Output-feedback controller design using hysteresis method for switched LPV systems with inexact parameters

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Abstract: This paper designs switched controllers for a class of linear parameter varying (LPV) systems with inexactly measured parameters. $H_\infty$ performance problem is solved, and a switched LPV controller is designed. By using the multiple Lyapunov functions method, Linear Matrix Inequality (LMI) conditions are proposed, and a switching law is designed based on the hysteresis method. An active magnetic bearing (AMB) example shows effectiveness of proposed method.

Key Words: LPV systems, switched controller, inexact parameters, Multi-Lyapunov functions, $H_\infty$ controller design.

1 Introduction

Over the last ten years, the switched LPV control has been researched in LPV systems [1–5]. LPV theory is practical to apply linear theory to a nonlinear system or a practical system such as missile [6–8], aircraft [2, 9], energy production systems [10–12], inverted pendulum [13].

Application of switching design on LPV systems makes it more practical [2, 8, 14]. In [8], a missile is modeled and controlled by a switched LPV controller. Paper [1] designs a switched LPV controller to improve the $H_\infty$ performance and apply the method to an AMB system. An F-16 aircraft model is controlled by a switching controller with state reset [2], and fault tolerance is considered [9]. Fast-varying and slowly-varying parameters are considered separately and a blending method is also used to an F-16 model [14].

In LPV system control field, the inexact parameter measuring has attracted research. It is usually assumed that parameters in systems are exactly provided, but this is generally not the case in real systems due to uncertainties caused by measurement errors, noise, or estimation inaccuracies [15]. Uncertainties on scheduling parameters are considered in [16, 17] by introducing an additional uncertainty block in $H_\infty$ control problem. An LMI condition is proposed to deal with the deviation of the parameters in [18], and the result is extended to discrete LPV system [19]. A new LMI solving method is proposed based on the Bounded Real Lemma (BRL) for $H_\infty$ performance problem, and solved Lyapunov function is parameter-dependent [15].

Inexactly measured parameters problem exists in both LPV systems and switched LPV systems, but there is no related result in switched LPV systems. Until now, all design methods in switched LPV control field assume that parameters can be exactly measured. Since the advantage of switching LPV controller is proven [1], it is necessary to research switching control design for systems in which parameters are inexactly measured. This is the motivation of this paper.

In this paper, the method in [15] is extended to switched LPV system. LMI conditions are proposed for $H_\infty$ performance and switched LPV controller is designed for an LPV system in which the parameter are not exactly provided.

Multiple Lyapunov functions method is used and a switching law is designed by hysteresis method. After the result is achieved, it is also applied to an AMB system.

This paper is organized as follows: Section 2 gives preliminaries for the main results; Section 3 gives the main results; In Section 4, a simulation of an AMB system is shown; The conclusion is given in Section 5.

The notations are as follows: $\langle X \rangle$ is a shorthand of $X + X^T$, $I_n$, $I$, and 0 denote an $n \times n$-dimensional identity matrix, identity and zero matrices of appropriate dimensions. $\mathbb{R}^n$, $\mathbb{R}^{n \times m}$ and $\mathbb{S}^n$ respectively denote sets of $n$-dimensional real vectors, $n \times m$-dimensional real matrices and $n \times n$-dimensional symmetric real matrices, $*$ denotes an abbreviated off-diagonal block in a symmetric matrix, and diag$(X_1, \cdots, X_k)$ denotes a block-diagonal matrix composed of $X_1, \cdots, X_k$.

2 Preliminaries

We consider the following LPV system $G(\rho)$

$$
\begin{align*}
\dot{x} &= A(\rho)x + B_1(\rho)\omega + B_2(\rho)u \\
z &= C_1(\rho)x + D_{11}(\rho)\omega + D_{12}u \\
y &= C_2(\rho)x + D_{21}\omega
\end{align*}
$$

where $x \in \mathbb{R}^n$ is the state, $z \in \mathbb{R}^n$ is the controlled output, and $w \in \mathbb{R}^{n_w}$ is the disturbance input, $y \in \mathbb{R}^m$ is the measurement for control, and $u \in \mathbb{R}^{n_u}$ is the control input. $\rho = [\rho_1, \cdots, \rho_k]$ is the parameter vector, which is in a hyper-rectangle set $\rho \in \Omega_\rho$. The vertexes of set $\Omega_\rho$ is denoted by ver$(\Omega_\rho)$. The system (1) is affinely dependent on the parameter $\rho$. The system is also proposed to satisfy the following assumption.

