COORDINATIVE DYNAMICS ANALYSIS OF A PARALLEL MANIPULATOR WITH THREE GRIPPERS

Yi Lu,* ** Peng Wang,* Zhuolei Hou,* Hu Bo,* Chumping Sui,*** and Jianda Han***

Abstract

A kinetostatic model and a dynamics model of a parallel manipulator (PM) with three grippers are established and analyzed in the light of its application. First, the coordinative kinematics of this manipulator and three grippers are analyzed. Second, the kinetostatic formulae are derived for transforming the coordinative kinematics and forces applied on the grippers into the workloads applied on the moving platform and the active/constrained forces and torques applied onto active legs. Third, the dynamics formulae are derived for solving dynamic active wrench of grippers and PM by considering inertial mass of the grippers, legs and platform. Finally, a numerical example is given for illustrating application of kinetostatic model and dynamics formulae, and the dynamics solutions of this manipulator are verified by simulation.

Key Words

Parallel manipulator with multi grippers, coordinative dynamics, coordinative kinestatics

Notation

DoF – degree of freedom; PM – parallel manipulator
m, B – platform and base of PM
bi, Bi – joints on m and B (i = 1, 2, 3)
w – the ith claw
S, P, R – spherical, prismatic, revolute joint
ri, δi – active legs of PM and its unit vector
{B}, {m} – coordinate frame of B and m
α, β, λ – three Euler angles of m in {B}
x1, x2, x3, y1, y2, y3, z1, z2, z3 – orientation parameters of m
Li, Li – side of m and side of B

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1. Introduction


In the aspect of dynamics of PMs, Legnani et al. [10], [11] proposed a homogeneous matrix approach to 3D kinematics and dynamics and applied it to the chains of rigid bodies and serial manipulators. Lu and Hu [12] established unified and simple dynamic model for PMs with linear active \( r_i \) and supposed that the angular velocity \( \omega_m \) of \( r_i \) satisfy \( \omega^2_m r_i = 0 \). Chen [13] studied optimal configuration of reconfigurable PMs for determining their maximum dynamic load capacity. Using Newton–Euler or the Lagrange methods, Mendes et al. [14], Li et al. [15] and Abdellatif and Heimann [16] derived dynamic models of the three-, four-, six-DoF PMs, respectively. Based on the principle of virtual work, Staicu [17], [18], Enferadi and Tootoonchchi [19] and Zhao and Gao [20]–[22] studied dynamic models of a three-, six-DoF PMs. Gallardo et al. [23] solved the dynamics of a modular spatial hyper-redundant manipulator by screw theory. As the PMs with multi-grippers are designed for high-speed and heavy-load coordinative operation or for high-speed and high-precision coordinative operation, it is significant to establish their coordinative dynamics model and determine their dynamic characteristics. However, up to now, there are no efforts towards the coordinative dynamics of the PMs with the multi-grippers. For this reason, this paper focuses on the cooperation dynamics analysis of a novel low-mobility PM with three grippers.

2. Prototype of PM with Three Grippers

A prototype of the novel 3SPR type PM with three grippers is built in Yanshan University, see Fig. 1(a). The 3SPR PM includes a moving platform \( m \), three SPR (spherical joint \( S \)-active prismatic joint \( P \)-revolute joint \( R \)) type active legs \( r_i \) (\( i = 1, 2, 3 \)) and a base \( B \). The three joint points \( b_i \) are distributed uniformly on \( m \) at the same circumference, the three joint points \( B_i \) are distributed uniformly on \( B \) at the same circumference and each of \( r_i \) is connected with \( m \) by \( R \) at \( b_i \) with \( B \) by \( S \) at \( B_i \). Each of the gripper mechanisms is formed by a planar four-bar mechanism which is composed of a frame, a claw \( w_i \) and a SPR-type active rod \( l_i \).

The three grippers can operate coordinately for grabbing and excluding the danger object. The active rod is divided as: the outside active rod and the inside active rod, see Fig. 1(b). The frame is fixed on \( m \) at a point \( d_m \); three point \( d_m \) are distributed uniformly on \( m \) at the same circumference, see Fig. 1(c). \( l_i \) is connected with \( w_i \) point \( G_i \), with \( m \) at point \( N_i \). \( w_i \) is connected with \( m \) at \( Q_i \). Let \( \perp \) be perpendicular constraints, \( || \) be parallel constraint, \( \{ B \} \) be a coordinate frame \( O-XYZ \) attached on \( B \) at \( O \). \( \{ m \} \) be a coordinate frame \( o-xyz \) attached on \( m \) at \( o \). \( \{ G_i \} \) be a coordinate frame \( o-x_iy_iz_i \) of gripper attached on \( m \) at \( o \), \( \phi_i \) be the angle between \( x_i \) and \( x \). \( X||B_1B_3 \), \( Z\perp B, z \perp m, z_i \perp m, r_i \perp l_i \) are satisfied.

