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Research on a Fast Algorithm of Model Predictive Control

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Abstract: The traditional control methods are not able to keep control performance in a high level because response speed becomes more important for industrial control. This paper presented a fast predictive control algorithm, which was easy and simple to calculate and the principle of algorithm was very clear and greatly improved the speed of response and calculation, as well as demonstrated the principles of fast algorithms and operation process of the algorithm. Finally, we analyzed the stability, dynamic characteristics and steady state characteristics of system, and used lead-lag correction link to improve the performance of system. The results of simulation demonstrated that all of methods and improved algorithms were feasibility.

Key Words: Fast Algorithm, Model Predictive Control, Steady-State Analysis, Dynamic Analysis, Lead-Lag Correction.

1 INTRODUCTION

Predictive control has been proposed since the 1970s, which has been kept rapid development for decades. Predictive control has been widely used in petrochemical, metallurgy, energy and other fields, which is a very effective multivariable control algorithm can solve the coupling problem of multivariable control system effectively[1-2]. Model Algorithmic Control (MAC)[3], Dynamic Matrix Control (DMC)[4], Generalized Predictive Control (GPC)[5] and State Space Predictive Control (SSPC) are common predictive control algorithm. In terms of theory, the predictive control need to solve a constrained optimization problem online that leading to increase amount of calculation, which makes the carrier of predictive control technology is often upper computer software, not make the DCS as a carrier for implementation directly. In practical applications, predictive control is a supervisory control layer in hierarchical control systems, the results of predictive control as a set point which is passed to PID controller in DCS via OPC communication.

MPC do one time on line optimization at each sampling period , resulting in an optimal performance index which is effect in future in the sampling period, the next sampling period to repeat the same process, which is known as rolling optimization. In the actual industrial production, every rolling optimization process takes about 1-2 two minutes, but PID control layer which is in the lower layer than MPC layer, performs control role a time has reached milliseconds. Obviously, MPC perform a control action need more time, because rolling optimization calculations will have several large dimension matrix computations to do and have a great amount of computation, so rolling optimization consumes a lot of time. With the continuous development of industrial technology, a lot of the production processes for the control have more high demand, some processes require extremely more high accuracy, and especially 1-2 minutes for some real process of change seems too long. Hence, a new fast response predictive control algorithm to be applied to control the process is necessary [6].

This paper is based on recent the progress of research and theoretical results of predictive control, the author presented a fast multivariable control method which is used to solve the problem of calculating very slow of predictive control. The remainder of this paper will discuss the principle of a fast algorithm, proof and execution step and the simulation the fast algorithm. The next parts will focus on analyzing and improving fast algorithm problems and applications of fast algorithm and their advantages and disadvantages.

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2 THE PRINCIPLE AND EXECUTION OF FAST ALGORITHM

First of all, we assume a linear process, by linear superposition theorem, we get the following theorem.

Theorem 1 If a linear, time invariant multivariable process (without regard to the integration process) have an initial steady state operating point ($\boldsymbol{u}_0$, $\boldsymbol{y}_0$), and control input is not changed in k time , so the steady-state value of process output $\boldsymbol{y}_{ss,k}$ is only related to $\boldsymbol{u}_k$ and $\boldsymbol{u}_0$, whereas changing path of input is not effect on $\boldsymbol{y}_{ss,k}$.

Proof. By using initial steady-state point ($\boldsymbol{u}_0$, $\boldsymbol{y}_0$) as the initial steady-state equilibrium point of process, assuming steady-state equation of the system is (1) as following.

