Dynamics Analysis of Wave-driven Unmanned Surface Vehicle in Longitudinal Profile

Baoqiang Tian1, Jiancheng Yu1, Aiqun Zhang1, Fumin Zhang2, Zhier Chen1 and Kai Sun1

Abstract—Wave-driven unmanned surface vehicle (WUSV) is a great success of application of solar and wave energy in the ocean robot. In this paper, the nonlinear dynamic model of WUSV in two dimension is established based on the analysis of its driving principle in the longitudinal profile. Then, we calculate the wave and driving force, and determine hydrodynamic coefficients according to the empirical data and experimental platform of WUSV. Finally, we simplify the nonlinear equations and present the simulation results of the model.

I. INTRODUCTION

With the development of economy and the improvement of people’s lives, the contradiction between land resources and social production growth is becoming more and more serious, which makes people have to turn our gaze to the ocean with the rich resource. It is estimated that the amount of ocean resource covered 71% of the earth’s surface is tens or hundreds times the land’s. So all kinds of Marine observation, development tools are emerging constantly, such as underwater and surface vehicles, including the AUV (autonomous underwater vehicle), ROV (remotely operated vehicle), manned submersible (for example JiaoLong), underwater glider and so on. But facing such a large area of the sea, the operating range of these tools is under restrictions on the amount of carrying energy (most batteries) and the endurance is very limited. Despite the energy ROV needs can be provided by mother-ship, but the use of the mother-ship greatly increased expenditure [1], [2].

American Liquid Robotics Inc has developed a wave driven unmanned surface vehicle(WUSV), also called wave-glider, composed of a floating body (2.1m x 0.6m) and underwater glider body (0.4m x 1.9m), through a flexible cable (7m) transferring the electric energy and communication[3],[4]. Its application has achieved a great success in the The PacX game from California to Australia, and set a new world record for the longest distance traveled by an autonomous vehicle this year, which immediately drawn many scholars’ attention around the world. As a new ocean robot dynamic model is generally divided into two categories depending on the different driving principle—one is the model of AUV, ROV and USV derived or simplified from ship model, and these theoretical researches have been very mature[5]; the other one is the model of underwater glider establish by Leonard and Graver from Princeton University[6], which has been verified by a large number of experiments. It is obvious that the robot model above is based on a single rigid body for dynamic analysis. But, WUSV is a two-body structure and there is a strong coupling relationship between float body and underwater glider body. So, The previous model cannot be used for reference here and we must establish the dynamics model of WUSV in order to meet the demands of motion control and make WUSV better application in ocean observation.

II. DRIVING PRINCIPLE

WUSV is an ocean wave-propelled USV with a two-body design, floating body and underwater glider body. There are two solar panels attached to the float portion of the WUSV that convert solar energy into electrical energy to supply electricity for the navigation and communication systems, as well as for the payload sensors installed onboard. The underwater glider body is the wave energy conversion mechanism converting the wave energy into forward force without middle transition conversion link, which can improve the conversion efficiency.

As shown in Fig. 1, the upper float body of the WUSV rises and falls with the wave action on the ocean surface. When the wave crest arrives, wave lifts float body and float body will pull the underwater glider. As a result, the whole robot will move up and the wing plate rotates downward under the action of hydrodynamic force. When the wave trough arrives, the underwater glider body will move downward due to its own gravity and the wing plate rotate upward under the action of hydrodynamic. Thus it can be seen that the horizontal component of the hydrodynamic wing plate is always toward the front whether it is rising or falling, which is the driving force in the process of WUSV motion. The underwater glider body, as the dynamic part, pulls the upper float body to move. The direction of WUSV is controlled by steering gear system, regardless of the direction of movement of waves.

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III. DYNAMIC OF WUSV

The force exerted on the float and underwater glider is mainly concentrated in the along-track and heave directions, so the two-dimensional dynamics model of WUSV is established in the longitudinal profile [7]. Meanwhile in order to simplify the challenge of modeling WUSV, we make the following assumptions about the vehicle itself and with respect to its environment:

- Environment assumption: because the wind and current forces bears randomness, their influences are ignored in the dynamic model;
- Float and underwater glider are strong coupled rigid bodies with constant mass;
- Cable should be always in tension, and its hydrodynamic drag is also ignored;
- Float is motivated by the Airy linear wave in deep water, and the effect of floating body rotation is ignored (Only up and down movement of float can contribute to the generation of driving force). Besides, Underwater glider only has translation movement, which can be achieved by balancing its mass;
- The distances between cable node and gravity center of float, gravity center of underwater glider are also ignored, because they are both very short compared with the length of umbilical;

A. Dynamic model

We assign the float coordinate frame $x_1z_1$ as with its origin $o_1$ fixed at gravity center of float, and the underwater glider coordinate frame as $x_2z_2$ with its origin $o_2$ fixed at gravity center of glider. Let $x_1$ and $x_2$ lie along the direction of forward motion, $z_1$ and $z_2$ points up and downwards respectively, and $y$ is ignored, as shown in Fig.2. we denote the inertial frame as $x_3$ with its origin at $E$. The float coordinate frame is mainly used in the analysis of wave forces acting on the float body, and the glider coordinate frame is for the lift and drag force calculation generated by the wings of underwater glider. We will transform all forces in the body-fixed frame ($x_1z_1$ and $x_2z_2$ ) into inertial coordinate frame, where we will establish the dynamics model of the entire WUSV finally.

The Euler angles: roll ($\phi$), pitch ($\theta$) and yaw ($\psi$) can be used to express the attitude of the vehicle in the inertial coordinate frame. The rotation matrix $R^e_b$ is then written in Euler angles from body frame to inertial frame [8]:

$$ R^e_b = \begin{bmatrix} \cos \theta & 0 & \sin \theta \\ 0 & 1 & 0 \\ -\sin \theta & 0 & \cos \theta \end{bmatrix} $$

The rotation matrix $R^e_f$ (from float frame to inertial frame) and $R^e_g$ (from glider frame to inertial frame) can also be written:

$$ R^e_f = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \text{ and } R^e_g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} $$

This implied force has the same representation form both in the float frame and inertial frame. But force in $x$ direction also has the same representation form in the glider frame and inertial frame, whereas force in $z$ direction has the opposite representation form in these two frame.

Assuming the displacement of center of gravity of WUSV are $x_0$, $z_0$, respectively, and the angle between the cable and the vertical direction is $\theta$. We obtain the dynamic model of WUSV in Longitudinal Profile by Newton-Euler equation:

$$ (M + m + M_{11} + m_{11}) \ddot{x}_o = F_{px} - D_f - D_g $$

$$ (M + m + M_{33} + m_{33}) \ddot{z}_o = F_w + B_f - G_f - F_{pz} + B_g - G_g $$

$$ (ML_1^2 + mL_2^2) \dot{\theta} = L_1 \sin \theta (G_f - F_w - B_f) + D_f L_1 \cos \theta + L_2 \sin \theta (B_g - F_{pz} - G_g) + L_2 \cos \theta (D_g - F_{pz}) $$

and geometric position relationship can be written:

$$ x_1 = x_o - L_1 \sin \theta $$
TABLE I

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Description of Meanings</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_n$</td>
<td>Displacement of WUSV CG(center of gravity)</td>
</tr>
<tr>
<td>$z_1$</td>
<td>Displacement of underwater glider CG in $z$ direction</td>
</tr>
<tr>
<td>$z_2$</td>
<td>Displacement of underwater glider CG in $z$ direction</td>
</tr>
<tr>
<td>$L_1$</td>
<td>Distance from center of gravity $o$ to $o_1$</td>
</tr>
<tr>
<td>$L_2$</td>
<td>Distance from center of gravity $o$ to $o_2$</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass of the float body in air</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of the underwater glider in air</td>
</tr>
<tr>
<td>$M_{11}$</td>
<td>Added mass of float body in $x$ direction</td>
</tr>
<tr>
<td>$m_{11}$</td>
<td>Added mass of underwater glider in $x$ direction</td>
</tr>
<tr>
<td>$M_{33}$</td>
<td>Added mass of float body in $z$ direction</td>
</tr>
<tr>
<td>$m_{33}$</td>
<td>Added mass of underwater glider in $z$ direction</td>
</tr>
<tr>
<td>$F_w$</td>
<td>Wave force on float</td>
</tr>
<tr>
<td>$F_{ps}$</td>
<td>Resultant force of lift and drag in $z$ direction</td>
</tr>
<tr>
<td>$D_f$</td>
<td>Hydrodynamic Drag Force of float body</td>
</tr>
<tr>
<td>$G_f$</td>
<td>Buoyancy of float body</td>
</tr>
<tr>
<td>$G_p$</td>
<td>Buoyancy of underwater glider body</td>
</tr>
<tr>
<td>$G_s$</td>
<td>Gravity of underwater glider body</td>
</tr>
</tbody>
</table>

