(s) = (T_{e,1}(s)T_{e,2}(s) \cdots T_{e,n}(s)T_{e,L}(s))^T \in \mathbb{R}^{n+1}, G_{y,b}(s) = \begin{pmatrix} G^{11}_{y,b}(s) & G^{12}_{y,b}(s) \\ G^{21}_{y,b}(s) & G^{22}_{y,b}(s) \end{pmatrix}, G_T(s) = \begin{pmatrix} G^{11}_{T}(s) \\ G^{21}_{T}(s) \end{pmatrix}; \]

\[ G^{11}_{y,b}(s) = \begin{pmatrix} D_y(s) - D_\Omega(s) & -D_\Omega(s) & \cdots & -D_\Omega(s) \\ -D_\Omega(s) & D_y(s) - D_\Omega(s) & \cdots & -D_\Omega(s) \\ \vdots & \vdots & \ddots & \vdots \\ -D_\Omega(s) & -D_\Omega(s) & \cdots & D_y(s) - D_\Omega(s) \end{pmatrix} \in \mathbb{R}^{n \times n} \]

\[ G^{12}_{y,b}(s) = \begin{pmatrix} D_{y,m}(s) + D_\Omega(s) & D_\Omega(s) & \cdots & D_\Omega(s) \\ D_\Omega(s) & D_{y,m}(s) + D_\Omega(s) & \cdots & D_\Omega(s) \\ \vdots & \vdots & \ddots & \vdots \\ D_\Omega(s) & D_\Omega(s) & \cdots & D_{y,m}(s) + D_\Omega(s) \end{pmatrix} \in \mathbb{R}^{n \times n} \]

\[ G^{21}_{y,b}(s) = \begin{pmatrix} -D_{y,m}(s) - D_\Omega(s) & -D_\Omega(s) & \cdots & -D_\Omega(s) \\ -D_\Omega(s) & -D_{y,m}(s) - D_\Omega(s) & \cdots & -D_\Omega(s) \\ \vdots & \vdots & \ddots & \vdots \\ -D_\Omega(s) & -D_\Omega(s) & \cdots & -D_{y,m}(s) - D_\Omega(s) \end{pmatrix} \in \mathbb{R}^{n \times n} \]

\[ G^{22}_{y,b}(s) = \begin{pmatrix} D_\lambda(s) + D_\Sigma(s) & D_\Sigma(s) & \cdots & D_\Sigma(s) \\ D_\Sigma(s) & D_\lambda(s) + D_\Sigma(s) & \cdots & D_\Sigma(s) \\ \vdots & \vdots & \ddots & \vdots \\ D_\Sigma(s) & D_\Sigma(s) & \cdots & D_\lambda(s) + D_\Sigma(s) \end{pmatrix} \in \mathbb{R}^{n \times n} \]

\[ G^{11}_{T}(s) = G^{21}_{T}(s) = \begin{pmatrix} D_\lambda(s) + D_\Sigma(s) & D_\Sigma(s) & \cdots & D_\Sigma(s) & D_L(s) \\ D_\Sigma(s) & D_\lambda(s) + D_\Sigma(s) & \cdots & D_\Sigma(s) & D_L(s) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ D_\Sigma(s) & D_\Sigma(s) & \cdots & D_\lambda(s) + D_\Sigma(s) & D_L(s) \end{pmatrix} \in \mathbb{R}^{n \times (n+1)} \]
large gear’s steady-state amplitude of vibration speed (SAVS) are about 0.002 mm and 0.3×10⁻³ mm respectively. Thus, the steady-state vibration speed of pinions and the large gear is small, and it could be inferred that the working states of the shield TBM cutterhead driving system can be reflected by pinions and large gear’s vibration.