![Figure 1. A PM with three grippers and its statics model.](image-url)
3. Kinestatic Analysis of Gripper and PM

3.1 Kinematics Analysis of Claw $w_i$ in $\{s_i\}$

The ith gripper mechanism is shown in Fig. 1(b). Let $P_i$ be the tip of the gripper in plane $x_i,y_i,z_i$; $g_i$ is the distance from $Q_i$ to $G_i$; $k_i$ is the distance from $Q_i$ to $P_i$; $h_i$ is the distance from $Q_i$ to $M_i$; $h_{ti}$ be the distance from $Q_i$ to $m_i$; $q_i$ be the distance from $Q_i$ to $Q_i$; $d_i$ be the distance from $M_i$ to $N_i$, $d_{ti}$; $p_i$ be the distance from $G_i$ to $P_i$. Let $\alpha_i$ be the angle between $q_i$ and $g_i$, $\beta_i$ be the angle between $q_i$ and $h_i$, $\theta_i$ be the angle between $g_i$ and $k_i$. Some geometric and differential formulae are expressed as follows:

$$D_0 = g_i^2 + q_i^2 - l_i^2$$
$$D'_0 = -2q_ig_i \cos \theta_i$$
$$\cos \theta_i = \frac{q_i^2 + k_i^2 - p_i^2}{2q_ik_i}$$
$$\sin \theta_i = \frac{2q_ig_i^2}{2q_ik_i}$$
$$\omega_{w_i} = \alpha_i = \frac{l_i V_{li}}{g_i \sin \alpha_i} = \frac{2I_i V_{li} V_{li}}{D_i}$$
$$\epsilon_{w_i} = \frac{2(V_{li}^2 + I_i a_{li})V_{li} - D_i V_{li}^{\ast} \omega_{w_i}}{D_i^2}$$

(1)

here, $l_i$, $v_{li}$ and $a_{li}$ are the length, the extension velocity and the acceleration of active rod $l_i$; $\omega_{w_i}$ and $\epsilon_{w_i}$ are the angular velocity and acceleration of $w_i$ about $q_i$ in $\{s_i\}$.

When given $q_i$, $v_{li}$ and $a_{li}$, the components of the position, the linear velocity and acceleration of point $P_i$, the angular velocity and acceleration of $w_i$ in $\{s_i\}$ are derived from (1) as:

$$P_{ix} = \rho_i \pm D(D_0 D_1 + D_2 D_{ti}), \quad P_{iy} = 0$$
$$P_{iz} = h_{ti} - D(D_0 D_2 - D_1 D_{ti})$$
$$D_1 = d_i + h_i \tan \theta_i, \quad D_2 = h_i - d_i \tan \theta_i$$
$$V_{pix} = \pm 2D_i V_{li} (-D_1 + D_0 D_2/D_i)$$
$$V_{piz} = 2D_i V_{li} (D_2 + D_0 D_{ti}/D_1), \quad \rho_i V_{piy} = 0$$
$$\omega_{wix} = \omega_{wiy} = 0, \quad \omega_{wiz} = 2l_i V_{li}/D_i$$
$$\alpha_{pix} = \pm 2D_i (V_{li}^2 + I_i a_{li})(-D_1 + D_0 D_{ti}/D_1)$$
$$\alpha_{piz} = 2D_i V_{li}^2 (D_2 + D_0 D_{ti}/D_1) - 2D_1 (D_0^2 + D_1^2) V_{li}^2 / D_i^2$$
$$\alpha_{piy} = \epsilon_{wix} = \epsilon_{wiy} = 0$$
$$\epsilon_{wiz} = 2[ (V_{li}^2 + I_i a_{li}) D_{ti} - D_0 l_i V_{li}^{\ast} \omega_{wiz} ] / D_i^2$$

(2)

Let $P_{ci}$ be the mass centre of $w_i$, $k_{ci}$ be the distance from $Q_i$ to $P_{ci}$, $P_{ci}$ be the distance from $G_i$ to $P_{ci}$ and $\theta_{ci}$ be the angle between $g_i$ and $k_{ci}$, see Fig. 1(b). The formulae for solving the position $\ast P_{ci}$, the general velocity $\ast V_{p_i}$ and the general acceleration $\ast A_{p_i}$ of $P_i$ are similar to (1)–(3), except that $P_i$, $k_i$, $p_i$ and $\theta_i$ are replaced by $P_{ci}$, $k_{ci}$, $p_{ci}$ and $\theta_{ci}$, respectively.