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$$\Delta \boldsymbol{y}_{ss} = \boldsymbol{G}\Delta \boldsymbol{u}_{ss} \tag{1}$$

Where, $\Delta \boldsymbol{y}_{ss} \in R^{p\times 1}$ are variation of steady-state value of the controlled outputs, and $\Delta \boldsymbol{y}_{ss} = \boldsymbol{y}_{ss} - \boldsymbol{y}_0$, $\boldsymbol{y}_{ss}$ are steady-state value of the controlled output variables, $\boldsymbol{y}_0$ are the initial steady-state value of controlled output variables, $\Delta \boldsymbol{u}_{ss} \in R^{m\times 1}$ are variation of steady-state value of the control inputs, $\Delta \boldsymbol{u}_{ss} = \boldsymbol{u}_{ss} - \boldsymbol{u}_0$, $\boldsymbol{u}_{ss}$ are steady-state value of the control inputs, $\boldsymbol{u}_0$ are the initial steady-state value of control input variables, $\boldsymbol{G} \in R^{p\times m}$ is steady-state gain matrix of process. Because of $\boldsymbol{u}$ are control variables, control inputs steady-state of k time $\boldsymbol{u}_{ss,k}$ is $\boldsymbol{u}(k)$.

From the initial time to k time, the controller apply any combination of control sequence $\Delta \boldsymbol{u}(1), \cdots \Delta \boldsymbol{u}(k)$, then after k time, the final steady-state value of process can be obtained using the principle of liner superposition. The process of proof is shown as following.

$$\begin{aligned} \boldsymbol{y}_{ss,k} &= \boldsymbol{y}_0 + \Delta \boldsymbol{y}_{ss,1} + \Delta \boldsymbol{y}_{ss,2} + \cdots \Delta \boldsymbol{y}_{ss,k} \\ &= \boldsymbol{y}_0 + \boldsymbol{G}\Delta \boldsymbol{u}(0) + \boldsymbol{G}\Delta \boldsymbol{u}(1) + \cdots \boldsymbol{G}\Delta \boldsymbol{u}(k) \\ &= \boldsymbol{y}_0 + \boldsymbol{G}\sum_{i=0}^{k} \Delta \boldsymbol{u}(i) \\ &= \boldsymbol{y}_0 + \boldsymbol{G}(\boldsymbol{u}_k - \boldsymbol{u}_0) \end{aligned} \tag{2}$$

For linear systems, the steady-state process of the final output value is only related to the initial steady state operating point of the process and the final control input steady state value, whereas nothing is not related to changing path of process input.

The proof of theorem 1 is finished.

As known from Theorem 1, for multivariable control system, according to the changing of set point we can solve the steady state operating point of control input, and then based on the operating point can solve the corresponding control law.

The detailed algorithm is written as following.

The algorithm contains two parameters, the steady-state time and the number of beats of control. Their definitions are divided into steady-state time (TTSS), which is elapsed time that the system reach steady state again by acting on step input, and controlling the number of beats (N), which decompose a control input into several beats to implement, the meaning of the number of beats is the number of beats of control. Algorithm includes both online and offline parts. By using analysis or identification, we can obtain steady-state mathematical models of multivariable process that is the major work of online part. Offline part is shown as following two parts.

1) Steady-state model predictions and feedback correction:

Steady predictive model is $\Delta \boldsymbol{y}_{ss} = \boldsymbol{G}\Delta \boldsymbol{u}_{ss}$, according to detective value of control input in current time, steady-state value of controlled output and detective value of control input in previous time, we can calculate the steady-state output value of the current time and the equation is written as following.

$$\boldsymbol{y}_{ss\_k} = \boldsymbol{y}_{ss\_k-1} + \boldsymbol{G}\Delta(\boldsymbol{u}_{ss\_k\_measure} - \boldsymbol{u}_{ss\_k-1}) \tag{3}$$

The output value of current time $\boldsymbol{y}(k)$ subtract steady-state predictive value of output before steady-state time $\boldsymbol{y}_{ss\_k-TTSS}$, we can get the error that is written as following.

$$\boldsymbol{e}(k) = \boldsymbol{y}(k) - \boldsymbol{y}_{ss\_k-TTSS} \tag{4}$$

Then, we use $\boldsymbol{e}(k)$ to adjust steady-state prediction of output, the equation is written as following.

$$\boldsymbol{y}_{ss\_k\_correction} = \boldsymbol{y}_{ss\_k} + \boldsymbol{e}(k) \tag{5}$$

2) Rolling optimization:

The difference between the steady-state predictive correction value and the current time set point of output, we can calculate the steady-state optimal control law $\Delta \boldsymbol{u}_{ss\_calc}$. According to control speed fast or slow in implementation process, we can change parameter N that the number of beats of control.