The Fourier Transform (FT) of vibration speeds for pinions and the large gear is presented in Figs. 8–9. The frequency spectrum of vibration speeds of pinions and the large gear in Figs. 8–9 reveals that vibration frequencies of the pinions and the large gear do not locate at the high frequency range and do locate at the low frequency range. The vibration speed’s maximum frequency range (MFR) is less than 400 Hz. The frequency spectrum in Figs. 8–9 shows that the noise signal is smaller and mainly focuses on the high frequency range. In addition, the frequency spectrum in Figs. 8–9 shows that the vibration speed of pinions and the large gear contains constant components and harmonic components. Pinions 1, 3, 5, 6, 8, and 10 have the same frequency spectrum and their corresponding frequency of maximum signal intensity (MSI) is about the 0.2883 Hz. However, pinions 2, 4, 7, and 9 have the same frequency spectrum and their corresponding frequency of MSI is about 0.2883 Hz. Furthermore, the frequency of MSI for the large gear’s vibration speed is about 0.2883 Hz and it is equal to the pinions’ frequency of MSI. In the frequency range of 0–10 Hz, it concentrates about 90 % of the signal energy in this frequency range, but it only concentrates about 10 % of the signal energy in the frequency range 10–400 Hz. Therefore, the pinions and the large gear’s vibration frequency spectrums are located at low frequency ranges, and the shield TBM cutterhead driving system’s vibration is also located at a low frequency range. In addition, the vibration speed and its frequency spectrum of pinions and the large gear implies that the shield TBM’s health status and work conditions can be reflected by detecting the vibration of the cutterhead driving system.

4.1. Physical parameters’ effects on dynamic vibration response

The proposed LTVMA dynamic vibration model of shield TBM cutterhead driving system is simulated and studied under different physical parameters, and the physical parameters’ effects on dynamic vibration response are investigated under various conditions. When discussing a specific physical parameter’s effects on the dynamic vibration response, other physical parameters are fixed. Physical parameters such as motor rotor inertia and motor rotor viscous damped and their effects on the vibration response of dynamic vibration model are investigated. The simulation results of the dynamic vibration model under different parameters are shown in Figs. 10–21. The motor rotor inertia’s effects on the shield TBM cutterhead driving system’s vibration response are investigated. Vibration response results of the dynamic vibration model under various different motor rotor inertia (in unload case) are shown in Figs. 10–15. The dynamic vibration response results in Figs. 10–15 obviously show that the vibration speed of pinions and the large gear contains harmonic components and oscillation behaviours in the initial stage. Figs. 10–11 show that the MVSA of pinions and the large gear is reduced when motor rotor inertia is increased from 2.1 kgm² to 3.1 kgm². The MVSA of pinions and the large gear is about 0.8911 mm and 0.0744 mm, respectively, when the motor rotor inertia is increased to 3.1 kgm².

The MVSA of pinions and the large gear appears in the initial stage. The pinions and large gear’s ATMVS and VST are increased when motor rotor inertia is increased to 3.1 kgm². The SAVS of pinions and the large gear is also reduced, and the SAVS of pinions and the large gear is about 1.5×10⁻³ mm and 0.18×10⁻³ mm, respectively, when the motor rotor inertia is increased to 3.1 kgm². The vibration response results in Figs. 12–13 show that pinions and large gear’s MVSA and SAVS are reduced when the motor rotor inertia is further increased from 3.1 kgm² to 4.1 kgm². The MVSA of pinions and the large gear is about 0.6937 mm and 0.0577 mm, respectively, when the motor rotor inertia is increased to 4.1 kgm². The MVSA of pinions and the large gear appears as oscillation behaviours in the initial stage. The vibration results in Figs. 12–13 also obviously show that the pinions and large gear’s VST and ATMVS are increased considerably when the motor rotor inertia is increased from 3.1 kgm² to 4.1 kgm².

When the motor rotor inertia is further increased from 4.1 kgm² to 5.1 kgm², the corresponding vibration response results in Figs. 14–15 obviously show that the vibration speed of the pinions and large gear appears as continuously oscillating behaviours. The vibration response results in Figs. 14–15 show that the pinions and large gear’s MVSA and SAVS are reduced when the motor rotor inertia is increased from 4.1 kgm² to 5.1 kgm². The MVSA of pinions and large gear is about 0.5679 mm and 0.0475 mm, respectively, when the motor rotor inertia is increased to 5.1 kgm². The MVSA of the pinions and large gear appears as oscillation behaviours in the initial stage. The vibration results in Figs. 14–15 obviously show that the pinions and large gear’s VST and ATMVS are increased considerably when the motor rotor inertia is increased from 4.1 kgm² to 5.1 kgm². Therefore, motor rotor inertia has important effects on the vibration response of the dynamic vibration model. The pinions and large gear’s MVSA and SAVS will be reduced, and pinions and the large gear’s VST and ATMVS will be increased when motor rotor inertia is increased. On the contrary, the MVSA and SAVS will be increased, and VST and ATMVS will be reduced when the motor rotor inertia is reduced.