3.2 Kinematics Analysis of Active Rod $l_i$ in $\{s_i\}$

The active rod $l_i$ in the ith gripper mechanism is shown in Fig. 1(b). Let $\gamma_i$ be the angle between $q_i$ and $l_i$. The formulae for solving angular velocity, acceleration of $l_i$ are derived as:

$$\omega_{lix} = \omega_{lix} = \omega_{lix} = 0, \quad \omega_{lix} = J_i V_{li}$$
$$\epsilon_{lix} = J_i a_{li} - \frac{V_{li}^2}{\sin \gamma_i} \left( \frac{q_i^2 - g_i^2}{q_i^2} + J_i \cos \gamma_i \right)$$

(4)

$$\cos \gamma_i = \frac{l_i^2 + q_i^2 - g_i^2}{2q_i}$$

Let $G_i$ be the connection point of $w_i$ with $l_i$; $N_i$ be the connection point of $l_i$ with $m$. When $\theta_i = 0$, the formulae for solving the position vectors $\ast G_i$ and $\ast N_i$ in $\{s_i\}$ are derived from (2) as follows:

$$G_i = \begin{bmatrix} \rho_i \pm (d_i D_0 + h_i D_{ti})/(2q_i^2) \\ 0 \\ (h_i - (h_i D_0 - d_i D_{ti})/(2q_i^2)) \end{bmatrix}$$
$$N_i = \begin{bmatrix} \rho_i + d_i \\ 0 \\ -(h_i - h_{ti}) \end{bmatrix}$$

(5)

Let $l_{ti}$ be the mass centre of piston of $l_i$ and the distance from $N_i$ to $l_{ti}$; $l_{ti}$ be the mass centre of cylinder of $l_i$ and the distance from $N_i$ to $l_{ti}$. The position vectors of $(l_i, l_{ti}, l_{2i})$ and their unit vector $\ast \delta_i$ are derived as follows:

$$\delta_{li} = \frac{1}{2q_i^2 l_i} \begin{bmatrix} \pm (d_i D_0 - q_i^2 d_i + h_i D_{ti}) \\ 0 \\ 2q_i h_i - D_0 h_i + d_i D_{ti} \end{bmatrix}$$

(6)

$$l_i = \ast G_i - \ast N_i, \quad \delta_{lij} = \ast \delta_{lij}, j = 1, 2$$

The linear velocity $\ast V_{lij}$, the angular velocity $\ast \omega_{lij}$, the linear acceleration $\ast a_{lij}$, the angular acceleration $\ast \alpha_{lij}$, the general velocity $\ast V_{lij}$ and general acceleration $\ast A_{lij}$ of $l_{ij}$ $(j = 1, 2)$ in $\{s_i\}$ are derived as follows:

$$\omega_{lij} = \omega_{lij} = \left[ \begin{array}{c} \omega_{lij} \\ 0 \end{array} \right], \quad \omega_{lij} = J_i V_{lij}$$
$$\alpha_{lij} = \alpha_{lij} = \left[ \begin{array}{c} \alpha_{lij} \\ 0 \end{array} \right]$$
$$\delta_{lij} = \delta_{lij} + \ast \alpha_{lij} \times \ast \delta_{lij}$$

$$\ast V_{lij} = (\ast V_{lij})^{\ast T}$$
$$\ast A_{lij} = (\ast A_{lij})^{\ast T}$$

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3.3 Kinematics Model of Platform of 3SPR PM

Let \( \alpha, \beta, \lambda \) be three Euler angles of \( m \) in \( \{B\} \); \( \varphi \) be one of \( \phi_i, \alpha, \beta, \lambda, s_\varphi = \sin \varphi, c_\varphi = \cos \varphi \). Let \( R \) be a rotational transformation matrix from \( \{s_i\} \) to \( \{m\} \), \( R \) be a rotational transformation matrix from \( \{m\} \) to \( \{B\} \) in Euler rotational order of \( X-Y-X' \). For \( \{s_i\} \) to \( \{B\} \), \( R \) be a rotational transformation matrix from \( \{B\} \) to \( \{s_i\} \); They are solved as:

\[
\begin{align*}
R_{\{m\}} &= \begin{bmatrix}
\cos \varphi & -\sin \varphi & 0 \\
\sin \varphi & \cos \varphi & 0 \\
0 & 0 & 1
\end{bmatrix} \\
R_{\{s_i\}} &= \begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3
\end{bmatrix} \\
R_{\{B\}} &= \begin{bmatrix}
x_1 & y_1 & z_1 \\
x_2 & y_2 & z_2 \\
x_3 & y_3 & z_3
\end{bmatrix}
\end{align*}
\]