\[ a_3(x_1,t) = \frac{\partial^2 \Phi}{\partial t^2} = -\frac{H \omega^2}{2} e^{i \omega t} \cos(kx_1 - \omega t) \]  

Where, $z_0$ is the $z$-coordinate of the bottom of the float body. According to the long wavelength assumption and strip theory, the vertical wave excitation load $dF_w$ on a strip of length $dx$ can be written as [12]:

\[ dF_w = (pA_p + M_{33}^{(2D)}) dx \]

Thus, the vertical wave forces $F_w$ can be written:

\[ F_w = \int \left( pA_p + M_{33}^{(2D)} \right) dx \]

Substituting (3), (4) and (5) into (6), then the wave forces $F_w$ can be expressed as:

\[ F_w = -(pBD + M_{33}^{(2D)}) H g e^{-\frac{1 \omega t}{2}} \sin\left(\frac{kL}{2}\right) \cos \omega t \]

B. The calculation of force

1) Wave Force: Float body is generally flat and the design of this shape depends on two main factors; one is that the float must have enough volume and horizontal area so that it can provide enough buoyancy and wave force to pull the underwater glider up; on the other hand, the cross-sectional area of the float in forward direction must be as small as possible to reduce the drag. So, in order to facilitate the calculation of wave force, we will simplify float into a rectangular structure, as shown in Fig. 3.

![Fig. 3. The equivalent diagram of float body](image)

Define the velocity potential of the incident waves $\Phi$ as in [9]-[11]:

\[ \Phi(x_1,t) = \frac{gH}{2 \omega} e^{i \omega t} \sin(kx_1 - \omega t) \]

And this means that the vertical undisturbed fluid acceleration $a_3$ at the geometrical center of the cross-sectional area is:

\[ z_1 = z_o + L_1 \cos \theta \]
\[ z_2 = z_o - L_2 \cos \theta \]

2) Driving Force: As known in the driving principle analysis above, the horizontal component of the hydrodynamic is produced by wing plates with a certain rotation angle. So We choose the wing plate for the force analysis in the process of up-and-down movement of WUSV like zigzag path, as shown in Fig.4.

![Fig. 4. The force diagram of wing plate](image)

With the float body rising and falling with the wave action, the wing plate will move obliquely up and down under the pull force of cable. In order to calculate the driving force, select a wing plate as the research object in the glider frame and the wing plate rotates around $y_2$ axis. The velocity $V$,
shown in Fig. 3, is the velocity of the water relative to the wing plate. The angle $\beta$ represents the direction of velocity $V$ with respect to the $z_2$ axis, while the angle $\gamma$ represents the angular position of the wing plate relative to that axis. The angle of attack $\alpha$ is the direction of flow relative to the wing plate. The angles above are all positive counter clockwise and can be calculated as $\alpha = \beta - \gamma$ in [13].

The direction of lift $L$ and drag forces $D$ generated by a single wing plate is shown in Fig. 4. The drag forces $D$ is in the same direction of velocity $V$, while the lift force $L$ is perpendicular to that velocity. They can be expressed as:

\[ L = 0.5 \rho V^2 S_w C_L(\alpha) \]  \hspace{1cm} (14)
\[ D = 0.5 \rho V^2 S_w C_D(\alpha) \]  \hspace{1cm} (15)

Where $\rho$ is the density of water, $S_w$ is the planform area of the wing plate, $\alpha$ is the angle of attack, $C_L(\alpha)$ and $C_D(\alpha)$ are the lift and drag coefficients respectively, and they are both the function of angle of attack. According to the geometric relationship in Fig. 4, we can draw the driving force in the glider reference frame:

\[ F_{px} = D \sin \beta + L \cos \beta \] \hspace{1cm} (16)
\[ F_{pc} = -L \sin \beta + D \cos \beta \] \hspace{1cm} (17)