The motor rotor viscous damp’s effects on the vibration response of the shield TBM cutterhead driving system are investigated, and the vibration response results of the dynamic vibration model under various motor rotor viscous damp (in the unload case) are shown in Figs. 16–21. The dynamic vibration response results in Figs. 16–21 obviously show that the vibration speeds of the pinions and large gear contain harmonic components and oscillation behaviours in the initial stage. Figs. 16–17 show that the MVSA of the pinions and large gear is reduced when the motor rotor viscous damp is increased from 0.25 rad s⁻³ to 0.35 rad s⁻³. The MVSA of the pinions and large gear is about 0.222 mm and 0.0277 mm, respectively, when the motor rotor viscous damp is increased to 0.35 rad s⁻³. The MVSA of the pinions and large gear appear in the initial stage. The SAVS of the pinions and large gear is about 0.46×10⁻³ mm and 0.09×10⁻³ mm respectively when the mo-
tor rotor viscous damp is increased to 0.35 kgm$^2$ rad$^{-2}$.

The pinions and large gear’s ATMVS and VST are also reduced when the motor rotor viscous damp is increased to 0.35 kgm$^2$ rad$^{-2}$. The vibration response results in Figs. 18–19 show that the pinion and large gear’s MVSA and SAVS are reduced when the motor rotor viscous damp is further increased from 0.35 kgm$^2$ rad$^{-2}$ to 0.45 kgm$^2$ rad$^{-2}$. The MVSA of the pinions and large gear is about 0.2059 mm/s and 0.0254 mm/s, respectively, when the motor rotor viscous damp is increased to 0.45 kgm$^2$ rad$^{-2}$. The MVSA of the pinions and large gear appears as oscillation behaviours in the initial stage. The vibration response results in Figs. 18–19 obviously show that pinions and large gear’s VST and ATMVS are reduced when the motor rotor viscous damp is increased from 0.35 kgm$^2$ rad$^{-2}$ to 0.45 kgm$^2$ rad$^{-2}$.

When motor rotor viscous damp is further increased from 0.45 kgm$^2$ rad$^{-2}$ to 0.55 kgm$^2$ rad$^{-2}$, the vibration response results in Figs. 20–21 obviously show that the vibration speed of the pinions and large gear appears as continuously oscillatory behaviours. The vibration response results in Figs. 20–21 obviously show that the pinions and large gear’s MVSA and SAVS are reduced when the motor rotor viscous damp is increased to 0.55 kgm$^2$ rad$^{-2}$. The MVSA of the pinions and large gear is about 0.192 mm/s and 0.0231 mm/s, respectively, when the motor rotor viscous damp is increased to 0.55 kgm$^2$ rad$^{-2}$. The MVSA of the pinions and large gear appears as oscillation behaviours in the initial stage.

The vibration response results in Figs. 20–21 obviously show that the pinions and large gear’s VST and ATMVS are reduced when the motor rotor viscous damp is reduced from 0.45 kgm$^2$ rad$^{-2}$ to 0.55 kgm$^2$ rad$^{-2}$. Therefore, the motor rotor viscous damp has important effects on the vibration response of the dynamic vibration model. The pinion and large gear’s MVSA and SAVS will be reduced, and the pinions and large gear’s VST and ATMVS will be reduced too when the motor rotor viscous damp is increased. On the contrary, the MVSA and SAVS will be increased, and the VST and ATMVS will be increased as well when the motor rotor viscous damp is reduced. Large MVSA and SAVS will cause damage to the cutterhead driving system in the long run. The vibration behaviours of the cutterhead driving system will reflect the actual working states and environment of the shield TBM to some degree.