The position vectors of \( m \) in \( \{B\} \) is expressed as:

\[
\begin{align*}
\mathbf{r}_{m} &= \begin{bmatrix}
x \\
y \\
z
\end{bmatrix} \\
\mathbf{r}_{B} &= \begin{bmatrix}
x_B \\
y_B \\
z_B
\end{bmatrix}
\end{align*}
\]

The linear velocity and the angular velocity of \( m \) at \( o \) in \( \{B\} \); \( \mathbf{v} \) and \( \boldsymbol{\omega} \) be the linear and the angular velocities of \( m \) at \( o \) in \( \{B\} \); \( \mathbf{E} \) be the unit matrix. They are expressed as follows:

\[
V = \begin{bmatrix}
\mathbf{v} \\
\boldsymbol{\omega}
\end{bmatrix}, \quad A = \begin{bmatrix}
\alpha \\
\varepsilon
\end{bmatrix}
\]

The formulae for solving \( V \) and \( A \) have been derived in [24] as:

\[
V = J^{-1}v_r, \quad v_r = J_{6 \times 6}V \\
A = J^{-1}(a_r - V^THV), \quad a_r = JA + V^THV
\]

Here, \( v_r \) and \( a_r \) be the general inverse (input) velocity and the general inverse acceleration of the three-SPR PM along active legs \( r_i \); \( J \) is a \( 6 \times 6 \) Jacobian matrix; \( H \) is a \( 6 \times 6 \times 6 \) Hessian matrix.

3.4 Kinematics Analysis of Gripper Mechanism in \( \{B\} \)

Let \( K_i \) \((K = P, P_a, W = \omega; K = l_1, l_2 \) for \( W = l) \) be a point of gripper. The formulae are derived for solving the position vector \( K_i \), the linear velocity \( \mathbf{v}_{Ki} \), the angular velocity \( \boldsymbol{\omega}_{Mi} \), the linear acceleration \( \mathbf{a}_{Ki} \), and the angular acceleration \( \varepsilon_{Mi} \) of \( \omega_{Ki} \) and \( l_i \) in \( \{B\} \) from (1)-(11) as follows:

\[
K_i = o + R_i^{R_s}K_i \\
(\mathbf{v}) = P, P_a, \omega, l, K = l_1, l_2 \) for \( W = l) \)

\[
\mathbf{v}_{Ki} = \mathbf{v} + R_i^{R_s}v_{Ki} - s_{R_i}R_i^{R_s}K_i \omega \\
= \mathbf{v} + J_{6 \times 6}v_r - s_{R_i}R_i^{R_s}K_i \omega
\]

\[
\omega_{W_i} = \omega + R_i^{R_s}\omega_{W_i} = \omega + J_{6 \times 6}v_r \omega \\
\mathbf{a}_{Ki} = a - s_{R_i}R_i^{R_s}K_i \mathbf{a}_{Ki} \\
\mathbf{a}_{Ki}^{(1)} = \mathbf{a} + R_i^{R_s}v_{Ki} + s_{R_i}R_i^{R_s}K_i
\]

\[
\mathbf{e}_{W_i} = \mathbf{e} + R_i^{R_s}\mathbf{e}_{W_i} + s_{R_i}R_i^{R_s}\mathbf{e}_{W_i}
\]
Here, $\frac{\partial R}{\partial s} v_{K_i}$ and $\frac{\partial R}{\partial s} \omega_{W_i}$ ($K = P, P_c$ as $W = w$; $K = l_1, l_2$ as $W = l$) can be expressed as follows:

$$\frac{\partial R}{\partial s} v_{K_i} = [v_{K_{i1}}, v_{K_{i2}}, v_{K_{i3}}]^{T} = J_{vK_i} v$$

$$\frac{\partial R}{\partial s} \omega_{W_i} = [\omega_{W_i1}, \omega_{W_i2}, \omega_{W_i3}]^{T} = J_{\omega W_i} \omega$$

$$v_{K_{ij}} = R_{ji} s^v v_{K_{i+}} + R_{ji} s^v v_{K_{i+}} + R_{ji} s^v v_{K_{i+}}$$

$$\omega_{W_{ij}} = R_{ji} s^\omega \omega_{W_{ij}} + R_{ji} s^\omega \omega_{W_{ij}} + R_{ji} s^\omega \omega_{W_{ij}}$$

$$J_{vK_i} = \begin{bmatrix} v_{K_{i1}}/v_x & 0 & 0 \\ v_{K_{i2}}/v_y & 0 & 0 \\ v_{K_{i3}}/v_z & 0 & 0 \end{bmatrix}$$

$$J_{\omega W_i} = \begin{bmatrix} \omega_{W_{i1}}/\omega_x & 0 & 0 \\ \omega_{W_{i2}}/\omega_y & 0 & 0 \\ \omega_{W_{i3}}/\omega_z & 0 & 0 \end{bmatrix}$$

(13)

Each item in $J_{vK_i}$ and $J_{\omega W_i}$ can be solved from (11) and (13).