The equation of current control increment is written as following.

$$\Delta \boldsymbol{u}_{ss\_calc\_now} = \Delta \boldsymbol{u}_{ss\_calc} / N \tag{6}$$

The control input value of current time is written as following,

$$\boldsymbol{u}_{ss\_k\_output} = \boldsymbol{u}_{ss\_k\_measure} + \Delta \boldsymbol{u}_{ss\_calc\_now} \tag{7}$$

which can pass to control system by OPC.

3 THE SIMULATION EXPERIMET OF FAST ALGORITHM.

We introduced the principle of fast algorithms and implementation steps in Section 2, then through simulation experiments to verify the feasibility of fast algorithm will be introduced in this section.

First of all, we assume the number of beats of control N=5, it means control increment $\Delta u = \Delta u / 5$ in next 5(N) time. After 5(N) time, we calculate control increment again. The result of simulation is shown in fig.1 as below.

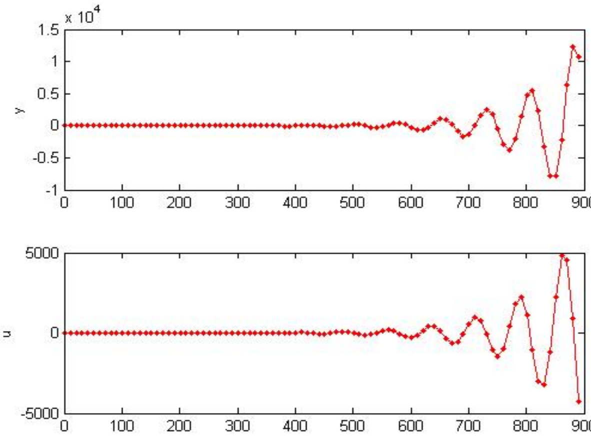

Fig. 1 The input and output curves when N=5.

As the fig.1 shown, input and output value are diverging shock, and amplitude is increasing more and more faster, so the curve can never converge. We assume N=10 next, and the result of simulation is shown in fig.2 as below.

As the fig.2 shown, input and output value are also diverging and but amplitude is decreasing, as well as the result is better than N=5. We assume N=20 and N=50 next, the result of simulation is shown in fig.3 as below, where the red curve and blue curve are the results of N=20 and N=50 respective.

As the fig.3 shown, the curve has a certain extent shock when N=20, but it can be convergence. The result of control is better when N=50. Obviously, the result of control will be more better when N continue increasing. We will discuss the reason of this result and how to deal with the problem and improve the algorithm in Section 4.

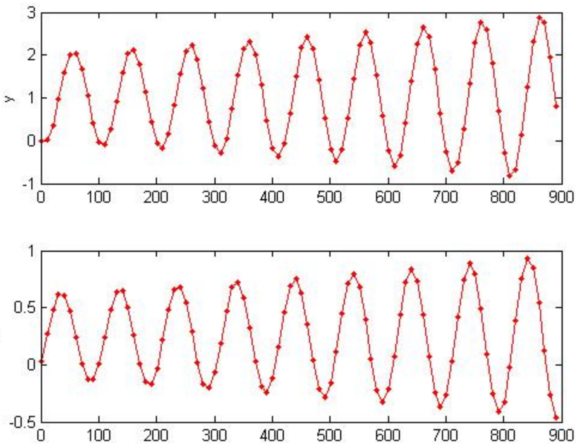

Fig. 2 The input and output curves when N=10.

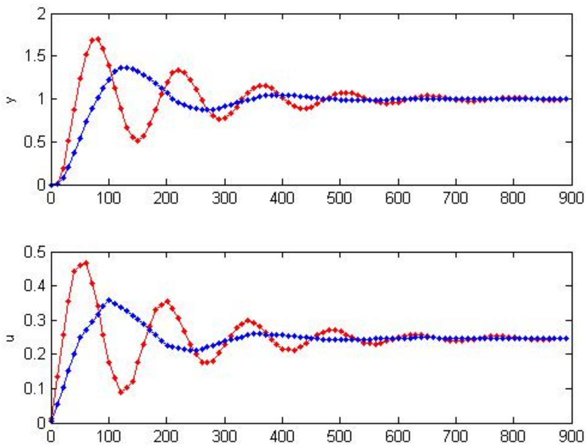

Fig. 3 The input and output curves when N=20 and N=50.

4 THE PROBLEM OF FAST ALGORITHM AND THE SIMULATION OF IMPROVED ALGORITHM

In Section 3, the implementation process of fast algorithm and simulation verification are introduced. By simulation, we find control effect will be more better when the number of beats of control N is increasing, the curve is diverging shock and never convergence when N is too small. In this section, we will discuss the reason of this result and how to deal with the problem and improve the algorithm.

In initial time, Δu is relative large. When N is small, $\Delta u = \Delta u / N$ is also large, and Δu will be control increment in next continuous N time, so the value of control variable will be more and larger. The curve is shown as fig.4 as below.

As the fig.4 shown, when N is small, Δu will be large and leading to control action more intense. $|\Delta u|$ is increasing continuously that leading to $|u|$ increasing too, the curve will diverge shock and the result is same as fig.1. When N increases, because of $\Delta u = \Delta u / N$, control increment Δu will decrease and control action will be stabilized gradually. While a new problem arises, if N value is too large, then the control process will take a long step and long time to reach steady-state. In order to keep value of Δu not too large, we need value of N to increase, however N value is too large will lead to increase step and time. The simplest method is to selected a value of N that simultaneously satisfy the two conditions, to changing value of N to adapt different application occasions. The N might be exist that satisfies all the conditions, but the author think this approach is not good enough. We will analyse how to make Δu not too large and meanwhile control step and time are not too long.

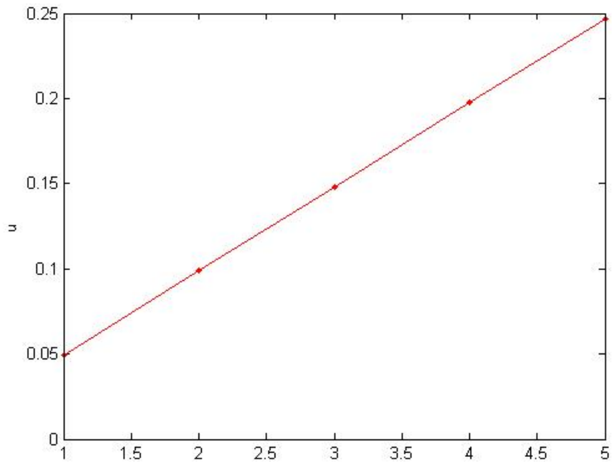

Fig.4 The trend of u.

For given step of fast algorithm, if the value of N increases, the number of calculations and the computation time will reduce. Conversely, increasing the number of calculations of Δu , the calculation time increases, but the N is smaller, Δu changes faster, and control action will be more precise, meanwhile will be earlier to steady-state.

The number of beat of control means every N time calculate Δu once, control increment is $\Delta u = \Delta u / N$ in every time within N time, where "N" in N time is written as N1 and "N" in $\Delta u = \Delta u / N$ is written as N2. N1=N and N1 is still the number of beat of control, N2 may or may not equal to N that is defined increment beat. We use N1 to control calculation accuracy and control time and to solve the problem of N and step size too large which discuss as above, so we can choose appropriate value of N1 to meet

the conditions of control time and accuracy. N2 is used to keep Δu in the appropriate region and prevent from control action too intense because of too Δu large. The result of simulation that is acted on two parameters N1 and N2, is shown as following.
The red curves are N1=10, the blue curves are N2=20 and N2=50 respective, the results of simulation are shown in fig.5 as following.

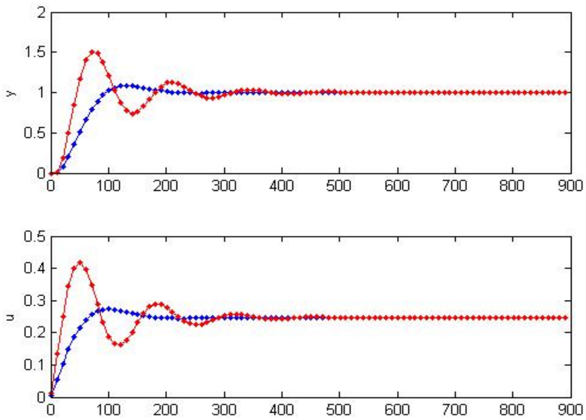

Fig. 5 The input and output curves when N2=20 and 50.

As fig.5 shown, increment range of Δu is improved and avoid control action overly drastic when keeping N1 and increasing N2, so control effect becomes better.
And then, the red curves are N1=2 and N1=10, the blue curves are N2=50 respective. The result of simulation is shown in fig.6 as following.

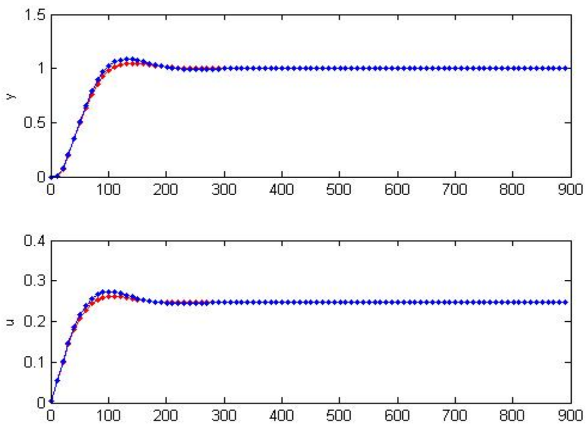

Fig.6 The input and output curves when N1=2 and N1=10.

As fig.6 shown, if N2 is invariant, reducing N1 will make invariant smaller and control accuracy better.
In this section we analysed the main problems of fast algorithm, and analysis and improvement of the problems and shortcomings are discussed, then the simulation results show the effectiveness of the improved algorithm. In Section 5, we will discuss advantage and disadvantage of fast algorithm and how to improve.

5 THE ADVANTAGE AND DISADVANTAGE OF FAST ALGORITHM AND THE IMPROVED APPROACH

The fast algorithm is different from other control algorithms; the main advantage is that the calculation is simple and easy. Similarly, the fast algorithm also has some disadvantages, such as anti-interference performance is not very strong, the system fluctuation may lead to decreasing steady-state error and increasing the dynamic characteristics of the system. The fast algorithm has only control beat N1 and increment beat N2 two parameters, if the fast algorithm will apply in more complicated region, just two adjustable parameters might affect dynamic performance and steady-state performance of system [7-8]. Based on the above problems, where the algorithm will be improved to overcome the shortcomings. The improved approach is described as following.
The approach is series connection of a lead-lag and a pure delay link, in other words that control targets (model) is series with function modules $e^{-Ts}[(1+T1*s)/(1+T2*s)]$, where e^{-Ts} is the pure lag link, $(1+T1*s)$ is the lead correction link and $1/(1+T2*s)$ is the lag correction link[9]. By adding lead-lag link, using its phase advanced features to increase the phase margin of the system and changing the open-loop frequency characteristics of system. Generally, we make the maximum phase leading angle of correction link appear in the new cross-over frequency point of system. Lag correction by adding lag correction link, that make the open-loop gain of system increase sharply, while also make the dynamic indicators of system keep in good condition in the original system. Lead correction link has a low pass filter characteristics, which not only reduce mid-high frequency gain of system, but also make crossover frequency of system forward, meanwhile that does not affect the low-frequency characteristics of the system, so adding lead link can increase the system phase margin. The lead-lag correction improves the dynamic and static performance of system so that corrected system will apply to more occasions. The improved algorithm has three new adjustable parameters T, T1 and T2, as well as improves the dynamic and static performance of system.
Holding N1 and N2, we adjust T at first and the curves are shown in fig.7 as following.

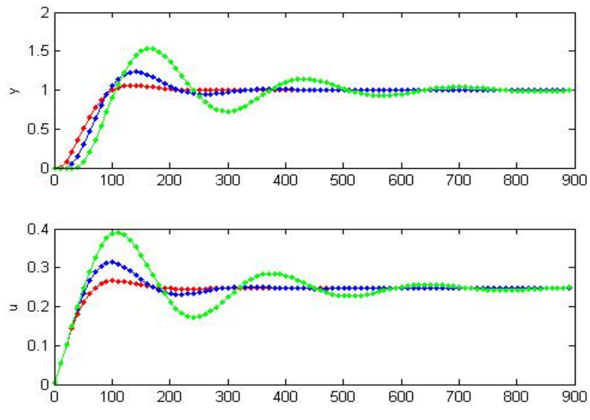

Fig.7 The input and output curves when changing values of T.

The red, blue, green curve represent the curve when T is increasing. When T=0, the system is a non-delay system, but system delay will increase when T increase. Although increasing T will increase overshoot and adjustment time will be longer, but the algorithm is proven to be ability to control large delay system and the control precision is also guaranteed.

We will adjust parameter T1 next. When T1=0, the system is a non-lead system. When T1 increase, the lead of system is increasing, the results of simulation are shown in fig.8 as following.

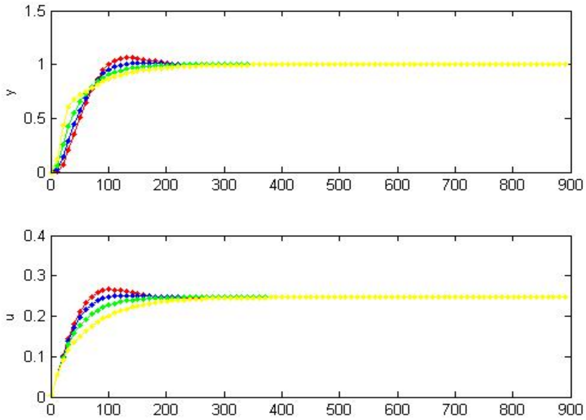

Fig.8 The input and output curves when T1 is increasing.

From red curve to yellow curve, which representing the T1 is increasing. It can be seen that the overshoot reduces significantly and the fast performance is also improved with the increase of T1, because the increased lead correction plays a role in improving the dynamic performance of the system significant [10].

Through the above analysis, lead correction link can improve the system dynamic performance, but improving steady state performance of the system is not obvious. We want to improve the steady state performance of the system too, so adding a lag correction link should be a choice. When T2 begins to increase from 0, lag correction link will work, and the result of simulation is shown in fig.9 as following.

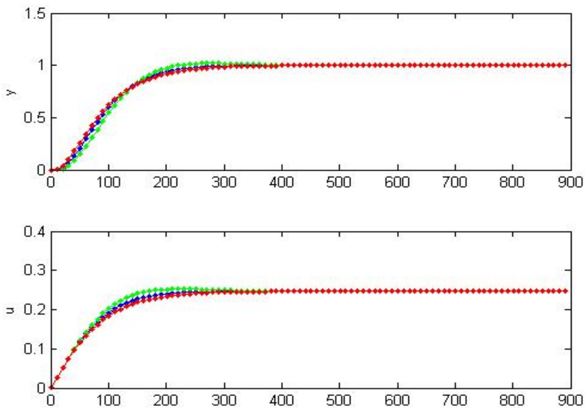

Fig.9 The input and output curves when T2 is increasing.

As fig.9 shown, the dynamic performance of system is not improved observably by changing T2, but the steady state performance of system is improved a lot. If adding a full lead-lag correction link, both the advantages of lead and lag correction will be used to improve the dynamic and steady state performance of the system [11-12]. The result of simulation is shown in fig.10 as following.

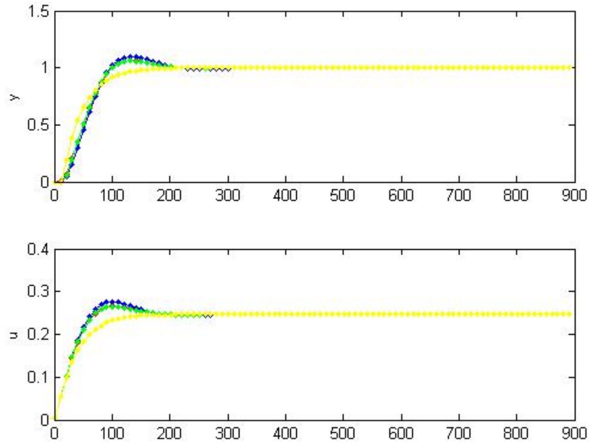

Fig.10 The curves of changing all of parameters.

As fig.10 shown, the dynamic and steady state performance of system are improved observably by using lead-lag correction link, meanwhile by adding T, T1 and T2 three adjustable parameters application flexibility of fast algorithm is improved. As the results of simulation seen, fast algorithm not only has a fast and simple features, but also the ability to handle large delay and lag complex processes have improved significantly by adding a lead-lag correction link.

Finally, we conducted a simulation experiment of process of heavy oil separation tower by using fast algorithm[13]. The results of simulation show that the control effect of heavy oil separation tower is very well and is shown in fig.11 as following.

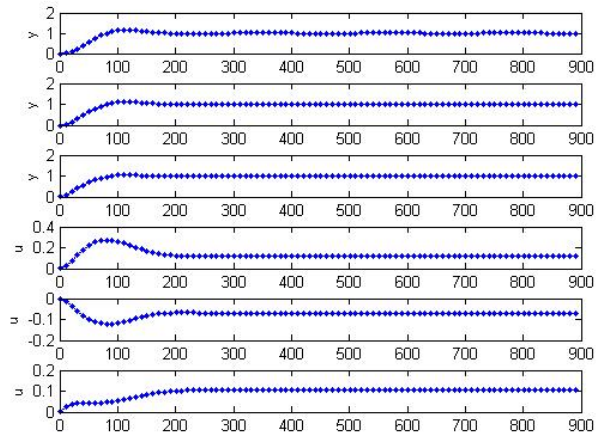

Fig.11 Control effect of heavy oil separation tower.

6 CONCLUTION

This paper presented a fast predictive control algorithm, introduced the principle and implementation step of fast algorithm, the effectiveness and problems of fast algorithm were shown by simulation. The problems and improved algorithms of fast algorithm were discussed in next and the results of simulation proved the correctness of the algorithm. Finally, the advantages and disadvantages of fast algorithm were analysed, we provided an improved method for overcoming shortcomings and was verified by simulation experiments. Although there are a variety of potential problems of fast algorithm, the author will continue to improve more problems in future work and to overcome the more shortcomings, the fast algorithm can be applied to a wider range of fields.

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