Combining equations (14), (15),(16), the driving force $F_{px}$ can be expressed as

\[ F_{px} = 0.5 \rho V^2 S_w (C_L(\alpha) \sin \beta + C_D(\alpha) \cos \beta) \] \hspace{1cm} (18)
\[ F_{pc} = 0.5 \rho V^2 S_w (C_D(\alpha) \cos \beta - C_L(\alpha) \sin \beta) \] \hspace{1cm} (19)

And the velocity $V^2$ of underwater glider can be written as

\[ V^2 = (x_0 + L_2 \cos \theta \cdot \dot{\theta})^2 + (z_0 + L_2 \sin \theta \cdot \dot{\theta})^2 \] \hspace{1cm} (20)

3) **Hydrodynamic Drag Force:** The hydrodynamic drag forces is a function of the vehicle motion and its geometry. If the float and underwater glider have velocity $u_1$, $u_2$ respectively in $x$ direction, the drag force can be expressed as [14]:

\[ D_f = 0.5 \rho C_{Dfx} B D u_1^2 \] \hspace{1cm} (21)
\[ D_g = 0.5 \rho C_{Dgx} A_g u_2^2 \] \hspace{1cm} (22)

$C_{Dfx}$ and $C_{Dgx}$ are the drag coefficients of float and underwater glider along $x_1$, $x_2$ axes can be taken as 1.39 in [15]. And the velocity $u_1$, $u_2$ can be written as

\[ u_1 = \dot{x}_0 - L_1 \cos \theta \cdot \dot{\theta} \] \hspace{1cm} (23)
\[ u_2 = \dot{z}_0 - L_1 \sin \theta \cdot \dot{\theta} \] \hspace{1cm} (24)

IV. Platform Design

The motion of ocean is random and the wave model in general is superposition of cosine waves. So we have great difficulty in its further research because the wave form, as the system input, is not determined. And allowing for the restriction on experimental condition, we designed this WUSV experiment platform in Fig. 5. There are a wave simulation mechanism mounted on the float, including motor, reducer, drum and so on. Motor drives drum through the gear reducer so that we can make use of the positive inversion of the motor to realize the simulation of the up-and-down motion of wave. We use PLC (for example SIEMENS S7-200) to control the rotation speed and running time of the motor to simulate wave motion period and amplitude, and eventually the float will realize forward movement under the driving force of underwater glider. The float no longer moves up and down and we can adjust the wing rotation angle through the bolt of underwater glider. Through this platform, we can simulate movement conditions of WUSV in a variety of wave parameters.

And the basic geometric parameters of WUSV experimental platform are listed in TABLE II. We can see that the size of float part is a little big, because it has to provide sufficient buoyancy for the underwater glider body and motor. We can obtain the added mass of underwater glider through this platform in the following way:

1) First, we can measure the displacement of floats body and underwater glider in vertical direction $z_1$, $z_2$:

\[ F_b = \rho g S_b z_1 \] \hspace{1cm} (25)

2) We can calculate the buoyancy changes of float body based on the its displacement in vertical direction $z_1$.

3) The displacement of underwater glider $z_2$ can be obtained by calculating the diameter $D$ and angular velocity $\omega_2$ of the drum driven by Motor,

\[ z_2 = \frac{D}{2} \int \omega_2 dt \] \hspace{1cm} (26)
The angular velocity \( \omega \) of the drum is set by the PLC program;

4) We can get the vertical added mass of underwater glider by the following formula,

\[
(m + m_{33})\ddot{z}_2 = F_b + B_g - G_g - F_{pc}
\]  

(27)

Where, \( F_{pc} \) is very small compared with \( F_b \) and can be ignored. The gravity and buoyancy of underwater glider \( B_g, G_g \) are constant.

V. Simulation

Let WUSV experimental platform travels in the wave for \( H = 0.3m \) and \( T = 3.2s \) of the typical secondary sea state, we can get approximate \( \beta = 35^0 \) and \( \alpha = 10^0 \), as shown in Fig. 4. Then, the lift and drag coefficients can be determined as \( C_L = 0.6, C_D = 0.07 \) respectively [16]. In the calculation of wave force, we simplify float into a rectangular structure in order to facilitate the calculation. So the equivalent size of the rectangular structure float is : \( L = 1.4m, B = 0.5m \) and \( D = 0.105m \). According to the added mass results based on Lewis [12], added mass of float body in heave is \( M_{33} = M_{33}^{(2D)} L = 2.1\rho BDL \). The added mass of the float body is 0.05 times of its mass, and the added mass of underwater glider is 0.10 times of mass in the X axis direction [8]. We can obtained the added mass of glider \( m_{33} = 16kg \) from the motion relationship between this two bodies of experiment platform.

All forces mentioned above are substituted into equations(1)-(3), which can be put in the form \( \dot{x} = f(x, u) \), where \( x = [x_0, z_0, \theta]^T \) represents the state of the system and \( u \) is a vector of input characterizing the wave force. For simplifying the challenge of simulation, force \( F_{pc} \) is assumed to be small relative to the wave force and therefore neglected. The gravity and buoyancy of system are considered to be equal, because the float body moves up and down with the wave action, the restoring force should be small. The outputs of the simulation are the position of the robot in the inertial frame \( xz \). In order to obtain \( x, \dot{x} \) is calculated and integrated. The numerical integration is performed using Simulinks, a software package for modeling, simulating, and analyzing dynamical systems.

In Fig. 6. The velocity simulation shows the average velocity of WUSV experimental platform in the x direction is about 0.13m/s, and the angular velocity of cable relative to vertical varies between -5 degree/s and 5 degree/s. The result of displacement simulation in Fig. 7. shows that the average angle \( \theta \) of cable is about 13 degrees and the heave displacement of the platform varies between -0.125m and 0.125m.

Compared with the declared data from wave glider of Liquid Inc [3], the velocity of WUSV experimental platform is a little smaller and the angle \( \theta \) of cable is relatively large. This results are mainly because that, in order to simulate the wave motion, the wave simulation mechanism of platform makes the floating body volume and mass become very large, which greatly increases the hydrodynamic drag of float. The angle \( \theta \) of cable is relevant to length of cable and through simulation, we found that its angle and angular velocity can be much decreased significantly when the length of the cable is set to be 7 meters. However, it is not clear that how the length of cable affects the movement performance of WUSV, So we need to do further research on it. There is no doubt that the design of the experimental platform of WUSV provides convenience for us to study the dynamics of wave glider.

### TABLE II

**Basic Geometric Parameters of WUSV Experimental Platform**

<table>
<thead>
<tr>
<th>Part Name</th>
<th>Basic Size (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Float Body</td>
<td>1.8 \times 0.5 \times 0.3</td>
</tr>
<tr>
<td>Underwater Glider Body</td>
<td>1.08 \times 0.62 \times 0.35</td>
</tr>
<tr>
<td>Cable</td>
<td>\phi 0.068 \times 3.5</td>
</tr>
<tr>
<td>Wing Plate</td>
<td>0.27 \times 0.12 \times 0.003</td>
</tr>
</tbody>
</table>

Fig. 6. The Simulation Velocity of WUSV

Fig. 7. The Simulation displacement of WUSV
VI. CONCLUSIONS AND FUTURE WORKS

In this work, the dynamic model of WUSV in two-dimension was established in the longitudinal profile. And then, wave and driving force was calculated and hydrodynamic coefficients are determined according to the empirical data and experimental platform. Finally, a simulation was developed in the wave for $H = 0.3m$ and $T = 3.2s$ of the typical secondary sea state.

A number of improvements could be made to the dynamic analysis of WUSV. Firstly, in order to consider steering performance of WUSV in the horizontal plane (for example, the control of the movement direction of WUSV by steering gear), a three-dimensional dynamic model has to be established. Secondly, We should further study the motion efficiency of WUSV under different ocean conditions, such as the influence of wave wave height and period, so that we can improve energy efficiency through optimization and make it more efficiently complete ocean measurement tasks. And finally, WUSV is an ocean wave powered USV and its driving force is also hydrodynamic force, so it is very important to identify the hydrodynamic coefficient in the future research.

REFERENCES