The general velocity $V_{K_i}$ and acceleration $A_{K_i}$ of $K_i$ in $B$ are derived from (13) as follows:

$$V_{K_i} = J_{vK_i} V$$

$$V_{K_i} = \begin{bmatrix} v_{K_{i1}} \\ v_{K_{i2}} \\ v_{K_{i3}} \end{bmatrix}, J_{vK_i} = E_{6\times6} + [0, J_{\omega W_i}^{T}]$$

$$A_{K_i} = \frac{E_{8\times3} - s(\frac{\partial R}{\partial s} R_{K_i})}{0_{3\times3} E_{8\times3}} A + \frac{\frac{\partial R}{\partial s} R_{K_i}}{0_{3\times3} E_{8\times3}} A_{K_i}$$

$$+ \frac{2s(\omega)^{\frac{\partial R}{\partial s}} R_{K_i}}{0_{3\times3} E_{8\times3}} \omega v_{K_i} + \frac{s(\omega)^{2\frac{\partial R}{\partial s}} R_{K_i}}{0_{3\times3} E_{8\times3}} \omega^2 v_{K_i}$$

(14)

When given ($v_{r_{i1}}, v_{r_{i2}}, v_{r_{i3}}, l_{i1}, l_{i2}, l_{i3}, \ldots i = 1, 2, 3$), $K_i$, $V_{K_i}$, $A_{K_i}$ can be solved from (12)–(14).

### 3.5 Kinematics Model of SPR-type Linear Leg in $B$

Let $\omega_{r_i}$ and $\varepsilon_{r_i}$ be the angular velocity and acceleration of the SPR-type linear leg, respectively, see Fig. 1(e). As $\omega_{r_i} \neq 0$ for SPR-type linear leg, formulae for solving $\omega_{r_i}$ and $\varepsilon_{r_i}$ in [12] is not suitable here. The formulae for solving $\omega_{r_i}$ and $\varepsilon_{r_i}$ have been derived in [25] as follows:

$$\omega_{r_i} = [\delta_i \times (\mathbf{v} + \omega \times e_i) + r_i \delta_i (\omega \times \delta_i)]/r_i$$

$$\varepsilon_{r_i} = \frac{1}{r_i} [(\omega_{r_i} \times \delta_i) \times \mathbf{v} + (\omega_{r_i} \times \delta_i) \times (\omega \times e_i) + \delta_i + \delta_i \times [e_i \times e_i + \omega \times (\omega \times e_i)]$$

$$+ r_i (\omega_{r_i} \times \delta_i) \delta_i^T \omega + r_i \delta_i (\omega_{r_i} \times \delta_i) \omega + r_i \delta_i (\omega_{r_i} \times \delta_i) T \omega$$

$$+ r_i \delta_i \delta_i^T \varepsilon - v_{r_i}(\omega_{r_i})$$

(15)

Each of the linear legs $r_i$ is composed of a piston and a cylinder. The piston does not spin about the cylinder's axis. The piston in $r_i$ is connected with $m$ at $b_i$. The cylinder in $r_i$ is connected with $B$ at $B_i$, see Fig. 1(e). Let $r_{i1}$ be the mass-centre of the $i$th piston and the distance from $B_i$ to $r_{i1}; r_{i2}$ be the mass-centre of the $i$th cylinder and the distance from $B_i$ to $r_{i2}$. Let $V_{r_{ij}}$ be a general velocity of $r_{ij}$ ($j = 1, 2$). Let $\omega_{r_{ij}}$ be the linear and angular velocities of $r_{ij}$; $\delta_{r_{ij}}$ and $\varepsilon_{r_{ij}}$ be the linear and angular accelerations of $r_{ij}$ ($r_{r_{ij}}, \omega_{r_{ij}}, \delta_{r_{ij}}, \varepsilon_{r_{ij}}, V_{r_{ij}}$) in $B$ are derived as follows:

$$v_{r_{i1}} = \mathbf{v} + \omega \times e_i - (r_i - r_{i1}) \omega_{r_{i1}} \times \delta_i = r_i D_v v - r_i D_v \omega$$

$$D_v = (r_i - r_{i1}) s(\delta_i)^2, D_\delta = s(\delta_i) s(e_i) + r_i \delta_i \delta_i^T$$

