Multipath time-delay estimate based on homomorphic filtering in logarithm domain

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ABSTRACT
It is important to estimate the multipath time delay in underwater communication and target positioning. Based on the good time delay resolution, wideband signal is used to estimate time delay conventionally, in some situation, it is hard to use wideband signals. Focused on the narrowband signal multipath time delay estimating, an algorithm based on spectral subtraction in logarithm domain was proposed. Compared to the complex cepstrum method, the proposed method effectively improved the anti-noise ability and adapted better to the doppler frequency shift of the signal.

KEY WORDS: time delay, estimate, logarithm domain, homomorphic filtering.

INTRODUCTION
In the underwater acoustic processing field, caused by the non-uniform of the channel, the received is sophisticated with multipath except the directive signal, it is especially obvious in the shallow-water, the multipath time delay is important in target positioning, multiple target estimating and underwater communication, the time-varying and space-varying characters of the underwater channel made it important to accurate get the multipath information in time. The conventional multipath evaluating methods mainly based on the good time delay resolution of wideband signals, by the correlation of the received signal to get the multipath output [1,2], but in some situation, restricted by the band wide or the hardware of the system, only the narrowband signal is available, the accuracy of the matched filtering can not satisfying the criteria. Focusing on the high-resolution time delay estimation of narrowband signals, researchers from many countries conducted extensive and in-depth study, maximum likelihood estimation method, alternating projection method, nonlinear least squares method, high-order statistics method have been proposed [3-6], though these methods have a better ability to estimate the delay than matched filtering, but also limited by large amount of calculation or sensitive to noise.

The cepstrum under the idea of homomorphic filtering is a nonlinear approach, it is calculated as convolution of the signal ($s(n)$) and the channel impulse response ($h(n)$), showed as:

$$x(n) = s(n) * h(n)$$

Where $s(n)$ is the input signal, $h(n)$ is the impulse responses of the channel, take the equation (2) into (1), and then get:

$$\hat{C}_c(n) = Z^{-1}(Ln\{Z[s(n) * h(n)]\})$$

$$= Z^{-1}\{Ln[S(Z) \cdot H(Z)]\}$$

$$= Z^{-1}\{Ln[S(Z)] + Ln[H(Z)]\}$$

It is obvious from the equation (3), the convolution relationship between the input signal and the channel has changed into addition after the complex cepstrum calculation. To filter out unwanted components by linear filtering in the cepstrum domain, and then transform the rest signal back to the time domain, this method is what we usually called homomorphic filtering.

If the $Z$ transform of the input signal includes the unit circle, its Fourier transforms can be used to instead of the $Z$ transform, the signals we used can fulfilled this condition.

1.2 Cepstrum’s character and the principles of multi-path delay estimation
If the signal is stable, its complex cepstrum can be showed as follow, $|a_i|, |b_j|, |c_k|, |d_l|$ are all less than 1, $Z = a_i$ and $Z = c_k$ are the zeros and poles inside the unit circle, $Z = b_j^{-1}$ and $Z = d_l^{-1}$ are the zeros and poles outside the unit circle, $A$ is the amplitude of the signal.
\[ \hat{C}_n(n) = \begin{cases} \frac{-M_a n^n}{n} + \frac{N_c n^n}{n}, & n < 0 \\ \frac{L_n A}{n}, & n = 0 \\ \frac{\sum_{i=1}^{M} h_{ij} n^i - \sum_{i=1}^{N} d_{ij} n^i}{n}, & n > 0 \end{cases} \] (4)

From the equation (4), the next characters can be got:

1. \( \hat{C}_n(n) \) is an infinity sequence, and its absolute value is decreased by the increase of \( n \);

2. For a series of impulse whose space is \( N_p \), it is still an impulse sequence in the cepstrum domain, and its space is still \( N_p \);

By deducing, the channel impulse \( h(n) \) can be expressed as equation (5):

\[ \hat{C}_n(n) = \sum_{i=1}^{N} a_i \delta(n-n_i) - \frac{1}{2} \left[ \sum_{i=1}^{N} a_i^2 \delta(n-2n_i) + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} a_i a_j \delta(n-n_i-n_j) \right] \\
+ \frac{1}{3} \left[ \sum_{i=1}^{N} a_i^3 \delta(n-3n_i) + 2 \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{k=1}^{N} a_i a_j a_k \delta(n-n_i-n_j-n_k) \right] + \ldots \] (5)

As can be seen from the above deduced, the impulse response of the channel in the cepstrum domain still showed a series of impulse response, this is concordance with the character (2), the position of the impulse is located in the time delay, its “frequency multiplication” and the “cross term” of different time delay.