For the system (1), the benefit of the switched controller is illustrated in [1]. To extend the effort into the inexact parameter systems, the switched LPV controller $K_\sigma(\hat{\rho})$ to be designed are as follows:

$$
\begin{align*}
\dot{x}_k &= A_{k,\sigma}(\hat{\rho})x_k + B_{k,\sigma}(\hat{\rho})u \\
u &= C_{k,\sigma}(\hat{\rho})x_k + D_{k,\sigma}(\hat{\rho})y
\end{align*}
$$

where $x_k \in \mathbb{R}^n$ denotes the state with $x_k = 0$ at $t = 0$. [This work is supported by China Postdoctoral Science Foundation under Grant No. 2014M551143 and No. 2013MS41258.]

Parameters are omitted for space in matrices $A_{cl,i}(\hat{\rho}), B_{cl,i}(\hat{\rho}), C_{cl,i}(\hat{\rho}), D_{cl,i}(\hat{\rho})$ are each subsystem’s controller matrices to be designed. For the close-loop system, it can be expressed as
\[
\begin{align*}
\dot{x}_{cl} &= A_{cl}(\hat{\rho})x_{cl} + B_{cl}(\hat{\rho})\omega, \\
x_{cl} &= C_{cl}(\hat{\rho})x_{cl} + D_{cl}(\hat{\rho})\omega,
\end{align*}
\]
where $x_{cl}$ denotes the state of the switched LPV system, and
\[
\begin{align*}
A_{cl,i} &= \begin{bmatrix} A(\rho) + B_{cl,i}(\rho)C_{cl,i}(\rho)C_{cl,i}(\rho) & A_{cl,i}(\hat{\rho}) \end{bmatrix}, \\
B_{cl,i} &= \begin{bmatrix} B_{cl,i}(\rho) + B_{cl,i}(\rho)D_{cl,i}(\rho)D_{cl,i}(\rho) \end{bmatrix}, \\
C_{cl,i} &= \begin{bmatrix} C_{cl,i}(\rho) + D_{cl,i}(\rho) \end{bmatrix}, \\
D_{cl,i} &= D_{cl,i}(\rho).
\end{align*}
\]

Proof: For each subsystem, the LMIs (6) and (7) can guarantee the $H_{\infty}$ performance of $G_{cl}(\rho, \hat{\rho})$ for the switching path, the LMI (8) guarantees descent of switching between different Lyapunov functions [1].

Lemma 2 ([15]): Suppose that, a symmetric matrix $\Sigma$ and matrices $Y_0$, $Y_1$, $Y_2$ with compatible dimensions are given. If one of either LMI (9) and (10) holds for some positive number $\varepsilon$,
\[
\begin{align*}
\left[ \begin{array}{cc} \varepsilon Y_0 & * \\
* & Y_0 & \end{array} \right] < 0, \\
\left[ \begin{array}{cc} \varepsilon Y_1 & * \\
* & \varepsilon Y_2 \end{array} \right] < 0,
\end{align*}
\]
then $Y_0 + \left[ \begin{array}{c} 0 \\
Y_2 Y_1 \end{array} \right] < 0$ holds.

3 Main Results

In this section, LMI conditions are proposed for Problem 1, and the controller design is given. To construct a suitable switched LPV controller, different strategies are used for different parameter subsets.

3.1 $H_{\infty}$ Performance Analysis

To solve Problem 1, the following theorem is proposed. Theorem 1: Consider the switched LPV system (1). Suppose that there exist positive matrices $X_i(\rho)$ and $Z_i(\rho)$, scalars $\gamma_{\infty,i}$ such that the following LMIs
\[
\begin{align*}
X_i(\rho)I_n < 0, \\
Y_0 + \left[ \begin{array}{cc} \varepsilon Y_1 & * \\
* & \varepsilon Y_2 \end{array} \right] < 0.
\end{align*}
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\end{align*}
\]
then $Y_0 + \left[ \begin{array}{c} 0 \\
Y_2 Y_1 \end{array} \right] < 0$ holds.

The following are two lemmas which is used in Section 3.

Lemma 1: For given positive numbers $\gamma_{\infty,i}$, if there exist parameter dependent matrices $X_{cl,i}(\rho, \hat{\rho}) \in S^{2n}$ such that the LMIs (6) and (7) hold for all combinations of $(\rho, \hat{\rho}, \delta, \hat{\delta}) \in \Omega_{\rho,i} \times \Omega_{\hat{\rho},i} \times \Omega_{\delta} \times \Omega_{\hat{\delta}}$ for the $i$th subsystem $(i = 1, \ldots, N)$, and the LMI (8) holds on the switching surface $S_{\rho,j}$, then the closed loop switched LPV system $G_{cl}(\rho, \hat{\rho})$ is asymptotically stable and satisfies (5) with
\[
X_i(\rho, \hat{\rho}) > 0, \\
\left[ \begin{array}{cc} A_{cl,i}X_{cl,i}(\rho, \hat{\rho}) & * \\
* & C_{cl,i}X_{cl,i}(\rho, \hat{\rho}) \end{array} \right] - \gamma_{\infty,i}I_n I_n < 0.
\]

Proof: For each subsystem, the LMIs (6) and (7) can guarantee the $H_{\infty}$ performance of $G_{cl}(\rho, \hat{\rho})$ for the switching path, the LMI (8) guarantees descent of switching between different Lyapunov functions [1].

Lemma 2 ([15]): Suppose that, a symmetric matrix $Y_0$ and matrices $Y_1$, $Y_2$ with compatible dimensions are given. If one of either LMI (9) and (10) holds for some positive number $\varepsilon$,
\[ Y_{\infty,i}(\rho, \hat{\rho}) = \begin{bmatrix} \{ A_i(\rho)X_i(\rho) + B_{2,i}\varphi_{k,i}(\rho) & A_i(\rho) + B_{2,i}\varphi_{h,i}(\rho)C_{2,i} \\ \hat{A}_{k,i}(\rho) & \hat{A}_{k,i}(\rho)C_{2,i} \} \end{bmatrix} * \begin{bmatrix} B_{1,i}(\rho) + B_{2,i}\varphi_{h,i}(\rho)C_{2,i} \\ Z_{i}(\rho)A_{i}(\rho) + \varphi_{k,i}(\rho)C_{2,i} \end{bmatrix} - \gamma_{\infty,i}I_n, \]

(15)

and the switched LPV controller is in the form of

\[ \begin{aligned}
& A_{k,i}(\hat{\rho}) = Z_{i}^{-1}(\hat{\rho})\{Z_{i}(\hat{\rho})A_{i}(\hat{\rho}) + Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) \\
& - \varphi_{k,i}(\hat{\rho}) - \{Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) - \varphi_{k,i}(\hat{\rho}) \}, \\
& B_{k,i}(\hat{\rho}) = Z_{i}^{-1}(\hat{\rho})\{Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) - C_{k,i}(\hat{\rho}) \}, \\
& C_{k,i}(\hat{\rho}) = \{\varphi_{k,i}(\hat{\rho}) - \varphi_{h,i}(\hat{\rho})C_{2,i}X_{i}(\hat{\rho}) \}Y_{i}^{-1}(\hat{\rho}), \\
& D_{k,i}(\hat{\rho}) = \varphi_{k,i}(\hat{\rho}),
\end{aligned} \]

(17)

with \(Y_{i}(\hat{\rho})\) being defined as \(Y_{i}(\hat{\rho}) = X_{i}(\hat{\rho}) - Z_{i}^{-1}(\hat{\rho})\).

**Proof:** The following inequality is achieved by applying Lemma 2 with inequality (9):

\[ Y_{\infty,i}(\rho, \hat{\rho}) + Y_{3,i}(\rho, \hat{\rho}) + \text{diag}\left( \begin{bmatrix} 0 & 0 \\ Z_{i}(\hat{\rho})A_{i}(\rho) - A_{i}(\rho)X_{i}(\hat{\rho}) \end{bmatrix} * \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right) < 0. \]

Substitute the following matrices into the above inequality,

\[ \begin{aligned}
& \varphi_{k,i}(\hat{\rho}) = -Z_{i}(\hat{\rho})A_{i}(\rho)Y_{i}(\hat{\rho}) + Z_{i}(\hat{\rho})A_{i}(\rho)X_{i}(\hat{\rho}) \\
& - \hat{Z}(\hat{\rho})Z_{i}^{-1}(\hat{\rho}) - Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho})C_{2,i}X_{i}(\hat{\rho}) \\
& + Z_{i}(\hat{\rho})B_{2,i}(C_{k,i}(\hat{\rho})Z_{i}(\hat{\rho}) + D_{k,i}(\hat{\rho})C_{2,i}X_{i}(\hat{\rho})) \}, \\
& \varphi_{k,i}(\hat{\rho}) = Z_{i}(\hat{\rho})B_{2,i}D_{k,i}(\hat{\rho}) - Z_{i}(\hat{\rho})B_{k,i}(\hat{\rho}) \}, \\
& \varphi_{k,i}(\hat{\rho}) = C_{k,i}(\hat{\rho})Y_{i}(\hat{\rho}) + D_{k,i}(\hat{\rho})C_{2,i}X_{i}(\hat{\rho}), \\
& D_{k,i}(\hat{\rho}) = \varphi_{k,i}(\hat{\rho}),
\end{aligned} \]

then we can obtain the following inequality:

\[ \text{diag}(\mathcal{Z}_{i}^{T}, I_{n_x}, I_{n_x})(L.H.S. of (7))\text{diag}(\mathcal{Z}_{i}, I_{n_x}, I_{n_x}) < 0, \]

(19)

where \( \mathcal{Z}_{i} = \begin{bmatrix} I_{n} & Z_{i}(\hat{\rho}) \\ 0 & -Z_{i}(\hat{\rho}) \end{bmatrix} \), and L.H.S is abbreviation of left hand side. We can conclude that the inequality (7) is satisfied for each subsystem.

The form of \(X_{i}(\hat{\rho})\) can be transformed as:

\[ \begin{bmatrix} X_{i}(\hat{\rho}) \\ Y_{i}(\hat{\rho}) \end{bmatrix} = \begin{bmatrix} I_{n} & 0 \\ I_{n} & I_{n} \end{bmatrix}^{T} \begin{bmatrix} X_{i}(\hat{\rho}) \\ 0 \end{bmatrix} \begin{bmatrix} I_{n} & 0 \\ I_{n} & I_{n} \end{bmatrix}, \]

then inequalities (13) and (14) guarantee the inequality (8) holds on the switching surface \(S_{ij}\). Descent property of \(V_{n}(\rho)\) can be guaranteed when switchings happen. Therefore, all conditions in Lemma 1 hold for the closed-loop system (3), and Problem 1 is solved. ☐

**Remark 1:** Because the parameter can not be measured exactly, the controller does not contain the value \(\rho\), but inexact values \(\hat{\rho}\). The LMIs (11)-(12) are constrained at the composition of \((\rho, \hat{\rho}, \delta, \hat{\delta}) \in \Omega_{\hat{\rho},\delta} \times \Omega_{\hat{\rho},\delta} \times \Omega_{\delta} \times \Omega_{\hat{\delta}}\), and the LMI (13) at the composition of \(S_{ij}(\rho) \times \Omega_{\delta}\). According to the relation of \(\rho, \hat{\rho}\) and \(\delta\), the character of the switched LPV system (1) with \(\rho\) can be guaranteed by the switched LPV controller (2) with \(\hat{\rho}\).

**Remark 2:** According to the term \(Y_{i}(\hat{\rho})\), the form of \(X_{i}(\hat{\rho})\) needs the inverse of \(Z_{i}(\hat{\rho})\) to guarantee the LMI (8). To solve LMIs in Theorem 1, it is set that \(Z_{i}(\hat{\rho}) = Z_{i}(\rho)\), and LMI (14) is as the same as LMI (13). The price is that the conservatism is increased [1].

### 3.2 Switched LPV Controller Construction

To construct a suitable switched LPV controller for the system (1), different characters are considered in different parameter subsets. In [1, 2], the controller design are the same in region \(\rho(t) \in \Omega_{\rho,\delta} - \Omega_{\hat{\rho},\delta}\) and \(\hat{\rho}(t) \in \Omega_{\hat{\rho},\delta} \cap \Omega_{\rho,\delta}\), because the parameter is in a compact set. The parameter is in a hyper-rectangle in this paper, so polytopic method is used to decrease the number of LMI constraints. In a hyper-rectangle set, the number of vertexes decides the number of LMI constraints [20], but it is not the case in a compact set [1].

In the following, two different strategies are applied for the different parameter regions. First, set Matrix \(X_{i}(\hat{\rho})\) as constant \(X_{i}(\hat{\rho})\) in each subset \(\Omega_{\rho,\delta}\). Second, solve parameter-dependent parameter matrix \(X_{i}(\hat{\rho})\) in subset \(\Omega_{\hat{\rho},\delta} \cap \Omega_{\rho,\delta}\) to construct the hysteresis regions and switching surfaces.

When the solved matrix \(X_{i}\) and \(Z_{i}\) are constant, the controller is the form of

\[ \begin{aligned}
& A_{k,i}(\hat{\rho}) = Z_{i}^{-1}(\hat{\rho})\{Z_{i}(\hat{\rho})A_{i}(\hat{\rho})X_{i} + Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) \\
& - \varphi_{k,i}(\hat{\rho}) - \{Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) - \varphi_{k,i}(\hat{\rho}) \}, \\
& B_{k,i}(\hat{\rho}) = Z_{i}^{-1}(\hat{\rho})\{Z_{i}(\hat{\rho})B_{2,i}\varphi_{k,i}(\hat{\rho}) - C_{k,i}(\hat{\rho}) \}, \\
& C_{k,i}(\hat{\rho}) = \{\varphi_{k,i}(\hat{\rho}) - \varphi_{h,i}(\hat{\rho})C_{2,i}X_{i} \}Y_{i}^{-1}(\hat{\rho}), \\
& D_{k,i}(\hat{\rho}) = \varphi_{k,i}(\hat{\rho}).
\end{aligned} \]

(20)

For the matrix \(X_{i}(\hat{\rho})\) in parameter subset \(\Omega_{\hat{\rho},\delta} \cap \Omega_{\rho,\delta}\) which is divided by switching surfaces \(S_{ij}(\rho)\) and \(S_{ij}(\hat{\rho})\), set \(X_{i}(\hat{\rho}) = X_{i}\) at \(S_{ij}(\rho)\), and apply the gridding method [21] in the region \(\Omega_{\hat{\rho},\delta} \cap \Omega_{\rho,\delta}\). Similarly, also set \(X_{i}(\hat{\rho}) = X_{i}\) at \(S_{ij}(\hat{\rho})\) and apply the gridding method in the adjacent region. Therefore, the controller in the parameter adjacent region is in the form of (17).

**Remark 3:** The gridding method makes the number of LMIs increases exponentially [22]. The above construction avoid the gridding method in the non-adjacent parameter region. Hence, it can also guarantee the character on the switching surface.

### 4 Example

In this section, we will apply the switching control method in the last section to an AMB system. Owing to the linear dependence of the plant dynamics on the rotor speed, the AMB plant can be simplified to a set of linear time-varying differ-
where the state-space data are denoted as electromagnetic pairs, 

\[ i = \rho J_l \dot{i} + \frac{1}{m} (-4c_2 i + 2c_1 \phi \theta + f_{dw}), \]

\[ \dot{\theta} = \frac{\rho J_l}{J_r} i + \frac{1}{m} (-4c_2 \theta + 2c_1 \phi + f_{d\theta}), \]

\[ \phi \theta = \frac{1}{N} (e_{\theta} + 2d_2 \theta - d_1 \phi), \]

\[ \phi \phi = \frac{1}{N} (e_{\phi} + 2d_2 \phi - d_1 \phi), \]

(21)

where \( \rho \) denotes the rotor speed, \( \theta, \psi \) are the Euler angles denoting the orientation of rotor centerline. \( J_s, J_r \) are the moment of inertia of the rotor in axial and radial directions, respectively. \( \phi_\theta, \phi_\phi \) are the differential magnetic flux from electromagnetic pairs, \( e_\theta, e_\phi \) are the corresponding differences of electric voltage. \( f_{dw}, f_{d\theta} \) are disturbance forces caused by gravity, modeling errors, imbalances, etc. The values of \( c_1, c_2, d_1, d_2 \) and \( m \) depend on the AMB’s geometry and parameters, which can be found in [23].

Let \( x^T = [i \ \dot{i} \ \phi \theta \ \phi \phi \ \phi_\theta \ \phi_\phi] \), \( w^T = [f_{dw} \ f_{d\theta}] \). In automatic balancing design, \( f_{dw}, f_{d\theta} \) are typically modeled as sensor noise on the measured rotor displacement. Under this assumption, the linearized equations (21) can then be written as the following LPV system

\[
\begin{bmatrix}
\dot{x} \\
y
\end{bmatrix} = \begin{bmatrix}
A(\rho) & B_1 & B_2 \\
C_1 & D_{11} & D_{12}
\end{bmatrix}
\begin{bmatrix}
x \\
w
\end{bmatrix},
\]

where the state-space data are

\[
A(\rho) = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
-4c_2 & 0 & 0 & -\rho J_l & 0 & 0 \\
0 & -4c_2 & \rho J_l & 0 & 0 & 2\alpha \rho J_l \\
2\alpha J_r & 0 & 0 & 0 & -\frac{d_1}{m} & 0 \\
0 & 2\alpha J_r & 0 & 0 & -\frac{d_1}{m} & -\frac{d_2}{m}
\end{bmatrix},
\]

\[
B_1 = b_0 x_2, \quad B_2 = \frac{1}{N} \begin{bmatrix}
0_4 & x_2 \\
\end{bmatrix},
\]

\[
C_1 = \begin{bmatrix}
I_2 & 0_2 & x_4 \\
0_2 & x_4
\end{bmatrix},
\]

\[
D_{11} = 0_4 \times 2, \quad D_{12} = \begin{bmatrix}
0_2 & x_2 \\
I_2
\end{bmatrix},
\]

\[
D_{21} = I_2, \quad D_{22} = 0_2 \times 2.
\]

The rotor speed \( \rho \) is assumed to be measured with some uncertainty \( \delta \), and it is assumed to vary between 315 and 1100 rad/s. In this example, the scale of uncertainty is \( \delta = [-5, 5] \). The bound of derivative of \( \rho \) is set to be \([ -40, 40 ] \).

The parameter set is divided into 2 subsets with adjacent set: [315, 720] and [700, 1100]. For each subsets, different LPV controller is designed as in [1]. The difference is that the controller in this paper is designed for parameters with uncertainty.

Following LMI solving method in Theorem 1, the switched LPV controller is constructed. In the subset [315, 700] and [720, 1100], the matrices \( X_i(\rho) \) and \( Z_i(\rho) \) are constant, i.e. \( X_i(\rho) = X_{i,0} \), \( Z_i(\rho) = Z_{i,0} \). In the adjacent set [700, 720], the matrices are parameter dependent as \( X_i(\rho) = X_{i,0} + \rho X_{i,1} \rho \) and \( Z_i(\rho) = Z_{i,0} + \rho Z_{i,1} \). Solving the LMIs, the achieved \( H_\infty \) performance is \( \gamma_{\infty} = 5.0126 \times 10^5 \).

In the simulation, the initial value of the states \( x_1, x_2 \) which is directly out as \( y_1, y_2 \) are set as \([-0.0025, 0.002] \). The other states’ initial value are 0. The disturbance are two impulse with magnitude 0.001 in opposite sign. The output response is showed in Fig. 1.

The parameter varying and the switching signal are shown in Fig. 2 and 3. In Fig. 2, the blue line is the real parameter \( \rho \), and the red line means the inexact parameter \( \hat{\rho} \) which the controller measure.

From the simulation result, the designed switching LPV controller can control the LPV plant with the inexact parameter measuring. Disadvantage is that the \( \gamma_{\infty} \) value of the \( H_\infty \) performance is high.

5 Conclusion

The \( H_\infty \) problem of the LPV system with inexact parameter has been studied. Switching LPV controller and switching law are designed based on the hysteresis method. The LMI tool is used to achieve the controller form. Two strategies are given for constructing the Lyapunov functions based on the multiple Lyapunov functions method.

The method proposed is applied on an AMB system which is a parameter affinely system. The AMB system can be controlled by the achieved switching LPV controller.

Fig. 1: Output response of the hysteresis switching LPV controller for a time-varying rotor speed trajectory.
References