$$a_{r_{i1}} = a + e \times e_i + \omega \times (\omega \times e_i)$$

$$- (r_i - r_{i1}) \varepsilon_{r_{i1}} \delta_i - (r_i - r_{i1}) \omega_{r_{i1}} \times (\omega_{r_{i1}} \times \delta_i)$$

$$\omega_{r_{i1}} = \omega_{r_{i1}} \omega_{r_{i1}} \omega_{r_{i1}} = e_{r_{i1}}$$

$$v_{r_{i2}} = \omega_{r_{i2}} \times r_{i2} \delta_i - r_{i2} s(\delta_i) \omega_{r_{i2}}$$

$$a_{r_{i2}} = r_{i2} \varepsilon_{r_{i2}} \delta_i + r_{i2} \omega_{r_{i2}} \times (\omega_{r_{i2}} \times \delta_i)$$

$$V_{r_{i1}} = \begin{bmatrix} v_{r_{i1}} \\ \omega_{r_{i1}} \end{bmatrix} = J_{r_{i1}} V, V_{r_{i2}} = \begin{bmatrix} v_{r_{i2}} \\ \omega_{r_{i2}} \end{bmatrix} = J_{r_{i2}} V$$

$$J_{r_{i1}} = \begin{bmatrix} D_v & D_\delta \\ s(\delta_i) & D_\delta \end{bmatrix}$$

$$J_{r_{i2}} = \begin{bmatrix} D_v & D_\delta \\ s(\delta_i) & D_\delta \end{bmatrix}$$

(16)

### 3.6 Coordinative Statics of Grippers and 3SPR PM

The statics model of the 3SPR PM is shown in Fig. 1(d). The dynamics model of active rod of gripper in shown in Fig. 2(a). The dynamics model of linear leg $r_i$ with piston/cylinder in PM is shown in Fig. 2(b).

Let $F_{K_i}$ and $T_{K_i}$ ($K = P, P_c$) be the concentrated force and the concentrated torque in $B$ applied at the tip point $P_i$ and the mass centre $P_{ci}$ or $w_i$, respectively. (*F_{K_i}, *T_{K_i}) and (*F_{K_i}, *m T_{K_i}) are the wrench ($F_{K_i}, T_{K_i}$) are represented in $\{s_i\}$ and $\{m\}$, respectively. When given (*F_{r_{i1}}, *m T_{r_{i1}}), the wrench (*F_{m}, *m T_{i}) and (F_{t_{i1}}, T_{t_{i1}}) exerted on $m$ at $a$ in $\{m\}$ and $\{B\}$ are derived as:

$$m F_{i} = \begin{bmatrix} m F_{i1} \\ m F_{t_{i1}} \end{bmatrix} = \begin{bmatrix} s(\omega)^T F_{P_i} \end{bmatrix}, m T_{i} = \begin{bmatrix} m T_{i1} \\ m T_{t_{i1}} \end{bmatrix} = \begin{bmatrix} s(\omega)^T T_{P_i} \end{bmatrix}$$

$$F_{i} = \begin{bmatrix} F_{i1} \\ F_{t_{i1}} \end{bmatrix} = \begin{bmatrix} s(\omega)^T F_{P_i} \end{bmatrix}, T_{i} = \begin{bmatrix} T_{i1} \\ T_{t_{i1}} \end{bmatrix} = \begin{bmatrix} s(\omega)^T T_{P_i} \end{bmatrix}$$

(17)
4. Dynamics of Parallel Manipulator with Three Grippers

The dynamics models of the active rod \( l_i \) with piston/cylinder in the \( i \)th gripper and the \( i \)th linear active leg \( r_i \) with piston/cylinder in PM are shown in Fig. 2. Let \((F_{qi}, T_{qi})\) and \(G_{qi}\) be the inertia wrench and gravity of the \( q \)th leg \((q = w, r_j, l_j)\), respectively; \((F_o, T_o)\) be the operating wrench exerted on \( m \) at \( o \); \((F_m, T_m)\) and \(G_m\) be the inertia wrench and the gravity of platform \( m \); \((F_d, T_d)\) be the end effector’s damping wrench exerted on \( m \) at \( o \); \( m_o\) and \( I \) be the mass and inertia moment tensor matrix of the platform about point \( o \); \( m_q \) and \( I_q \) be the mass and inertia moment tensor matrix of \( q \); \( \mu \) be a damping coefficient; \( g \) be a gravity acceleration. These dynamic parameters are expressed as follows [12]:

\[
\begin{align*}
F_m &= -m_o a, G_m = m_o g \\
T_m &= -I e - \omega \times (I \omega), F_d = \mu v, T_d = \mu \omega \\
F_{qi} &= -m_{qi} q_{qi}, G_{qi} = m_{qi} g_{qi} \\
T_{qi} &= -I_{qi} e_{qi} - \omega_{qi} \times (I_{qi} \omega_{qi}), (q = w, r_j, l_j) \\
\omega_{jii} &= \omega_{ii}, \varepsilon_{jii} = \varepsilon_{ii}, \varepsilon_{rij} = \varepsilon_{ri}, \omega_{rij} = \omega_{ri} \\
I &= \begin{bmatrix} I_{11} & -I_{12} & -I_{13} \\
-I_{21} & I_{22} & -I_{23} \\
-I_{31} & -I_{32} & I_{33} \end{bmatrix}, I_{qi} = \begin{bmatrix} 0 & 0 & 0 \\
0 & I_{qi22} & 0 \\
0 & 0 & I_{qi33} \end{bmatrix}
\end{align*}
\]

When ignoring the friction of all the joints in the mechanism, the dynamic workload wrench \((F, T)\) includes the inertia wrench and the gravity of platform and grippers, the end effector’s damping wrench, the equivalent inertia wrench and the gravity of legs, and the operating wrench \((F_o, T_o)\) [see Figs. 1(d) and 2(b)]. Thus, based on the principle of virtual work, a power equation is derived:

\[
\begin{align*}
\begin{bmatrix} F \\ T \end{bmatrix} V &= (\Gamma_m + \Gamma_{Pci} + \Gamma_{li}) V \\
\Gamma_m &= \begin{bmatrix} F_o + F_m + G_m + F_d \\
T_o + T_m + T_d \end{bmatrix}, \\
\Gamma_{Pci} &= \sum_{i=1}^{3} \begin{bmatrix} J_{Pci}^T F_{Pci} + G_{Pci} \\
T_{Pci} \end{bmatrix}, \\
\Gamma_{li} &= \sum_{i=1}^{3} \sum_{j=1}^{2} \begin{bmatrix} J_{lji}^T F_{lji} + G_{lji} \\
T_{lji} \end{bmatrix} + J_{rji}^T \begin{bmatrix} F_{rji} + G_{rji} \\
T_{rji} \end{bmatrix}
\end{align*}
\]

When considering the friction of all the joints in a PM, the friction loads of the joints should be transformed into a part of the dynamic workload wrench \((F, T)\) by counting the efficiency \( \eta \) of PM. Thus, the dynamic workload wrench \((F, T)\) is derived from (19)–(21):

\[
\begin{align*}
\begin{bmatrix} F \\ T \end{bmatrix} &= \frac{1}{\eta} (\Gamma_m + \Gamma_{Pci} + \Gamma_{li})
\end{align*}
\]
The active forces $F_{ci}$ ($i = 1, 2, 3$) and the constrained forces $F_{c\dot{a}}$ in $\{B\}$ of the 3SPR PM can be solved using a statics model of the 3SPR PM [22]:

$$\begin{bmatrix} F_{a1} & F_{a2} & F_{a3} & F_{c1} & F_{c2} & F_{c3} \\ \end{bmatrix}^T = -(J^T)^{-1} \begin{bmatrix} F \\ T \end{bmatrix} \tag{23}$$

After $V$ and $A$ of $m$ are solved, when the masses, the mass-centres, the inertial moments and the damping coefficients of platform and legs, and the efficiency of PM are given, the dynamic workload wrench $(F, T)$ can be solved by (20) and (22). Finally, the dynamic active forces and the dynamic constrained wrench can be solved from (23).

Similarly, based on the principle of virtual work, a power equation of the ith gripper in $\{s_i\}$ is derived as follows:

$$F_{iis}v_{iit} = \sum_{j=1}^{2} \left( s^i F_{ijt} \right) \delta_{iij} + \sum_{j=1}^{2} \left( s^i T_{ijt} \right) \delta_{iij}$$

The active force of $l_i$ with piston/cylinder in gripper is derived from (2), (7) and (24) as follows:

$$F_{li} = \left| \pm F_{Piz} \left( D_{o}D_{i} - D_{1i}D_{li} \right) + F_{Piz} D_{o}D_{i} + D_{2i}D_{li} \right|$$

$$+ F_{Pci} D_{o}D_{ci} + D_{ci}D_{li} + \sum_{j=1}^{2} \left( s^i T_{ijt} \right)$$

$$\cos \theta_{ci} = \frac{g_i^2 + k_i^2 - p_i^2}{2g_i k_i}, D_c = \frac{k_c \cos \theta_{ci}}{2g_i q_i^2}$$

$$D_{ci} = d_i + h_i \tan \theta_{ci}, D_{cl} = h_i - d_i \tan \theta_{ci}$$

When ignoring the inertia wrench of grippers, a static force of $F_{li}$ is obtained from (25):

$$F_{li} = \frac{2l_i}{D_{li}} \left| \pm F_{Piz} \left( D_{o}D_{i} - D_{1i}D_{li} \right) + F_{Piz} D_{o}D_{i} + D_{2i}D_{li} + s^i T_{ijt} \right|$$

5. A Numerical Example of Kinetostatic Model and Dynamics Model

Some given dimensions of grippers with the inside active rod and platform and the symmetry grabbing forces and a torque applied on the tip of grippers for this manipulator are listed in Table 1. The inertia moment tensor matrices of $w_i$, $l_i$ and $r_{ji}$ ($i = 1, 2, 3; j = 1, 2$) are given as follows:

<table>
<thead>
<tr>
<th>$g_i$</th>
<th>0.2 m</th>
<th>$Q_i$ to $G_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_i$</td>
<td>1.1 m</td>
<td>$G_i$ to $P_i$</td>
</tr>
<tr>
<td>$h_i$</td>
<td>0.46 m</td>
<td>$Q_i$ to $M_i$</td>
</tr>
<tr>
<td>$q_i$</td>
<td>0.48 m</td>
<td>$Q_i$ to $N_i$</td>
</tr>
<tr>
<td>$h_{1i}$</td>
<td>0.32 m</td>
<td>$Q_i$ to $m$</td>
</tr>
<tr>
<td>$l_i$</td>
<td>0.35 m</td>
<td>$l_i$ to $t_i$</td>
</tr>
<tr>
<td>$k_i$</td>
<td>0.98 m</td>
<td>$Q_i$ to $P_i$</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>0.5 m</td>
<td>$o$ to $Q_iN_i$</td>
</tr>
<tr>
<td>$e_i$</td>
<td>0.5 m</td>
<td>$o$ to $b_i$</td>
</tr>
<tr>
<td>$E$</td>
<td>0.6 m</td>
<td>$O$ to $B_i$</td>
</tr>
</tbody>
</table>

| $m_{uni}$ | 15 kg | $\phi_1 = 30^\circ$, angle of $(x_1, x)$ |
| $m_{uni}$ | 1 kg | $\phi_2 = 120^\circ$, angle of $(x_2, x)$ |
| $m_{uni}$ | 0.5 kg | $\phi_3 = 270^\circ$, angle of $(x_3, x)$ |
| $m_o$ | 70 kg | $s^i F_{p1} = [-1000, 0, 700]^T$ kN |
| $m_{uni}$ | 3 kg | $s^i F_{p1} = [100, 0, 0]^T$ N-m |
| $m_{uni}$ | 2 kg | $s^i F_{p2} = [-1000, 0, 700]^T$ kN |
| $\alpha$ | 0.25$^\circ$ | $s^i T_{p2} = [100, 0, 0]^T$ N-m |
| $\beta$ | 2.5$^\circ$ | $s^i T_{p3} = [-1000, 0, 700]^T$ kN |
| $Z_o$ | 2$^\circ$ m | $s^i T_{p3} = [100, 0, 0]^T$ N-m |

The extensions, velocity, acceleration of $l_i$ ($i = 1, 2, 3$) of three grippers and $(\alpha, \beta, Z_o)$ of $m$ are given, see Fig. 3(a) and (b). The analytic solutions of $F_{ci}$ ($i = 1, 2, 3$) and $F_{c\dot{a}}$ and $(F, T)$ applied on $m$ at $o$ of the PM and their components are obtained from relative equations, see Fig. 3(c). The analytic solutions of the dynamic active forces $F_{li}$ ($i = 1, 2, 3$) of three grippers are obtained from relative equations, see Fig. 3(d).
6. Conclusion

The formulae for solving the coordinative dynamics of the 3SPR (PM with three grippers) are derived. When given the workload applied on the grippers at their tip, the coordinative dynamic active force and dynamic constrained force exerted on PM can be solved. The coordinative dynamic active force applied on active rod of grippers can be solved by considering inertia wrench and the friction of all the joints in the mechanism. The analytic solutions of coordinative dynamics for this manipulator are verified by its simulation solutions.

The derived formulae and solutions can be used for analyzing the coordinative dynamics of the other PMs with the multi-grippers in the light of their application. The 3SPR PM with three grippers has potential applications for the rescue missions, industry pipe inspection, manufacturing and fixture of parallel machine tool, CT-guided surgery, health recover and training of human neck or waist and micro–nano operation of bio-medicine.

The stiffness of this PM with three grippers should be studied in the future.

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References


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