5. CONCLUSIONS

A general LTVMA dynamic vibration model is established for the shield TBM cutterhead driving system, and the corresponding MIMO state-space model is also presented. The linear vibration model is analysed to obtain the vibration-torque transfer function matrix and vibration-torque static gain matrix. The LTVMA vibration model is simulated under various physical parameters conditions. Physical parameters such as motor rotor inertia and motor rotor viscous damp and their effects on the dynamic vibration response of linear vibration model are investigated and analysed. A preliminary approach is proposed to reduce the vibration amplitude and vibration intensity of the cutterhead driving system by increasing motor rotor inertia and motor rotor viscous damp. Through numerically studying and analysing the LTVMA vibration model, the simulation results reveal the following: (1) The vibration speeds of pinions and the large gear appear as complex harmonic components. The frequency spectrum of the vibration speed reveals that vibration frequencies of pinions and large gears are located at a low frequency range, and the shield TBM cutterhead driving system’s vibration is located at a low frequency range. Vibration noises concentrate on relatively high frequency ranges. For the vibration frequency spectrum of the pinions and large gear, the vibration signal concentrates about 90% of signal energy in a low frequency range. The vibration speed reveals that health status and working states of the shield TBM can be reflected by detecting the cutterhead driving system’s vibrations. (2) The motor rotor inertia has important effects on the dynamic vibration response of the LTVMA vibration model. The pinions and large gear’s MVSA and SAVS will be reduced when the motor rotor inertia is increased; however pinions and large gear’s VST and ATMVS will be reduced when the motor rotor inertia is increased. On the contrary, the MVSA and the SAVS will be increased when the motor rotor inertia is reduced; however, VST and ATMVS will be reduced when the motor rotor inertia is reduced. (3) The motor rotor viscous damp has important effects on the dynamic vibration responses of the LTVMA vibration model. The pinions and large gear’s MVSA and SAVS will be reduced when motor rotor viscous damp is increased, and the pinions and large gear’s VST and ATMVS will be reduced too when the motor rotor viscous damp is increased. On the contrary, the MVSA and SAVS will be increased when the motor rotor viscous damp is reduced, and the VST and ATMVS will be increased when the motor rotor viscous damp is reduced. In all, the vibration response of the cutterhead driving system will reveal actual working states and environment of the shield TBM to some degree. The LTVMA vibration model provides a basis for FDD and healthy monitoring of the shield TBM cutterhead driving system.

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APPENDIX 1: MATRICES $A_{\rho}$, $A_{\delta}$, $B_{\rho}$, $B_{\delta}$, $C_{\rho}$, AND $C_{\delta}$ FOR LTVMA DYNAMIC VIBRATION MODEL