From the above properties, the following conclusions can be got: the amplitude of the signal decreased rapidly with \( n \), the energy of the signal is mainly concentrated in the “low-frequency” part of the cepstrum domain, and so calculate cepstrum of limited point can retain the basic information of a signal, the impulse response in the cepstrum domain still showed as a series of impulse, its location only is associated with time delay, so the multipath can be abstracted or be erased in the cepstrum domain.

2 TIME DELAY ESTIMATION ALGORITHM BASED ON THE SPECTRAL SUBTRACTION IN THE LOGARITHM DOMAIN

2.1 The proposed algorithm

The above analysis showed that the multipath component can be abstracted or be erased in the cepstrum domain, but it ignores a very important parameter: noise, if the received signal is:

\[ x(n) = A \cdot s(n) \cdot h(n) + z(n) \]

\( z(n) \) is noise, its Fourier transform is:

\[ X(\omega) = AS(\omega) \cdot H(\omega) + N(\omega) \]

\[ = A|S(\omega)| \cdot |H(\omega)| e^{j(\theta_s + \theta_h)} + N(\omega) \] (6)

Logarithmic transform of \( X(\omega) \) is:

\[ \log[X(\omega)] = \left\{ \log|S(\omega)| + \log|H(\omega)| + \ldots \right\} + \log A + \log[N(\omega)] + j(\theta_s + \theta_h) \] (7)

In equation (7), \( N'(\omega) = 1 + \frac{N(\omega)}{AS(\omega) \cdot H(\omega)} \); it can be ignored when the SNR is big, but it can not be ignored if the SNR is not big enough. Assuming the transmitted signal is CW signal with rectangular envelope, its duration is \( T = 200ms \), the center frequency is \( f_0 = 2kHz \), the sampling rate is \( f_s = 10kHz \), there are two multipath in the received signal, the time the cepstrum result in different SNR are showed in figure(1), X Axis represent time, its unit is millisecond(ms), Y Axis represent normalized amplitude, the other figure has the same unit with figure (1).

![Fig 1 The cepstrum results versus SNR](image)

When the SNR is high, cepstrum can obtained the time delay (as shown by the figure a), it has been difficult to accurately distinguish the delay peaks when the SNR drops to 20dB (as shown by the figure b). In the underwater acoustic channel, the SNR is not big enough sometimes, then the cepstrum is disabled.

If the priori information of the transmitting signal can be used, delete the signal in the cepstrum domain, recover the remaining components back to time domain, so the multipath time delay can be acquired. But if the frequency of the received signal and copy signal is not the same, it will lead to the signal component can not be eliminated, thus affected the result of the multipath time delay estimation.

2.2 Algorithm description

According to previous analysis, a multipath time delay estimation method called spectral subtraction and homomorphic filtering in logarithm domain, the specific implementation methods are:

1) Calculate the short-time Fourier transform \( X(f) \) of the received signal \( x(n) \), get its logarithmic transformation \( L_n(\omega) \), and calculate the logarithmic transformation of the copy signal \( L_n'(\omega) \).

2) \( L_n(\omega) \) deduced by \( L_n'(\omega) : L_n(\omega) = L_n(\omega) - L_n'(\omega) \), then \( L_n(\omega) \) deduced by its mean \( M_n \), get the result \( L_n(\omega) \).

3) Search the sequence \( L_n(\omega) \) one by one, if its absolute value is
greater than the threshold $d$, replace it by 0, then smooth the sequence after searching: calculate the mean $M_i$ by each $N$ points, then the $N$ points all deduced by $M_i$ until the entire sequence be smoothed, then the sequence noted as $L_n'(\omega)$.

4) Turn the sequence $L_n'(\omega)$ back to time domain, filter it by a band-pass filter.

The main purpose of the step (3) is to erase the signal’s energy if the received signal is different from the local copy signal, and it can eliminate the energy of noise due to its unevenness also. By filtering the result in step (4), the noise be eliminated once more.

