Flame Front Detection and Curvature Calculation Using Level Set

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Abstract—A level set method is proposed for flame front detection and curvature calculation on turbulent premixed OH Planar Laser Induced Fluorescence (OH-PLIF) images. The flame front on an OH-PLIF image is detected by our proposed curve evolution model. In this model, a global term is designed to make the evolution curve converge to the flame front and a Gaussian kernel function is used to detect the flame front more accurately. For the detected flame front contour, we calculate its curvature using its level set function which represents the contour as the zero level of a higher dimensional embedding function. The precision of our method is verified by the comparison with a common curve-fitting method on synthetic images. The results on different flame images also demonstrate that our method can efficiently detect the flame fronts and get more accurate curvature on OH-PLIF images. The calculated curvature, as an important factor in understanding turbulence and chemistry interaction phenomena, can be used to perform quantitative analysis of flame fronts.

Index Terms—level set, flame shape, combustion, flame structure

I. INTRODUCTION

The future development of combustion devices with improved efficiency, flame stabilization and pollutant emissions relies on a better understanding of the underlying chemical and physical processes in flame. Premixed combustion is of great interest for industrial applications with the potential of low pollutant emissions, and the detailed investigation of its fundamental mechanisms is still undergoing. Flame front, as an indispensable role in flame burning, provides important information for understanding turbulence and chemistry interaction phenomena. Since accurate flame front detection and curvature calculation clearly show the interaction between the turbulent flow field and chemical reactions, we focus on accurately detecting flame fronts from Planar Laser Induced Fluorescence (PLIF) images [1] and calculating curvatures with a higher precision by the level set theory.

To calculate the curvature of flame front on PLIF images, segmenting the flame images is an indispensable step. Some traditional methods usually use edge detection operators [2], [8] to detect flame front. They always have the problems of low efficiency for noisy flame images and still suffer from the difficulties of determining the suitable global or local thresholds from intensity inhomogeneous images. In [3], we propose a curve evolution method for homogeneous image segmentation, which uses image data and inherent constraint of curve to get the continuous and smooth object boundaries. Different from the popular curve evolution models [5], [6], [7] that rely on intensity homogeneity in each region to be segmented, we use a kernel function to extract the local image information, which enables our segmentation method to accurately detect flame front for various types of flame images with sharp edges, cusps and long cracks.

Usually, it is difficult to get the accurate curvature of flame front, especially for noisy images. Some proposed methods [8] on curvature calculation use the arc length as a parameter and express a flame front as a curve function \((x(s), y(s))\). They fit the curve by a cubic spline interpolation and calculate the curvature using the curvature formula. These methods usually perform well on less noisy images. However, with images containing sharps, cracks or heavy noise, their calculated curvature is imprecise. Moreover, the precision relies on the image size and the curve fitting points. In this paper, we use the level set function, whose zero level set is the flame front, to calculate the curvature of flame front contour. We establish the level set function of flame front according to the segmented flame front contour and calculate the curvature based on the level set function which is independent of image noise. Our experiments on synthetic images validate that our curvature calculation method is much more precise than the curve-fitting methods. The results on different OH-PLIF image sequences also demonstrate that our method can efficiently detect the flame fronts and get the accurate curvature of the flame fronts on PLIF images.

II. EXPERIMENT SETUP

The image data sequences are obtained through controlled PLIF imaging of OH radicals in combustion processes. The data includes premixed V-flame images under microgravity
environment[9], as well as conical premixed flame images subject to varying degrees of premixed turbulence. In PLIF imaging, suitable optics are used to transform a laser beam into a light sheet that traverses the flame. If a wavelength is tuned to match a molecular resonance line of OH, the light from the sheet is emitted from the OH radicals in the interaction region. This scattered light is detected using an intensified CCD camera arrangement.

The V-flame images of CH4 air premixed flames are generated from the Center of Applied Space Technology and Microgravity (ZARM, the University of Bremen, Germany). The image resolution is 256 × 256 pixels and 0.2617 mm/pixel. For more information about the V-flame experimental setups, please refer to [10]. The conical turbulence premixed flames are conducted in the Institute of Engineering Thermophysics, Chinese Academy of Sciences. The image resolution is 650 × 650 pixels and 0.2145 mm/pixel.

III. IMAGE PROCESSING AND ANALYSIS

A. Flame front detection

In [13], Tony Chan and Luminita Vese proposed an active contour model to detect objects in an image, based on techniques of curve evolution and level sets. The level set method, proposed by Osher and Sethian [12], has been proven to have the ability to handle topological change. Denote \( \Omega \) be a bounded open subset of \( \mathbb{R}^2 \) and \( C \) be a curve representing the boundary of the open subset \( \omega \) of \( \Omega \) (i.e. \( C = \partial \omega \)). In the level set method, the curve \( C \) is represented implicitly by the zero level set of a Lipschitz function \( \phi : \Omega \to \mathbb{R} \)(called level set function):

\[
\begin{aligned}
C = \partial \omega &= \{(x, y) | \phi(x, y) = 0\} \\
\text{inside}(C) &= \{(x, y) | \phi(x, y) > 0\} \\
\text{outside}(C) &= \{(x, y) | \phi(x, y) < 0\}
\end{aligned}
\]

Unlike the classic active contour based on the gradient information, the Chan-Vese Model (CV Model) can detect the contours without using gradient information. In addition, it has stronger anti-noise performance and weaker dependence on the initial curve. Assume that the image \( I(x, y) \)(shown in Fig. 1) is formed by two regions of approximatively piecewise-constant intensities. The CV model is formulated by the minimizer of the following energy functional \( E \):

\[
E = \frac{\mu_1}{2} \iint_{\Omega} |\nabla H(\phi)| dx dy + \int_{\Omega} |I - c_1| H(\phi) dx dy + \lambda_1 \int_{\Omega \cap C} I - c_1|H(\phi)| dx dy + \lambda_2 \int_{\Omega \cap C} (I - c_2)(1 - H(\phi)) dx dy 
\]

where \( \lambda_1, \lambda_1, \mu_1 \) and \( \mu_2 \) are positive parameters. \( c_1 \) and \( c_2 \) are the mean intensity of the region inside and outside of the evolution curve \( C \), respectively. \( H(\phi) \) is the Heaviside function of \( \phi \). The regularization term keeps the evolution curve smooth, and \( F_1 \) and \( F_2 \) are the data fitting terms, which are minimized when the curve is on the boundary of the object (shown in Fig. 1). The energy functional \( E \) can be minimized using the associated Euler-Lagrange equation. From Fig. 1, its steady solution is the object contour which is the zero-level of \( \phi \).

![Fig. 1. The energy functional is minimized only when the curve is on the boundary of the object.](image)

Since the PLIF images are not piecewise-constant images, the CV model generally fails to detect flame fronts with intensity inhomogeneity. In [11], Li et al. propose a new model that draws upon intensity information in local regions to segment images with intensity inhomogeneity:

\[
E = \int e^{F_{11}} I dx + \mu_1 \int |\nabla H(\phi)| dx + \frac{\mu_2}{2} \int |\nabla \phi| - 1|^2 dx 
\]

where \( e^{F_{11}} I \) is the local energy function for point \( x \) to capture its local intensity information:

\[
e^{F_{11}} I = \sum_{i=1}^{n} K_\sigma(x - y) |I(y) - f_i(x)|^2 M_i(\phi(y)) dy 
\]

where \( K_\sigma \) is the Gaussian kernel function with scale \( \sigma \), \( M_i(\phi) = H(\phi) \) and \( M_2(\phi) = 1 - H(\phi) \). \( f_i(x) \) and \( f_2(x) \) are the approximate image intensity inside and outside the kernel region centered at the point \( x \), whose size is controlled by the scale parameters \( \sigma \). Given a center point \( x \), the local energy \( e^{F_{11}} I \) can be minimized when the evolution curve is exactly on the object boundary.

The model in [11] can detect accurate flame front but it always produces over-segmented results in PLIF images. Therefore, we add a global term acting on the outside region of the evolution curve according to the PLIF image characteristics that the background is almost black. This term will generate a global force to overcome local minimum and over-segmentation problems. The energy functional \( E \) in our segmentation model [3] is defined as follows:

\[
E = \int e^{F_{11}} I dx + \nu \int |I(x) - c_2|^2 (1 - H(\phi)) dx + \mu_1 \int |\nabla H(\phi)| dx + \mu_2 \int \frac{1}{2} |\nabla \phi| - 1|^2 dx 
\]

The first term in Eq. (5) is called the local data fitting term that collects the local intensity information to guarantee the accuracy of our segmentation model. The second term is
the global term that generates a global force to overcome local minimum and over-segmentation problems, and the third term is the length regularization term that has a length shortening or smoothing effect on the zero level contour. The last term is the level set regularization term which maintains the level set function to be a signed distance function in the evolution process. For a class of images, parameter values in this model are all fixed after experimentally deciding the optimal values on one image. In this paper, non-negative weighting parameters are set as follows in all experiments:

\[ \lambda_1 = \lambda_2 = 0.03, \mu_1 = \mu_2 = 0.03, \nu = 1 \text{ and } \sigma = 4. \]

Parameterizing the gradient descent direction by an artificial time \( t \geq 0 \), the curve evolution equation can be derived by minimizing the energy functional \( \mathcal{E} \) with respect to \( \phi \):

\[
\frac{\partial \phi}{\partial t} = \delta_\varepsilon(\phi)(-\lambda_1 e_1 + \lambda_2 e_2) + \delta_\varepsilon(\phi)(\mu_1 \nabla \cdot (\frac{\nabla \phi}{|\nabla \phi|})) + \mu_2 (\nabla^2 \phi - \nabla \cdot (\frac{\nabla \phi}{|\nabla \phi|})) + \delta_\varepsilon(\phi)(-\nu (I - e_2)^2)
\]

\[
e_1 = \int_{\Omega} K_s(x) I(x) M_s^1(\phi(x)) dx,
\]

\[
e_2 = \int_{\Omega} K_s(x) [M_s^1(\phi(x)) I(x)] dx,
\]

\[
e_3 = \frac{1}{2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{2} \left( \frac{\partial \phi}{\partial y} \right)^2
\]

\[\phi(x, t = 0) = \phi_0(x)\]  

(6)

where \( H_s(\phi) \) and \( \delta_\varepsilon(\phi) \) are respectively the smooth approximation of Heaviside function and Dirac function given in Eq. (8). \( e_1 \) and \( e_2 \) are defined as follows:

\[
e_1(x) = \int_{\Omega} K_s(y - x) [I(x) - f_i(y)]^2 dy, i = 1, 2
\]

\[H_s(\phi) = \frac{1}{2} (1 + \frac{2}{\pi} \arctan(\frac{\phi}{\varepsilon})), \quad \delta_\varepsilon(\phi) = \frac{1}{\pi} \frac{\varepsilon}{\varepsilon^2 + \phi^2}
\]

(7)

(8)

The explicit finite difference method is adopted to discretize the partial differential equation in Eq. (6) for the numerical solution of \( \phi(x, y, t) \). The steady solution of the discrete function is acquired when the stop criterion \( \max |\phi^{n+1} - \phi^n| < 0.01 \) is satisfied. Interested readers may refer to [3] for more details.

Fig. 2 compares the flame front detection results of our model, the CV model and Li’s model. The results show that the flame front can be exactly detected by our segmentation model. Our model is robust to image noise. In addition, cusps and cracks can be exactly detected to guarantee the calculated curvature characteristics are much more accurate.

B. Curvature calculation

Let \( \phi(x) \) be the steady solution of Eq. (6). Its zero level set \( C = \{(x, y) | \phi(x, y) = 0\} \) is the contour of a flame front on a PLIF image. To get the accurate curve curvature, the level set function \( \phi \) needs to be re-initialized as a signed distance function \( \phi_s \):

\[
\phi_s(x) = \begin{cases} 
\text{dist}(x, C), & x \in \text{inside}(C) \\
-d\text{dist}(x, C), & x \in \text{outside}(C)
\end{cases}
\]

(9)

where \( x = (x, y) \) and \( \text{dist}(x, C) \) denotes the Euclidean distance function between \( x \) and \( C \). The gradient of the level set function \( \phi_s \) is:

\[
\nabla \phi_s = \left( \frac{\partial \phi_s}{\partial x}, \frac{\partial \phi_s}{\partial y} \right)
\]

(10)

The gradient \( \nabla \phi_s \) is perpendicular to the isocontours of \( \phi_s \) and points in the direction of increasing \( \phi_s \). If \( \phi_s \) is a signed distance function, \( |\nabla \phi_s| = 1 \). The unit (outward) normal vector \( \vec{N} \) is:

\[
\vec{N} = \frac{\nabla \phi_s}{|\nabla \phi_s|} = \nabla \phi_s
\]

(11)

for points on the flame front contour. The curvature \( k \) of the contour is calculated as the divergence of the normal vector \( \vec{N} \):

\[
k = \nabla \cdot \vec{N} = \nabla \cdot (\nabla \phi_s) = \Delta (\phi_s) = (\phi_s)_{xx} + (\phi_s)_{yy}
\]

(12)

We compare our curvature calculation method with the curve-fitting method [1, 8], which chooses nine adjacent points from the contour and fits them to be a cubic spline function \( y = f(x) \). The curvature of the compared curve-fitting method can be calculated by

\[
k = \left[ \left( \frac{d^2 x}{ds^2} \right)^2 + \left( \frac{d^2 y}{ds^2} \right)^2 \right]^{\frac{1}{2}}
\]

(13)

We further verify our level set-based curvature results on a digitized circle with the radius of 150 pixels. The curvature of any point on the circle is -0.00667. Fig. 3 shows the digitized circle together with its level set function. Fig. 4 shows the comparison of the results obtained by our method and the curve-fitting method on the digitized circle shown in Fig. 3. It clearly shows that our method achieves significantly better curvature estimation.
result than the curve-fitting method since the bold line is more stable and is close to the theoretical value (i.e., -0.00667) of the curvature.

We also verify our level set-based curvature results on a two-overlapping-circle image (shown in Fig. 5), where the radius of the top circle is 60 pixels and the radius of the bottom circle is 80 pixels. The theoretical curvature values are respectively -0.01667 and -0.0125 for the top and bottom circles, and infinity at two intersection points. The aim of this experiment is to testify the accuracy of our curvature calculation method on a singular point where the curvature values are very large and the errors tend to be large in the most methods. Its level function is shown in the right figure of Fig. 5. We compare the difference of curvature values of our method and theoretical curvature values with the difference of curvature values of curve-fitting method and theoretical curvature values in Fig. 6. It can be seen that our method is much more accurate. Moreover, our method relies on the level set function of object contour detected by the curve evolution model, which is robust to noise and image size.

We conduct an experiment on V-flame video (named as drop4, which contains 200 gravity environment images, 200 micro-gravity images and 200 transition images) with the equivalence ratio $\Phi=0.75$, the speed velocity of laminar flow $S_L=23.09$ cm/s, the flame thickness $\delta=1.5$ mm, and the turbulent intensity $u'_t = 6.6$ cm/s. The curvature is defined to be positive for flame front convex towards the reactants and the efficient height range of curvature calculation is set to be 90~130 pixels. The probability distribution functions (pdf) of this flame front curvature for three gravity environments are shown in Fig. 7(a). It can be seen that the flame front curvature pdf at the micro-gravity condition shows the broadest of the three conditions. This corresponds to the enlarged wrinkles of the flames under micro-gravity.

When changing the turbulent intensity $u'_t$ to 8.2 cm/s, we generated another V-flame image sequence (named as drop7). Fig. 8 shows the curve evolution process of an image in drop7. The PDFs of its flame front curvature for three gravity environments are shown in Fig. 7(b). Compared with the PDFs of drop4, drop7 shows a broader shape under micro-gravity. It clearly shows that the increase of turbulence inten-
sity makes the flame front curvature distributions broader and the influence of gravity on flame makes the wrinkle weaker.

![Fig. 10. pdf comparison results of C02 and C04. ('-in' denotes the flame front interface and '-out' denotes the smoke-air interface.)](image)

Fig. 10. pdf comparison results of C02 and C04. ('-in' denotes the flame front interface and '-out' denotes the smoke-air interface.)

Fig. 9 shows the curvature results on two conical turbulent premixed flames, which contain 300 images, together with their PDFs. The efficient height range of curvature calculation is 400 to 650 pixels (shown in Fig.10). For C02 ($v_j'=1.8$) and C04 ($v_j'=1.5$) conditions, C02 has a larger grid turbulence than C04. Results in Fig. 10 indicate that the PDF distribution of C02 is broader and flatter near the region of combustor exit. It can be concluded that grid turbulence generates flame wrinkles and hence enhances combustion via applying disturbance on flame roots.

![Fig. 9. Results on C02 and C04 conical flames: the bold curves are the effective flame front we used to calculate pdfs and the thin curves are the effective smoke-air interface we used.](image)

Fig. 9. Results on C02 and C04 conical flames: the bold curves are the effective flame front we used to calculate pdfs and the thin curves are the effective smoke-air interface we used.

IV. CONCLUSION

In summary, we propose a new level set-based flame front detection and curvature calculation method for PLIF images. In order to obtain accurate curvature of flame front, we use a Gaussian kernel and a global term in the curve evolution model to improve its capability to extract the actual geometrical structures of flame front. The experiments on both premixed V-flames and conical-flames show that the sharps and cracks in flame front can be exactly detected. The level set function in the detection stage is used to calculate the curvature of the flame front. The experiments on synthetic images verified that our level set-based curvature calculation method produces more accurate curvature results than the popular curve-fitting method. We further apply our method on premixed V-flame and conical-flame images to achieve accurate segmentation results without requiring any preprocessing step or gradient information. By analyzing the statistical parameters of the curvature of both premixed V-flame and conical-flame images, we conclude that wrinkles of the flames are enlarged under micro-gravity under weak turbulence intensity for V-flames. Furthermore, the grid turbulence generates wrinkles and therefore enhances combustion by applying disturbance on flame roots for conical turbulent premixed flames.

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