$$A_{11}^\rho = 0_{(n+1) \times (n+1)}, \quad A_{12}^\rho = I_{(n+1) \times (n+1)},$$
$$A_{21}^\rho = \begin{pmatrix} A_{11}^\rho & A_{12}^\rho \\ A_{21}^\rho & A_{22}^\rho \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)},$$
$$A_{22}^\rho = \begin{pmatrix} A_{11}^\rho & A_{12}^\rho \\ A_{21}^\rho & A_{22}^\rho \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)},$$
$$A_{21}^{11} = \text{ diag } \left( -\frac{k_{1,1}}{J_1}, -\frac{k_{1,2}}{J_2}, \ldots, -\frac{k_{1,n}}{J_n} \right) \in \mathbb{R}^{n \times n},$$
$$A_{22}^{11} = \sum_{k=1}^{m} i_{k} \frac{2}{r_{m,k} J_m} \in \mathbb{R},$$
$$A_{21}^{12} = \begin{pmatrix} i_{m,1} k_{1,1} \frac{1}{J_1} & i_{m,2} k_{1,2} \frac{1}{J_2} & \cdots & i_{m,n} k_{1,n} \frac{1}{J_n} \end{pmatrix} \in \mathbb{R}^{n}.$$
\[
A^{11}_{22} = \begin{pmatrix}
\frac{i_{m,1}k_{t,1}}{J_m} & \frac{i_{m,2}k_{t,2}}{J_m} & \cdots & \frac{i_{m,n}k_{t,n}}{J_m}
\end{pmatrix}^T \in \mathbb{R}^n,
\]
\[
A^{11}_{22} = \begin{pmatrix}
-b_1 + c_{t,1} & -b_2 + c_{t,2} & \cdots & -b_n + c_{t,n}
\end{pmatrix}^T \in \mathbb{R}^{n \times n},
\]
\[
A^{22}_{22} = -\frac{b_n + \sum_{k=1}^{n} i_{m,k}c_{t,k}}{J_m} \in \mathbb{R}
\]
\[
A^{12}_{22} = \begin{pmatrix}
\frac{i_{m,1}c_{t,1}}{J_m} & \frac{i_{m,2}c_{t,2}}{J_m} & \cdots & \frac{i_{m,n}c_{t,n}}{J_m}
\end{pmatrix}^T \in \mathbb{R},
\]
\[
A^{22}_{22} = \begin{pmatrix}
\frac{i_{m,1}c_{t,1}}{J_m} & \frac{i_{m,2}c_{t,2}}{J_m} & \cdots & \frac{i_{m,n}c_{t,n}}{J_m}
\end{pmatrix}^T \in \mathbb{R},
\]
\[
B^{11}_{12} = \begin{pmatrix} B^{11}_{11} \\ B^{11}_{12} \\ \vdots \\ B^{11}_{12} \end{pmatrix} \in \mathbb{R}^{(2n+2) \times (n+1)},
\]
\[
B^{12}_{12} = \begin{pmatrix} B^{12}_{11} & B^{12}_{12} & B^{12}_{13} & \cdots & B^{12}_{12} \\ B^{12}_{12} & B^{12}_{12} & B^{12}_{13} & \cdots & B^{12}_{12} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ B^{12}_{12} & B^{12}_{12} & B^{12}_{13} & \cdots & B^{12}_{12} \end{pmatrix} \in \mathbb{R}^{(2n+2) \times 4n}
\]
\[
B^{11}_{11} = 0_{(n+1) \times (n+1)},
\]
\[
B^{12}_{12} = \begin{pmatrix} B^{12}_{11} \\ B^{12}_{12} \\ \vdots \\ B^{12}_{12} \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)}
\]
\[
B^{11}_{11} = \begin{pmatrix} q \\ \vdots \\ q \\ \frac{q - 1}{J_m} \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)},
\]
\[
B^{12}_{12} = \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)},
\]
\[
B^{11}_{11} = \begin{pmatrix} 1 \\ \vdots \\ q \\ \frac{q - 1}{J_m} \end{pmatrix} \in \mathbb{R}^{(n+1) \times (n+1)}.
\]
Figure 8. Frequency spectrum of each pinion's vibration speed for the cutterhead driving system.

Figure 9. Frequency spectrum of large gear's vibration speed for the cutterhead driving system.

Figure 10. Pinion vibration speed with motor rotor inertia $3.1 \text{ kgm}^2$.

Figure 11. Large gear vibration speed with motor rotor inertia $3.1 \text{ kgm}^2$. 
Figure 12. Pinion vibration speed with motor rotor inertia $4.1 \text{ kgm}^2$.

Figure 13. Large gear vibration speed with motor rotor inertia $4.1 \text{ kgm}^2$.

Figure 14. Pinion vibration speed with motor rotor inertia $5.1 \text{ kgm}^2$.

Figure 15. Large gear vibration speed with motor rotor inertia $5.1 \text{ kgm}^2$. 
Figure 16. Pinion vibration speed with motor rotor viscous damped $0.35 \frac{\text{kgm}^2}{\text{rad s}^{-1}}$.

Figure 17. Large gear vibration speed with motor rotor viscous damped $0.35 \frac{\text{kgm}^2}{\text{rad s}^{-1}}$.

Figure 18. Pinion vibration speed with motor rotor viscous damped $0.45 \frac{\text{kgm}^2}{\text{rad s}^{-1}}$.

Figure 19. Large gear vibration speed with motor rotor viscous damped $0.45 \frac{\text{kgm}^2}{\text{rad s}^{-1}}$. 
Figure 20. Pinion vibration speed with motor rotor viscous damped 0.55 \( \frac{\text{kgm}^2}{\text{rad}^2\text{s}^{-2}} \).

\[ V_{21}^{11} = \begin{pmatrix} \frac{k_{y,1}r_1}{m_p,1r_1} & \frac{k_{y,2}r_2}{m_p,2r_2} & \cdots & \frac{k_{y,n}r_n + k_{f,n}}{m_p,nr_n} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ V_{21}^{12} = \begin{pmatrix} \frac{k_{f,1}}{m_p,1r_1} & \frac{k_{f,2}}{m_p,2r_2} & \cdots & \frac{k_{f,n}}{m_p,nr_n} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ V_{21}^{21} = \begin{pmatrix} \frac{k_{f,1}}{m_g,r_1} & \frac{k_{f,2}}{m_g,r_2} & \cdots & \frac{k_{f,n}}{m_g,r_n} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ V_{21}^{22} = \begin{pmatrix} \frac{k_{y,m}r_1 + k_{f,1}}{m_g,r_1} & \frac{k_{y,m}r_2 + k_{f,2}}{m_g,r_2} & \cdots & \frac{k_{y,m}r_n + k_{f,n}}{m_g,r_n} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ B_3^{11} = \begin{pmatrix} \Psi_{11}^{11} \\ \Psi_{11}^{12} \\ \vdots \\ \Psi_{11}^{11} \\ \Psi_{12}^{11} \end{pmatrix} \in \mathbb{R}^{4n \times (n+1)}, B_3^{12} = \begin{pmatrix} \Psi_{12}^{11} \\ \Psi_{12}^{12} \end{pmatrix} \in \mathbb{R}^{4n \times (n+1)}, \]

\[ \Phi_{11}^{11} = \Phi_{11}^{21} = \Phi_{11}^{22} = 0_{n \times (n+1)}, \]

\[ \Psi_{11}^{11} = \left( \Phi_{11}^{11} \Phi_{12}^{11} \right) \in \mathbb{R}^{n \times (n+1)}, \]

\[ \Phi_{11}^{11} = \begin{pmatrix} -k_{f,1} \frac{m_p,1}{m_p,2} & \cdots & -k_{f,n} \frac{m_p,n}{m_p,2} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ \Psi_{11}^{11} = \left( \Pi_{11}^{11} \Pi_{12}^{11} \right) \in \mathbb{R}^{n \times (n+1)}, \]

\[ \Pi_{11}^{11} = \begin{pmatrix} k_{f,1} & k_{f,2} & \cdots & k_{f,n} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ \Pi_{11}^{12} = \begin{pmatrix} \frac{i_{n,k} k_{f,1}}{m_p,1} & \cdots & \frac{i_{n,k} k_{f,n}}{m_p,2} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ \Psi_{11}^{12} = \left( \Pi_{11}^{12} \Pi_{12}^{12} \right) \in \mathbb{R}^{n \times (n+1)}, \]

\[ \Phi_{11}^{12} = \begin{pmatrix} \frac{i_{n,k} k_{f,1}}{m_p,1} & \cdots & \frac{i_{n,k} k_{f,n}}{m_p,2} \end{pmatrix} \in \mathbb{R}^{n \times n}, \]

\[ \Psi_{11}^{12} = \left( \Pi_{11}^{11} \Pi_{12}^{12} \right) \in \mathbb{R}^{n \times (n+1)}, \]

\[ \Pi_{11}^{12} = \begin{pmatrix} \frac{i_{n,k} k_{f,1}}{m_p,1} & \cdots & \frac{i_{n,k} k_{f,n}}{m_p,2} \end{pmatrix} \in \mathbb{R}^{n \times n}. \]
APPENDIX 2: SIMULATION PARAMETERS

\[
\Pi_{11} = \left( -\frac{i_{m,1}c_{f,1}}{m_g} - \frac{i_{m,2}c_{f,2}}{m_g} - \ldots - \frac{i_{m,n}c_{f,n}}{m_g} \right)^T \in \mathbb{R}^n
\]

\[
C_{11} = I_{2n \times 2n} + C_{d} = (C_{1}^{d})^{T} C_{1}^{d} \in \mathbb{R}^{2n \times 2n}
\]

\[
\kappa_{0} = I_{4n \times 4n}, \quad \kappa_{1} = (2n+2) \times 2(n+2),
\]

\[
J_{0} = \left( 0_{(n+1) \times (n+1)} \right) \in \mathbb{R}^{(n+1) \times (n+1)}
\]

\[
J_{i} = (q^{2} d_{i} + c_{i}), \quad b_{i} = (q^{2} b_{di} + b_{c,i}) \quad (i = 1, 2, \ldots, n)
\]

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