3 SIMULATIONS ANALYSIS

3.1 Simulation Analysis

Experiment Description: the carrier frequency transmitting signal is $f_0 = 2kHz$, its envelope is rectangular, the sound speed is $c = 1500m/s$ (ignore its difference in different depth), $f_r = 20kHz$, the reflection coefficient of the surface and the floor is -0.9 and 0.6, pass band of the band-pass filter is [1500 2500]Hz, the threshold is $d = 4$, the interval is $N = 5$.

Analysis the performance of the proposed algorithm first, $T=30ms$, SNR=20dB, the receiver located in (0, 0, 30) m, the coordinates of the target (1200, 500,100) m, depth of water is 200m. Consider there are two reflections by the sea surface and the seafloor, the time delay are 3.1ms and 17.3ms, the normalized amplitude are 0.84 and 0.45, FFT number is 1024, and the estimated results are shown below:

As can be seen from the figure (2-a), due to the difference of signal and the influence of noise, due to the large unwanted energy that left after deduction, the estimation result is not good; figure(2-b) showed that by homomorphic filtering and band-pass filtering after deduction, the energy of signal and noise are eliminated effectively, there are two peaks, the estimated time delay is [3.15, 17.2]ms, the maximum evaluated error is 0.1ms, it is very accurately; the estimated result of normalized magnitude is [0.80, 0.49], the difference is small too.

Assuming that there are five multipaths, their time delay is [4.8, 11.15, 15.55, 23.3, 31.15]ms, corresponding normalized amplitude is [-0.8, 0.65, -0.54, -0.45, -0.35], SNR=20dB, the multipath estimation result is showed as below:

As can be seen from the figure (3), though there are many multipaths sophisticated in the receiving signal, the proposed algorithm can still be able to ensure a high accuracy.

Analyzing the effectivity of the proposed algorithm when received doppler signal. The simulation condition is the same as before, away form the receiver is defined as the positive direction, the frequency of the target is 15m/s, the frequency shift is 40Hz according to the equation above, multipath time delay estimation results versus different SNR are showed in figure (4):

Figure 6-a shows that, if the SNR is high enough, the proposed algorithm can still evaluate the effectively even when the target is in a big radial velocity, the estimation ability of the algorithm degrading as the SNR is falling, showed in figure 6-b.

Then simulate the anti-noise ability of the algorithm, there are three multipaths (denoted as d1, d2, d3), their time delay distributed
randomly, normalized amplitude distributed randomly between 0.2 and 0.7, T = 30ms, simulate 200 times, the mean multipath time delay estimation error (denoted as $M_{et}$) and the mean multipath amplitude estimation error (denoted as $M_{ea}$) versus SNR is showed in Table 1.

<table>
<thead>
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<th>SNR (dB)</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
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<td>0.21</td>
<td>0.09</td>
<td>0.08</td>
<td>0.10</td>
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<tr>
<td>12</td>
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<td>0.31</td>
<td>0.26</td>
<td>0.10</td>
<td>0.12</td>
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<td>0.38</td>
<td>0.14</td>
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<td>0.18</td>
</tr>
</tbody>
</table>

As can be seen from Table 1, the time delay and amplitude estimation error degrading as the SNR is falling, the maximum average time delay and amplitude estimation errors were (0.29, 0.31, 0.33, 0.41) ms and (0.10, 0.12, 0.13, 0.18), both of them keep in a high-level.

4 CONCLUSIONS

Since the bandwidth of line spectrum signal is narrow, its time resolution is weak, so the multipath time delay estimation is limited, especially in underwater acoustic channel. Cepstrum is a nonlinear analysis method, it can change the convolution into addition, and the impulse still shows as impulse sequence in the cepstrum domain, but this method is sensitive to noise which has limited its applications. In this paper, according to the character of active sonar, a multipath time delay estimation algorithm based on spectral subtraction and homomorphic filtering in logarithm domain was proposed, eraser the energy of signal and noise in the logarithmic domain by deduced the copy signal and homomorphic filtering, abstracted the multipath time delay by transform the rest back to time domain. Simulation shows that the proposed method has a good accuracy and adapted to the doppler frequency shift, it improved the anti-noise ability compared to cepstrum.

REFERENCES:


