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Adaptive neural control for cooperative path following of marine surface vehicles: state and output feedback

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This paper addresses the cooperative path-following problem of multiple marine surface vehicles subject to dynamical uncertainties and ocean disturbances induced by unknown wind, wave and ocean current. The control design falls neatly into two parts. One is to steer individual marine surface vehicle to track a predefined path and the other is to synchronise the along-path speed and path variables under the constraints of an underlying communication network. Within these two formulations, a robust adaptive path-following controller is first designed for individual vehicles based on backstepping and neural network techniques. Then, a decentralised synchronisation control law is derived by means of consensus on along-path speed and path variables based on graph theory. The distinct feature of this design lies in that synchronised path following can be reached for any undirected connected communication graphs without accurate knowledge of the model. This result is further extended to the output feedback case, where an observer-based cooperative path-following controller is developed without measuring the velocity of each vehicle. For both designs, rigorous theoretical analysis demonstrate that all signals in the closed-loop system are semi-global uniformly ultimately bounded. Simulation results validate the performance and robustness improvement of the proposed strategy.

Keywords: cooperative path following; marine surface vehicles; neural networks; observer; uncertainties

1. Introduction

Recently, studies on the cooperative control of multi-vehicle systems have attracted great attention from various research communities. Successful applications can be found in diverse areas, which include formation control of mobile robots (Fax & Murray, 2004), formation flight of unmanned aerial vehicles (Ren & Beard, 2004), coordinated control of marine surface vehicles (Peng, Wang, Chen, Hu, & Lan, 2013; Chen & Tian, 2012; Hou, Cheng, & Tan, 2009; Wang, Wang, Peng, & Wang, 2013; Wang, Wang, & Peng, 2014) , and so on. In order to achieve cooperative control, several methods have been proposed, which include cooperative target tracking (Lili & Hovakimyan, 2011), cooperative trajectory tracking (Federico & Jess, 2010), and cooperative path following (Skjetne, Ihle, & Fossen, 2003; Skjetne, Moi, & Fossen, 2002; Thorvaldsen & Skjetne, 2011). In particular, the objective of cooperative path following is to steer a group of vehicles along predefined paths while keeping a desired spatial formation.

During the past few years, cooperative path-following problem of marine surface vehicles has been studied by many researchers. A variety of approaches to this problem have been proposed in the literature, using a wide range of analytic tools. For example, in Skjetne et al. (2002), cooperative path-following problem is first presented and solved by defining a geometric task and a dynamic task. The desired formation is maintained by the proposed Formation Reference Point (FRP) method. In Ghabcheloo et al. (2006), the problem of temporary communication losses is solved by putting together the path-following and path-coordination strategies as a cascade system form. In Ihle, Arcak, and Fossen (2007), a passivity-based approach for cooperative path following is developed. A major advantage of this approach is that it allows the designer to construct filters that preserve the passivity properties of the closed-loop system and this additional flexibility is capable of improving the system performance and robustness. In these studies, the uncertainties existing in the vehicle systems are not treated as an important problem. Therefore, additional schemes should be developed. Decentralised cooperative control for uncertain plants are developed by Fradkov, Junussov, and Ortega (2009), Fradkov, Grigoriev, and Selivanov (2011), and Selivanov, Fradkov, and Fridman (2011). Besides, it is well known that neural network (NN) has been found to be a particularly powerful tool for modelling uncertain nonlinear systems due to its universal approximation property (Hornik, 1991). Based on the universal approximation property, analysis and design of control systems with NN

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adaptive controllers have been reported in Tee and Ge (2006) for tracking control of an ocean surface vessel, in Tong, Li, and Zhang (2011) for tracking control of uncertain SISO nonlinear systems, and in Chen, Ge, and Ren (2013) for position mooring control of a marine surface vessel. However, such technique has not been explored for cooperative path following of marine surface vehicles.

A key feature of the aforementioned results in Fax and Murray (2004), Ren and Beard (2004), Wang et al. (2013), Peng et al. (2013), Lili and Hovakimyan (2011), Federico and Jess (2010), Skjetne et al. (2002), Skjetne et al. (2003), Thorvaldsen and Skjetne (2011), Ghabcheloo et al. (2006), Ihle et al. (2007), and Chen et al. (2013) is that the state variables of each vehicle are all assumed to be measurable. However, some state variables may not be available in practice due to technical reasons or saving of implementing cost. Therefore, some researchers developed controllers based only on position information. In Do and Pan (2004), the proposed controllers are designed by backstepping and cascade system theory. The unmeasured sway and yaw velocities are estimated by a nonlinear passive observer. In Fossen and Grϕvlen (1998), a globally exponentially stable nonlinear control law is proposed. A nonlinear observer is included in the design such that only position measurements are required. To tackle the problem of uncertainties and unmeasured states, Tong et al. (2011), Tong, Li, and Shi (2012), Tong and Li (2013), and Li, Tong, Li, and Jing (2014) also provide many good approaches. Despite these efforts, there are still no results on output feedback-based cooperative path following of marine surface vehicles, especially with the case the vehicle dynamics are totally unknown.

Motivated by the aforementioned observations, this paper considers the cooperative path-following problem of multiple marine surface vehicles subject to dynamical uncertainties and environmental disturbances induced by unknown wind, wave and constant ocean current. The solution to this problem unfolds into two basic aspects. First, in order to force each marine surface vehicle to follow a predefined path, an NN adaptive path-following controller is designed. Second, in order to maintain a desired formation, the along-path speed and path variables are synchronised to each vehicle owing to the proposed decentralised synchronisation control law building on graph theory and Lyapunov theory. Moreover, an extension to the output feedback case is further studied. An observer-based cooperative path-following scheme is developed without measuring the velocities of the vehicles. Based on Lyapunov analysis, it is proved that with the developed algorithms, all signals in the closed-loop system are semi-global uniformly ultimately bounded (SGUUB), and the path-following error and path-coordination error converge to a small neighbourhood of origin by choosing appropriate control parameters. The main advantages of the proposed algorithms are summarised as follows: (1) compared with the previous works (Aguiar & Hespanha, 2007; Almeida, Silvestre, & Pascoal, 2010; Chen & Tian, 2012; Ghabcheloo et al., 2006; Ihle et al., 2007) of cooperative path-following problem, in this paper, the uncertainties including model parametric uncertainty, unmodelled dynamic and time-varying environmental disturbances are compensated by NNs; (2) compared with our previous works of cooperative control for marine surface vehicles (Peng et al., 2013; Peng, Wang, Shi, Wang, & Wang, 2015; Peng, Wang, Zhang, & Sun, 2014), the assumption of measurable velocities is relaxed by the observer-based control. At the same time, the output feedback control for cooperative path-following problem of multiple marine surface vehicles is the first trial in the control field, which was not followed in the previous works (Almeida et al., 2010; Chen & Tian, 2012; Ghabcheloo et al., 2006; Ihle et al., 2007; Wang et al., 2013); (3) the amount of communications is reduced effectively due to the proposed decentralised synchronisation control law.

2. Preliminaries and problem formulation

2.1. *Graph theory*

The definitions in this section borrow from Balakrishnan and Ranganathan (2000), Biggs (1996), and Godsil and Royle (2001). An undirected graph $\mathcal{G} = \mathcal{G}(\mathcal{V}, \mathcal{E})$ consists of a finite set $\mathcal{V} = \{1, 2, \ldots, n\}$ of n vertices and a finite set $\mathcal{E}$ of m pairs of vertices $\{i, j\} \in \mathcal{E}$ named edges. If $\{i, j\}$ belongs to $\mathcal{E}$, then i and j are said to be adjacent. A path from i to j is a sequence of distinct vertices called adjacent. If there is a path in $\mathcal{G}$ between any two vertices, then the graph $\mathcal{G}$ is said to be connected. The adjacency matrix of the graph $\mathcal{G}$, denoted by $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{n\times n}$, is a square matrix with rows and columns indexed by the vertices, such that a_{ij} equals one if $\{j, i\} \in \mathcal{E}$ and zero otherwise. The degree matrix $\mathcal{D} = [d_{ij}] \in \mathbb{R}^{n\times n}$ of the graph $\mathcal{G}$ is a diagonal matrix where d_{ij} equals to the number of adjacent vertices of vertex i. The Laplacian associated with the graph $\mathcal{G}$ is defined as $L = \mathcal{D} - \mathcal{A}$. If the graph $\mathcal{G}$ is connected, then zero is an eigenvalue of L and all nonzero eigenvalues are positive. If each vehicle can be represented by a vertex, then the communication relationship between any two vehicles can be described by an edge between the corresponding vertices.

Lemma 1: *If $\mathcal{G}$ is a connected undirected graph, then there exists a positive definite matrix P such that $\theta^T L\theta = s^T Ps$, where $\theta = [\theta_1, \ldots, \theta_n]^T \in \mathbb{R}^n$, $s = [s_1, \ldots, s_n]^T \in \mathbb{R}^n$, $s_i = \sum_{i=1}^{n} a_{ij}(\theta_i - \theta_j)$.*

Proof: The proof can be found in Hou et al. (2009), and thus omitted here for brevity.

2.2. *Neural networks*

Before introducing our control design method, let us first recall the NNs (Ge & Wang, 2002; Lewis, Yesildirek, &

Liu, 1996). The NNs take the form of $W^T\sigma(\xi)$ where $W \in \mathbb{R}^{\ell\times m}$ is called ideal weight matrix and satisfy $\|W\| \le W_M$ (Wang & Huang, 2002; Young & Lewis, 1999), with ℓ being the number of NN nodes. $\sigma(\xi) \in \mathbb{R}^{\ell}$ is a vector valued function defined in $\mathbb{R}^{\ell}$, with $\xi = [\xi_1, \ldots, \xi_q]^T \in \mathbb{R}^q$ being the NN input vector. Denote the components of $\sigma(\xi)$ by $\rho_l(\xi_i)$, $l = 1, \ldots, \ell$, $i = 1, \ldots, q$, and $\rho_l(\xi_i)$ is a basis function. In this work, $\rho_l(\xi_i)$ is chosen as the commonly used hyperbolic tangent function, which has the following form $\rho_l(\xi_i) = \frac{1-\exp(-p\xi_i)}{1+\exp(-p\xi_i)}$, where $p \in \mathbb{R}$ represents the slope of the hyperbolic tangent function.

Lemma 2 (Universal Approximation Theorem (Hornik, 1991; Wang & Huang, 2005))**:** *Given a continuous functions $f(\xi): \Omega \to \mathbb{R}^s$ with a compact set $\Omega \in \mathbb{R}^k$, and a constant real number $\varepsilon_M > 0$, there exists an ideal weight matrix W such that*

$$f(\xi) = W^T\sigma(\xi) + \varepsilon(\xi), \xi \in \Omega, \tag{1}$$

where $\|\varepsilon(\xi)\| \le \varepsilon_M$. In the NN, we should like to adapt the weights online in real time to provide suitable performance of the net. That is, the NN should exhibit learning while controlling behaviour. Note that NN approximation is only guaranteed within some compact sets in the derivation of the adaptive neural controller. Accordingly, the stability results obtained in this work are SGUUB (Ge & Wang, 2002).

2.3. Problem formulation

Consider a class of networked multi-vehicle systems consisting of n marine surface vehicles, where the dynamical model of the ith vehicle is given by (Breivik & Fossen, 2006)

$$\dot{\eta}_i = J_i(\psi_i)\nu_i + \nu_{ic}, \tag{2}$$

$$\begin{aligned} M_i\dot{\nu}_i &= \tau_i - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - \Delta_i(\nu_i) \\ &\quad +\tau_{iw}(t), \qquad (3) \\ y_i &= \eta_i, \end{aligned}$$

where $\eta_i = [x_i, y_i, \psi_i]^T \in \mathbb{R}^3$ represents the three degree-of-freedom pose vector in the earth-fixed reference frame, with (x_i, y_i) being the position and ψ_i being the heading angle. $\nu_i = [u_i, v_i, r_i]^T \in \mathbb{R}^3$ denotes the relative velocity between the vehicle and the fluid in the body-fixed reference frame. (In fact, $\nu_i = \nu_{iv} - \nu_{if}$, where ν_{iv} represents the generalised velocity of the vehicle and ν_{if} is the velocity of the fluid. Let $\nu_{ic} = [u_{ic}, v_{ic}, 0]^T \in \mathbb{R}^3$ represent the velocity of the ocean current which is assumed to be constant. According to Almeida et al. (2010), we have $\nu_{if} = J^T\nu_{ic}$.) $\tau_i = [\tau_{iu}, \tau_{iv}, \tau_{ir}]^T \in \mathbb{R}^3$ is the control input vector with τ_{iu} being the surge force, τ_{iv} being the sway force, and τ_{ir} being the yaw moment. $M_i \in \mathbb{R}^3$ denotes the constant positive definite mass matrix. $C_i(\nu_i) \in \mathbb{R}^{3\times3}$ is the skew-symmetric matrix of Coriolis and centripetal terms. $D_i(\nu_i) \in \mathbb{R}^{3\times3}$ is the nonlinear damping matrix. $\Delta_i(\nu_i) \in \mathbb{R}^3$ represents the unmodelled dynamic. $\tau_{iw}(t) = [\tau_{iuw}, \tau_{ivw}, \tau_{irw}]^T \in \mathbb{R}^3$ denotes the time-varying ocean disturbance induced by wind and wave satisfying $|\tau_{iw}(t)| \le \tau_{iwM}$, where $\tau_{iwM} \in \mathbb{R}^3$ is a positive constant vector. The rotation matrix $J_i(\psi_i)$ is given by $J_i(\psi_i) = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$. The inertial and body-fixed coordinate frames are shown in Figure 1.

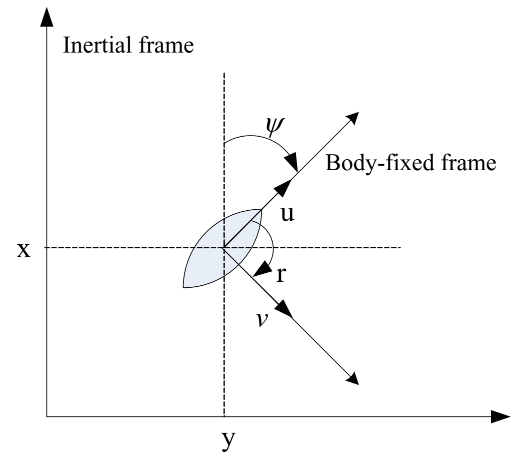

Figure 1. Inertial and body-fixed coordinate frames.

Assumption 1: *For the special case of marine surface vessels, M_i and $C_i(\nu_i)$ also both include the rigid-body mass matrix M_{iRB}, $C_{iRB}(\nu_i)$, and hydrodynamic added-mass matrix M_{iA}, $C_{iA}(\nu_i)$, i.e. $M_i = M_{iRB} + M_{iA}$, $C_i(\nu_i) = C_{iRB}(\nu_i) + C_{iA}(\nu_i)$, and satisfy $M_i = M_i^T > 0$, $C_i(\nu_i) = -C_i(\nu_i)^T$. M_i is considered to be a known matrix similarly to Do and Pan (2004).*

Cooperative path-following problem. For a system output $y_i \in \mathbb{R}^n$, the desired path is represented by the set $\mathbf{y}_{id} := \{y_i \in \mathbb{R}^n : \exists\theta_i \in \mathbb{R}, y_i = y_{id}(\theta_i)\}$, where y_{id} is continuously parametrised by the path variable $\theta_i \in \mathbb{R}$. Given the desired path $\mathbf{y}_{id}$ and a reference speed assignment $v_{id} \in \mathbb{R}$, design a controller τ_i such that all signals in the closed-loop networked system are SGUUB , and

(1) Force the output y_i to follow the desired geometric path $y_{id}(\theta_i)$, i.e.,

$$\lim_{t\to\infty} \|y_i - y_{id}(\theta_i)\| \le \epsilon_{i1}, \tag{4}$$

(2) Force the along-path speed $\dot{\theta}_i$ to follow a desired speed v_{id}, i.e.,

$$\lim_{t\to\infty} \left\|\dot{\theta}_i - v_{id}\right\| \le \epsilon_{i2}, \tag{5}$$

(3) Force each path variable to satisfy

$$\lim_{t\to\infty} \left\|\theta_i - \theta_j\right\| \leq \epsilon_{i3}, \tag{6}$$

where $\epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3} \in \mathbb{R}$ are some small constants.

Assumption 2: *The desired path $y_{id}(\theta_i)$ is sufficiently smooth and its second derivative $y_{id}^{\theta_i^2}$ is bounded, where $y_{id}^{\theta_i^2} = \partial y_{id}^{\theta_i}/\partial\theta_i$, $y_{id}^{\theta_i} = \partial y_{id}/\partial\theta_i$, $\left\|y_{id}^{\theta_i}\right\| \leq y_{idM}^{\theta_i}$, $\left\|y_{id}^{\theta_i^2}\right\| \leq y_{idM}^{\theta_i^2}$.*

Remark 1: The first two control objectives stated above can be considered as the manoeuvering problem (Skjetne et al., 2002) involving a geometric task (4) and a dynamic task (5). For the geometric task, the vehicle is required to follow and remain on the desired path. The dynamic task specifies the speed assignment along the path. The control objective (6) is the coordination goal among vehicles while taking into account the constraints imposed by the communication network .

3. State feedback controller design

3.1. Individual path-following design

In this section, we go through individual path-following design for each vehicle by employing the backstepping technique in two steps. The decentralised synchronisation control law for cooperative path following will be derived in the next section.

Step 1. The error variables are defined as

$$z_{i1} := J_i^T(y_i - y_{id}), \tag{7}$$

$$z_{i2} := \nu_i - \alpha_i, \tag{8}$$

$$\tilde{\nu}_{ic} := \hat{\nu}_{ic} - \nu_{ic}, \tag{9}$$

where $\hat{\nu}_{ic}$ is an estimate of ν_{ic}; α_i is a virtual control law to be specified later.

Differentiating z_{i1} with respect to time and using (2) yield

$$\begin{aligned}\dot{z}_{i1} &= \dot{J}_i^T(y_i - y_{id}) + J_i^T\left(\dot{\eta}_i - y_{id}^{\theta_i}\dot{\theta}_i\right)\\ &= -rSz_{i1} + \nu_i + J_i^T\left(\hat{\nu}_{ic} - y_{id}^{\theta_i}\dot{\theta}_i\right) - J_i^T\tilde{\nu}_{ic},\end{aligned} \tag{10}$$

where S is defined as

$$S = \begin{bmatrix} 0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}. \tag{11}$$

Let $\omega_{is} := \dot{\theta}_i - \upsilon_{id}$ be the along-path speed tracking error, and it follows that

$$\begin{aligned}\dot{z}_{i1} = &-rSz_{i1} + \nu_i + J_i^T\left(\hat{\nu}_{ic} - y_{id}^{\theta_i}\upsilon_{id}\right)\\ &+ J_i^T\left(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}\right).\end{aligned} \tag{12}$$

Consider a scalar function

$$V_{i1} = \frac{1}{2}z_{i1}^T z_{i1}. \tag{13}$$

Taking the time derivative of (13), we obtain

$$\begin{aligned}\dot{V}_{i1} = z_{i1}^T\big[&-rSz_{i1} + \nu_i + J_i^T(\hat{\nu}_{ic} - y_{id}^{\theta_i}\upsilon_{id})\\ &+ J_i^T\left(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}\right)\big].\end{aligned} \tag{14}$$

Choose a virtual control law α_i as

$$\alpha_i = -K_{i1}z_{i1} - J_i^T\left(\hat{\nu}_{ic} - y_{id}^{\theta_i}\upsilon_{id}\right), \tag{15}$$

where $K_{i1} \in \mathbb{R}^{3\times 3}$ is a diagonal matrix and its diagonal elements are positive constants. It follows from (8) and (14) that

$$\dot{V}_{i1} = -z_{i1}^T K_{i1}z_{i1} + z_{i1}^T z_{i2} + z_{i1}^T J_i^T(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}). \tag{16}$$

Step 2. Taking the time derivative of z_{i2} along (3) gives

$$\begin{aligned}M_i\dot{z}_{i2} = &\tau_i - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - \Delta_i(\nu_i)\\ &+ \tau_{iw}(t) - M_i\dot{\alpha}_i - M_iK_{i1}J_i^T\tilde{\nu}_{ic},\end{aligned} \tag{17}$$

where $M_i\dot{\alpha}_i = M_i[\alpha_{i1} + \alpha_{i2}(\omega_{is} + \upsilon_{id})]$ with

$$\begin{aligned}\alpha_{i1} = &-K_{i1}\left(\nu_i - rSz_{i1} + J_i^T\hat{\nu}_{ic}\right)\\ &+ rSJ_i^T(-\hat{\nu}_{ic} - y_{id}^{\theta_i}\upsilon_{id}) - J_i^T\dot{\hat{\nu}}_{ic},\\ \alpha_{i2} = &K_{i1}J_i^T y_{id}^{\theta_i} + J_i^T y_{id}^{\theta_i^2}\upsilon_{id}.\end{aligned}$$

Define the second scalar function as

$$V_{i2} = V_{i1} + \frac{1}{2}z_{i2}^T M_i z_{i2}, \tag{18}$$

whose time derivative with (17) can be written as

$$\begin{aligned}\dot{V}_{i2} = &-z_{i1}^T K_{i1}z_{i1} + z_{i1}^T J_i^T(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}) - z_{i2}^T M_i\alpha_{i2}\omega_{is}\\ &+ z_{i2}^T[z_{i1} - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i - \Delta_i(\nu_i)\\ &- M_i(\alpha_{i1} + \alpha_{i2}\upsilon_{id}) + \tau_i + \tau_{iw}(t)]\\ &- z_{i2}^T M_iK_{i1}J_i^T\tilde{\nu}_{ic}.\end{aligned} \tag{19}$$

Then,

$$\begin{aligned}\dot{V}_{i2} \leq & -z_{i1}^T K_{i1} z_{i1} + z_{i1}^T J_i{}^T\big(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}\big) \\ & - z_{i2}^T M_i \alpha_{i2}\omega_{is} + z_{i2}^T\big[z_{i1} - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i \\ & - \Delta_i(\nu_i) - M_i(\alpha_{i1} + \alpha_{i2}v_{id}) + \tau_i\big] \\ & + \big|z_{i2}^T\big|\,\tau_{iwM} - z_{i2}^T M_i K_{i1} J_i^T \tilde{\nu}_{ic}. \end{aligned} \tag{20}$$

Using the inequality $|\chi_1| - \chi_1 \tanh(\chi_1/\chi_2) \leq 0.2478\chi_2$ with $\chi_1, \chi_2 \in \mathbb{R}$, one has

$$\begin{aligned}\dot{V}_{i2} \leq & -z_{i1}^T K_{i1} z_{i1} + z_{i1}^T J_i{}^T(-\tilde{\nu}_{ic} - y_{id}^{\theta_i}\omega_{is}) - z_{i2}^T M_i \alpha_{i2}\omega_{is} \\ & + z_{i2}^T[z_{i1} - f_i(\cdot) + \tau_i] + 0.2478\delta_i^T \tau_{iwM} \\ & - z_{i2}^T M_i K_{i1} J_i^T \tilde{\nu}_{ic}. \end{aligned} \tag{21}$$

where $\delta_i = [\delta_{i1}, \delta_{i2}, \delta_{i3}]^T$ with δ_{i1}, δ_{i2}, and δ_{i3} being positive constants; $f_i(\cdot)$ is written as

$$\begin{aligned} f_i(\cdot) = & C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + \Delta_i(\nu_i) + M_i(\alpha_{i1} + \alpha_{i2}v_{id}) \\ & - \tanh\left(\frac{z_{i2}^T}{\delta_i{}^T}\right)\tau_{iwM}, \end{aligned} \tag{22}$$

where $\tanh(\frac{z_{i2}^T}{\delta_i{}^T}) = \mathrm{diag}[\tanh\frac{(z_{i2}^T)_1}{\delta_{i1}}, \tanh\frac{(z_{i2}^T)_2}{\delta_{i2}}, \tanh\frac{(z_{i2}^T)_3}{\delta_{i3}}]$.

Consider a desired control law τ_i as

$$\tau_i = -z_{i1} - K_{i2} z_{i2} + f_i(\cdot), \tag{23}$$

where $K_{i2} \in \mathbb{R}^{3\times3}$ is a diagonal matrix and its diagonal elements are positive constants. In practice, $f_i(\cdot)$ is very hard to obtain accurately. Hence, the controller (23) cannot be implemented. To overcome this problem, a NN is employed to approximate $f_i(\cdot)$ as follows:

$$f_i(\cdot) = W_i^T \sigma(\xi_i) + \varepsilon_i, \tag{24}$$

where $\xi_i = [1, \nu_i^T, \alpha_{i1}^T, \alpha_{i2}^T]^T \in \mathbb{R}^{10}$ is the NN input; W_i is the NN weight, $\|W_i\|_F \leq W_{iM}$; ε_i is the approximation error satisfying $\|\varepsilon_i\| \leq \varepsilon_{iM}$ with ε_{iM} being a positive constant.

Select a control law τ_i as

$$\tau_i = -z_{i1} - K_{i2} z_{i2} + \hat{W}_i^T \sigma(\xi_i), \tag{25}$$

where $\hat{W}_i$ is an estimate of W_i and its update law will be specified later. Substituting (25) into (21) yields

$$\begin{aligned}\dot{V}_{i2} \leq & -z_{i1}^T K_{i1} z_{i1} - z_{i2}^T K_{i2} z_{i2} - \mu_i \omega_{is} \\ & + (z_{i1}^T J_i^T + z_{i2}^T M_i K_{i1} J_i^T)\tilde{\nu}_{ic} \\ & + z_{i2}^T\big[\tilde{W}_i{}^T \sigma(\xi_i) - \varepsilon_i\big] + 0.2478\delta_i{}^T \tau_{iwM}, \end{aligned} \tag{26}$$

where $\mu_i = z_{i1}^T J_i{}^T y_{id}^{\theta_i} + z_{i2}^T M_i \alpha_{i2}$ and $\tilde{W}_i = \hat{W}_i - W_i$.

Define the third scalar function as

$$V_{i3} = V_{i2} + \frac{1}{2} tr(\tilde{W}_i^T \Gamma_W^{-1} \tilde{W}_i). \tag{27}$$

Its time derivative along (26) is

$$\begin{aligned}\dot{V}_{i3} \leq & -z_{i1}^T K_{i1} z_{i1} - z_{i2}^T K_{i2} z_{i2} - \mu_i \omega_{is} \\ & + \big(z_{i1}^T J_i^T + z_{i2}^T M_i K_{i1} J_i^T\big)\tilde{\nu}_{ic} + z_{i2}^T\big[\tilde{W}_i^T \sigma(\xi_i) - \varepsilon_i\big] \\ & + 0.2478\delta_i{}^T \tau_{iwM} + tr(\tilde{W}_i^T \Gamma_W^{-1} \dot{\hat{W}}_i). \end{aligned}$$

Design the NN adaptive law as

$$\dot{\hat{W}}_i = \Gamma_W[-\sigma(\xi_i) z_{i2}^T - k_W \hat{W}_i], \tag{28}$$

where $\Gamma_W \in \mathbb{R}$ and $k_W \in \mathbb{R}$ are positive constants. Then, $\dot{V}_{i3}$ can be rewritten as

$$\begin{aligned}\dot{V}_{i3} \leq & -z_{i1}^T K_{i1} z_{i1} - z_{i2}^T K_{i2} z_{i2} - \mu_i \omega_{is} \\ & + (z_{i1}^T J_i^T + z_{i2}^T M_i K_{i1} J_i^T)\tilde{\nu}_{ic} - k_W tr(\tilde{W}_i^T \hat{W}_i) \\ & + 0.2478\delta_i{}^T \tau_{iwM} + z_{i2}^T \varepsilon_i. \end{aligned} \tag{29}$$

Now, it is the time to design the update law for $\hat{\nu}_{ic}$. Define the fourth scalar function

$$V_{i4} = V_{i3} + \frac{1}{2}\tilde{\nu}_{ic}^T \Gamma_c^{-1} \tilde{\nu}_{ic}, \tag{30}$$

and its time derivative with (29) is written as

$$\begin{aligned}\dot{V}_{i4} \leq & -z_{i1}^T K_{i1} z_{i1} - z_{i2}^T K_{i2} z_{i2} - \mu_i \omega_{is} \\ & + (z_{i1}^T J_i^T + z_{i2}^T M_i K_{i1} J_i^T)\tilde{\nu}_{ic} - k_W tr\big(\tilde{W}_i^T \hat{W}_i\big) \\ & + 0.2478\delta_i{}^T \tau_{iwM} + z_{i2}^T \varepsilon_i + \tilde{\nu}_{ic}^T \Gamma_c^{-1} \dot{\hat{\nu}}_{ic}. \end{aligned}$$

Consider the following ocean current model update law

$$\dot{\hat{\nu}}_{ic} = -\Gamma_c[(J_i z_{i1} + J_i K_{i1} M_i z_{i2}) + k_c \hat{\nu}_{ic}], \tag{31}$$

where $\Gamma_c \in \mathbb{R}$ and $k_c \in \mathbb{R}$ are positive constants. Then, the time derivative of V_{i4} can be rewritten as

$$\begin{aligned}\dot{V}_{i4} \leq & -z_{i1}^T K_{i1} z_{i1} - z_{i2}^T K_{i2} z_{i2} - \mu_i \omega_{is} - k_c \tilde{\nu}_{ic}^T \hat{\nu}_{ic} \\ & - k_W tr\big(\tilde{W}_i^T \hat{W}_i\big) + 0.2478\delta_i{}^T \tau_{iwM} + z_{i2}^T \varepsilon_i. \end{aligned} \tag{32}$$

3.2. *Cooperative path-following design*

Consider the fifth scalar function as

$$V = \frac{1}{2}\theta^T L \theta + \frac{1}{2}\gamma^T \gamma + \sum_{i=1}^{n} V_{i4}, \tag{33}$$

where $\theta = [\theta_1, \ldots, \theta_n]^T \in \mathbb{R}^n$, $\gamma = [\gamma_1, \ldots, \gamma_n]^T \in \mathbb{R}^n$ is an auxiliary state to be specified later. The time derivative of

V along (32) is given by

$$\dot{V} \le \theta^T L\dot{\theta} + \gamma^T\dot{\gamma} + \sum_{i=1}^{n}[-z_{i1}^T K_{i1}z_{i1} - z_{i2}^T K_{i2}z_{i2} - \mu_i\omega_{is} - k_c\tilde{\upsilon}_{ic}^T\hat{\upsilon}_{ic} - k_W tr(\tilde{W}_i^T\hat{W}_i) + 0.2478\delta_i{}^T\tau_{iwM} + z_{i2}^T\varepsilon_i].$$

Cooperative control strategies for multiple vehicles are supported by the communications network over which the vehicles exchange information. Because more than one vehicle is involved, the along-path speed and the path variables should be synchronised to each vehicle, in order to maintain the desired formation. From a graph theoretical point of view, each vehicle is represented by a vertex, and a bidirectional communication link between two vehicles is represented by an edge between the corresponding vertices. Let $\mathcal{N}_i$ be the set of vehicles with which the vehicle i communicates. In order to satisfy the constraints imposed by the communication network, the synchronisation control law for vehicle i can only depend on its own local states and on the information exchanged with its neighbours, so that the synchronisation control law is decentralised. The amount of communications is reduced effectively due to the decentralised synchronisation control law. To this end, choose the following decentralised synchronisation control law with an auxiliary state γ_i as

$$\omega_{is} = -\kappa_{i1}^{-1}\left[\sum_{j\in\mathcal{N}_i} a_{ij}(\theta_i - \theta_j) + \mu_i\right] - \gamma_i,$$
$$\dot{\gamma}_i = -(\kappa_{i1} + \kappa_{i2})\gamma_i - \sum_{j\in\mathcal{N}_i} a_{ij}(\theta_i - \theta_j) - \mu_i, \quad (34)$$

where $\kappa_{i1} \in \mathbb{R}$ and $\kappa_{i2} \in \mathbb{R}$ are positive constants. Let $\omega_s = [\omega_{1s}, \ldots, \omega_{ns}]^T \in \mathbb{R}^n$, $\mu = [\mu_1, \ldots, \mu_n]^T \in \mathbb{R}^n$, $\mathcal{K}_1 = \mathrm{diag}[\kappa_{i1}] \in \mathbb{R}^{n\times n}$, $\mathcal{K}_2 = \mathrm{diag}[\kappa_{i2}] \in \mathbb{R}^{n\times n}$. Then, (34) can be written as

$$\dot{\theta} = \upsilon_{id}\mathbf{1}_n - \mathcal{K}_1^{-1}(L\theta + \mu) - \gamma,$$
$$\dot{\gamma} = -(\mathcal{K}_1 + \mathcal{K}_2)\gamma - L\theta - \mu. \quad (35)$$

Substituting (35) into $\dot{V}$, one has

$$\dot{V} \le -\omega_s^T\mathcal{K}_1\omega_s - \gamma^T\mathcal{K}_2\gamma + \sum_{i=1}^{n}[-z_{i1}^T K_{i1}z_{i1} - z_{i2}^T K_{i2}z_{i2} - k_c\tilde{\upsilon}_{ic}^T\hat{\upsilon}_{ic} - k_W tr(\tilde{W}_i^T\hat{W}_i) + 0.2478\delta_i{}^T\tau_{iwM} + z_{i2}^T\varepsilon_i]. \quad (36)$$

3.3. Stability analysis

Theorem 1: *Consider a network of marine surface vehicles with the vehicles dynamics given in* (2) *and* (3)*. Suppose the communication network is undirected and connected. Design the control law* (25) *with the NN adaptive law* (28)*, ocean current model update law* (31)*, and decentralised synchronisation control law* (34)*. Select the control parameter $\lambda_{\min}(K_{i2}) > \frac{1}{2}$. Then, for bounded initial conditions, all signals in the closed-loop system are SGUUB, and the path-following error $y_i - y_{id}$, along-path speed tracking error $\dot{\theta}_i - \upsilon_{id}$, and path variable coordination error $\theta_i - \theta_j$ satisfy*

$$\lim_{t\to\infty} \|y_i - y_{id}\| \le \epsilon_{i1}, \quad (37)$$

$$\lim_{t\to\infty} \left\|\dot{\theta}_i - \upsilon_{id}\right\| \le \epsilon_{i2}, \quad (38)$$

$$\lim_{t\to\infty} \left\|\theta_i - \theta_j\right\| \le \epsilon_{i3}, \quad (39)$$

where $\epsilon_{i1}, \epsilon_{i2}, \epsilon_{i3} \in \mathbb{R}$ are some small constants.

Proof: Consider the scalar function (33) and use the following Young's inequalities

$$-k_W tr(\tilde{W}_i^T\hat{W}_i) \le -\frac{k_W}{2}\|\tilde{W}_i\|_F^2 + \frac{k_W}{2}\|W_{iM}\|_F^2, \quad (40)$$

$$-k_c\tilde{\upsilon}_{ic}^T\hat{\upsilon}_{ic} \le -\frac{k_c}{2}\|\tilde{\upsilon}_{ic}\|^2 + \frac{k_c}{2}\|\upsilon_{ic}\|^2, \quad (41)$$

$$\left|z_{i2}^T\varepsilon_i\right| \le \frac{1}{2}\left\|z_{i2}^T\right\|^2 + \frac{1}{2}\|\varepsilon_{iM}\|^2. \quad (42)$$

Then (36) can be rewritten as

$$\dot{V} \le -\lambda_{\min}(\mathcal{K}_1)\|\omega_s\|^2 - \lambda_{\min}(\mathcal{K}_2)\|\gamma\|^2 + \sum_{i=1}^{n}\left[-\lambda_{\min}(K_{i1})\|z_{i1}\|^2 - \left(\lambda_{\min}(K_{i2}) - \frac{1}{2}\right)\|z_{i2}\|^2 - \frac{k_W}{2}\|\tilde{W}_i\|_F^2 - \frac{k_c}{2}\|\tilde{\upsilon}_{ic}\|^2 + H_i\right], \quad (43)$$

where $H_i = 0.2478\delta_i{}^T\tau_{iwM} + \frac{k_W}{2}\|W_{iM}\|_F^2 + \frac{k_c}{2}\|\upsilon_{ic}\|^2 + \frac{1}{2}\|\varepsilon_{iM}\|^2$. Note that, either $\|\omega_s\| > \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(\mathcal{K}_1)}}$, or $\|\gamma\| > \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(\mathcal{K}_2)}}$, or $\|z_{i1}\| > \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(K_{i1})}}$, or $\|z_{i2}\| > \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(K_{i2})-\frac{1}{2}}}$, or $\left\|\tilde{W}_i\right\|_F > \sqrt{\frac{2\sum_{i=1}^n H_i}{k_W}}$, or $\|\tilde{\upsilon}_{ic}\| > \sqrt{\frac{2\sum_{i=1}^n H_i}{k_c}}$ renders $\dot{V} < 0$. This proves that all signals in the closed-loop system are SGUUB. Moreover, when $t \to \infty$, the path-following error $y_i - y_{id}$ and along-path speed tracking error $\dot{\theta}_i - \upsilon_{id}$ satisfy

$$\lim_{t\to\infty} \|y_i - y_{id}\| \le \epsilon_{i1}, \quad (44)$$

$$\lim_{t\to\infty} \left\|\dot{\theta}_i - \upsilon_{id}\right\| \le \epsilon_{i2}, \quad (45)$$

where $\epsilon_{i1} = \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(K_{i1})}}$, $\epsilon_{i2} = \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(\mathcal{K}_1)}}$. Let $s = L\theta$, from (35) and (45) we have

$$\begin{aligned}\|s\| &= \|-\mu + \mathcal{K}_1\omega_s - \mathcal{K}_1\gamma\| \\ &\leq \|\mu\| + \lambda_{\max}(\mathcal{K}_1)\|\omega_s\| + \lambda_{\max}(\mathcal{K}_1)\|\gamma\| \\ &\leq \epsilon_{is},\end{aligned} \tag{46}$$

where $\epsilon_{is} = \sqrt{\epsilon_{i1}^2(y_{idM}^{\theta_i})^2 + \lambda_{\max}(K_{i1})\lambda_{\max}(M_i)} \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(K_{i2})-\frac{1}{2}}(y_{idM}^{\theta_i})^2} + v_{id}\lambda_{\max}(M_i)\sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(K_{i2})-\frac{1}{2}}(y_{idM}^{\theta_i^2})^2} + \lambda_{\max}(\mathcal{K}_1)(\sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(\mathcal{K}_1)}} + \sqrt{\frac{\sum_{i=1}^n H_i}{\lambda_{\min}(\mathcal{K}_2)}})$. In addition, since $\frac{1}{2}\lambda_2(L)\|\theta - Ave(\theta)\mathbf{1}_n\|^2 \leq \frac{1}{2}\theta^T L\theta$, where $Ave(\theta) = \frac{1}{n}\sum_{i=1}^n \theta_i$ (Ceragioli, De Persis, & Frasca, 2011), then by Lemma 1 we further obtain

$$\frac{1}{2}\lambda_2(L)\|\theta - Ave(\theta)\mathbf{1}_n\|^2 \leq \frac{1}{2}s^T Ps. \tag{47}$$

It follows from (46) that

$$\lim_{t\to\infty} \left\|\theta_i - \theta_j\right\| \leq \epsilon_{i3}, \tag{48}$$

where $\epsilon_{i3} = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_2(L)}}\epsilon_{is}$, i.e., $\theta_i \to \theta_j \to Ave(\theta)$. Furthermore, since ω_s is bounded, from (38) we know that $\dot{\theta}$ is bounded. Thus, θ_i is also bounded.

This concludes the proof. □

Remark 2: The uncertainties existing in Almeida et al. (2010) and Fuyuki, Satoshi, and Minoru (2004) are considered as model parametric uncertainty, which takes the form of $\phi^T F(\cdot)$, where ϕ is unknown constant vector, $F(\cdot)$ is a known nonlinear function. In our design, the uncertainties including model parametric uncertainty, unmodelled dynamic and time-varying ocean disturbances induced by wind and wave are considered together. Thus, the adaptive methods given in Almeida et al. (2010) and Fuyuki et al. (2004) cannot deal with the case when $f_i(\cdot)$ are totally unknown.

4. Output feedback controller design

4.1. Observer design

In the previous section, the proposed algorithm requires that all states of the vehicles are measured, which may not be available in many circumstances due to technical reasons. The objective of this section is to design a cooperative path-following controller without measuring ν_i. To move on, let us recall the observer structure. In the earlier studies, many linear observers had been developed, such as Ge and Wang (2002) and Tee and Ge (2006). In order to handle the nonlinear case, nonlinear adaptive observers have been developed (Fossen, 2002; Tong et al., 2011; Young & Lewis, 1999). Inspired by Tong et al. (2011), Tong, Li, Feng, and Li (2011), Tong et al. (2012), Tong and Li (2013) and Young and Lewis (1999), to facilitate the observer design, rewrite the dynamical model (2) and (3) as

$$\dot{\eta}_i = J_i\nu_i + \nu_{ic}, \tag{49}$$

$$\dot{\nu}_i = M_i^{-1}\tau_i + f_{i1o}(\cdot) + M_i^{-1}\tau_{iw}(t), \tag{50}$$

where $f_{i1o}(\cdot) = -M_i^{-1}C_i(\nu_i)\nu_i - M_i^{-1}D_i(\nu_i)\nu_i - M_i^{-1}\Delta_i(\nu_i)$ is an unknown function.

Let $\tilde{\eta}_i = \hat{\eta}_i - \eta_i$, $\tilde{\nu}_i = \hat{\nu}_i - \nu_i$, $\tilde{\nu}_{ic} = \hat{\nu}_{ic} - \nu_{ic}$. Here, the proposed observer has the form of

$$\dot{\hat{Z}}_{i1} = J_i\hat{\nu}_i - k_{i1o}\tilde{\eta}_i + \hat{\nu}_{ic}, \tag{51}$$

$$\dot{\hat{Z}}_{i2} = \hat{W}_{i1o}^T\sigma(\hat{\xi}_{i1o}) + M_i^{-1}\tau_i - K_{i2o}J_i^T\tilde{\eta}_i, \tag{52}$$

where $k_{i1o} \in \mathbb{R}$ is a positive constant, $K_{i2o} \in \mathbb{R}^{3\times3}$ is a diagonal matrix and its diagonal elements are positive constants, $\hat{\xi}_{i1o} = [1, \hat{\nu}_i^T, \hat{\eta}_i^T, \tilde{\eta}_i^T]^T$, $\|W_{i1o}\|_F \leq W_{i1M}$, and the estimates $\hat{\eta}_i$ and $\hat{\nu}_i$ are defined as

$$\hat{\eta}_i = \hat{Z}_{i1}, \tag{53}$$

$$\hat{\nu}_i = \hat{Z}_{i2} - k_{i3o}J_i^T\tilde{\eta}_i, \tag{54}$$

where $k_{i3o} \in \mathbb{R}$ is a positive constant. Using the estimate states in (53) and (54), the dynamic equations of the observer can be written in terms of $\hat{\eta}_i$, $\hat{\nu}_i$, as

$$\dot{\hat{\eta}}_i = J_i\hat{\nu}_i - k_{i1o}\tilde{\eta}_i + \hat{\nu}_{ic}, \tag{55}$$

$$\begin{aligned}\dot{\hat{\nu}}_i = {} & \hat{W}_{i1o}^T\sigma(\hat{\xi}_{i1o}) + M_i^{-1}\tau_i - K_{i2o}J_i^T\tilde{\eta}_i \\ & - k_{i3o}\dot{J}_i^T\tilde{\eta}_i - k_{i3o}J_i^T\dot{\tilde{\eta}}_i.\end{aligned} \tag{56}$$

The adaptive law of $\hat{\nu}_{ic}$ is designed as

$$\dot{\hat{\nu}}_{ic} = -K_{i2o}\tilde{\eta}_i - k_c\hat{\nu}_{ic}. \tag{57}$$

The observer error dynamics is obtained by subtracting (49) and (50) from (55) and (56).

$$\dot{\tilde{\eta}}_i = J_i\tilde{\nu}_i - k_{i1o}\tilde{\eta}_i + \tilde{\nu}_{ic}, \tag{58}$$

$$\begin{aligned}\dot{\tilde{\nu}}_i = {} & \hat{W}_{i1o}^T\sigma(\hat{\xi}_{i1o}) - M_i^{-1}\tau_{iw}(t) - K_{i2o}J_i^T\tilde{\eta}_i \\ & - k_{i3o}J_i^T\dot{\tilde{\eta}}_i - f_{i2o}(\cdot),\end{aligned} \tag{59}$$

where $f_{i2o}(\cdot) = f_{i1o} + k_{i3o}\dot{J}_i^T\tilde{\eta}_i$. Let $f_{i2o}(\cdot)$ be approximated by an NN as follows:

$$f_{i2o} = W_{i1o}^T\sigma(\xi_{i1o}) + \varepsilon_{i1o}, \tag{60}$$

where $\|\varepsilon_{i1o}\| \leq \varepsilon_{i1M}$ with ε_{i1M} is a positive constant, $\|W_{i1o}\| \leq W_{i1M}$. $\xi_{i1o} = [1, \nu_i^T, \tilde{\eta}_i^T]^T$. Then, we have the observer error dynamics by adding and subtracting $W_{i1o}^T\sigma(\hat{\xi}_{i1o})$.

$$\dot{\tilde{\eta}}_i = J_i\tilde{\nu}_i - k_{i1o}\tilde{\eta}_i + \tilde{\nu}_{ic}, \tag{61}$$

$$\begin{aligned}\dot{\tilde{\nu}}_i = &-k_{i3o}\tilde{\nu}_i - K_{i2o}J_i^T\tilde{\eta}_i + k_{i1o}k_{i3o}J_i^T\tilde{\eta}_i - k_{i3o}J_i^T\tilde{\nu}_{ic}\\ &- M_i^{-1}\tau_{iw}(t) + \tilde{W}_{i1o}^T\sigma(\hat{\xi}_{i1o}) - \varepsilon_{i1o} - w_{io},\end{aligned} \tag{62}$$

where $w_{io} = W_{i1o}^T[\sigma(\xi_{i1o}) - \sigma(\hat{\xi}_{i1o})]$, $\|w_{io}\| \leq w_{ioM}$ (Young & Lewis, 1999) with w_{ioM} being a positive constant.

Theorem 2: *Consider the NN observer defined in* (55) *and* (56)*, ocean current adaptive law* (57) *and let the NN adaptive law be*

$$\dot{\hat{W}}_{i1o} = \Gamma_W[-\sigma(\hat{\xi}_{i1o})\tilde{\eta}_i^T - k_W\hat{W}_{i1o}]. \tag{63}$$

Select the control parameters such that

$$k_{i1o}\lambda_{\min}(K_{i2o}) > \frac{k_{i1o}}{2} + \frac{1}{2}, k_{i3o} > 1, k_W > \ell + \frac{1}{2}, \frac{k_c}{2} > \frac{1}{2}.$$

Then, the state estimate errors $\tilde{\eta}_i$, $\tilde{\nu}_i$, ocean current estimate error $\tilde{\nu}_{ic}$, and NN observer weight errors $\tilde{W}_{i1o}$ are SGUUB.

Proof: Consider the following Lyapunov candidate function:

$$\begin{aligned}V_{io} = &\frac{1}{2}\tilde{\eta}_i^TK_{i2o}\tilde{\eta}_i + \frac{1}{2}\tilde{\nu}_i^T\tilde{\nu}_i + \frac{1}{2}tr(\tilde{W}_{i1o}^T\Gamma_W^{-1}\tilde{W}_{i1o})\\ &+ \frac{1}{2}\tilde{\nu}_{ic}^T\tilde{\nu}_{ic}.\end{aligned} \tag{64}$$

Differentiating (64) along (57), (61), (62) and (63), using $\|\sigma(\xi_{i1o})\| \leq \sqrt{\ell}$ (Young and Lewis, 1999) and Young's inequalities, we obtain

$$\begin{aligned}\dot{V}_{io} \leq &-\left[k_{i1o}\lambda_{\min}(K_{i2o}) - \frac{k_{i1o}}{2} - \frac{1}{2}\right]\|\tilde{\eta}_i\|^2\\ &- (2k_{i3o} - 2)\|\tilde{\nu}_i\|^2 - \left(k_W - \ell - \frac{1}{2}\right)\left\|\tilde{W}_{i1o}\right\|_F^2\\ &- \left(\frac{k_c}{2} - \frac{1}{2}\right)\|\tilde{\nu}_{ic}\|^2 + \Pi_i,\end{aligned} \tag{65}$$

where $\Pi_i = \frac{1}{2}\lambda_{\min}{}^2(M_i^{-1})\tau_{iwM}{}^2 + \frac{1}{2}\varepsilon_{i1M}^2 + \frac{1}{2}w_{ioM}^2 + \frac{1}{2}k_WW_{i1M}^2 + \frac{1}{2}k_c\|\nu_{ic}\|^2$. Note that, either $\|\tilde{\eta}_i\| > \sqrt{\frac{\Pi_i}{k_{i1o}\lambda_{\min}(K_{i2o}) - \frac{k_{i1o}}{2} - \frac{1}{2}}}$, or $\|\tilde{\nu}_i\| > \sqrt{\frac{\Pi_i}{2k_{i3o} - 2}}$, or $\left\|\tilde{W}_{i1o}\right\|_F > \sqrt{\frac{\Pi_i}{k_W - \ell - \frac{1}{2}}}$, or $\|\tilde{\nu}_{ic}\|_F > \sqrt{\frac{\Pi_i}{\frac{k_c}{2} - \frac{1}{2}}}$ renders $\dot{V}_{io} < 0$. It follows that the state estimate errors $\tilde{\eta}_i$, $\tilde{\nu}_i$, ocean current estimate error $\tilde{\nu}_{ic}$, and NN observer weight errors $\tilde{W}_{i1o}$ are SGUUB.

This concludes the proof. □

Remark 3: Note that W_{i1o} and $\sigma(\cdot)$ are bounded by W_{i1M} and $\sqrt{\ell}$, respectively. Thus, $\|w_{io}\| \leq w_{ioM}$ is a reasonable condition (Young & Lewis, 1999).

4.2. *Individual path-following design*

The aim of this section is to design an observer-based individual path-following controller where the observer has been developed in the previous section.

Step 1. Consider the marine surface vehicle dynamical system (3) and (4). Similar to Section 3.1, (12) can be expressed by

$$\begin{aligned}\dot{z}_{i1} = &-rSz_{i1} + \nu_i + J_i{}^T\left(\hat{\nu}_{ico} - y_{id}^{\theta_i}v_{id}\right)\\ &+ J_i{}^T\left(\tilde{\nu}_{ico} - y_{id}^{\theta_i}\omega_{is}\right),\end{aligned} \tag{66}$$

where $\hat{\nu}_{ico}$ is an estimate of ν_{ico}. Consider the first scalar function as

$$V_{i1o} = \frac{1}{2}z_{i1}^Tz_{i1}, \tag{67}$$

whose time derivative along (66) is given by

$$\begin{aligned}\dot{V}_{i1o} = &z_{i1}^T[-rSz_{i1} + \nu_i + J_i{}^T(\hat{\nu}_{ico} - y_{id}^{\theta_i}v_{id})\\ &+ J_i{}^T(\tilde{\nu}_{ico} - y_{id}^{\theta_i}\omega_{is})].\end{aligned}$$

Since ν_i cannot be used for feedback, we define $\hat{z}_{i2} = \hat{\nu}_i - \alpha_{io}$, where α_{io} is a virtual control and is designed as

$$\alpha_{io} = -K_{i1o}z_{i1} - J_i{}^T(\hat{\nu}_{ico} - y_{id}^{\theta_i}v_{id}), \tag{68}$$

where $K_{i1o} \in \mathbb{R}^{3\times3}$ is a diagonal matrix and its diagonal elements are positive constants.

Then, $\dot{V}_{i1o}$ can be rewritten as

$$\begin{aligned}\dot{V}_{i1o} = &-z_{i1}^TK_{i1o}z_{i1} + z_{i1}^T(\hat{z}_{i2} - \tilde{\nu}_i)\\ &+ z_{i1}^TJ_i^T(\tilde{\nu}_{ico} - y_{id}^{\theta_i}\omega_{is}).\end{aligned} \tag{69}$$

Step 2. Noting that $\hat{z}_{i2} = (\nu_i - \alpha_{io}) + \tilde{\nu}_i$, taking the time derivative of $\hat{z}_{i2}$, we have

$$\begin{aligned}M_i\dot{\hat{z}}_{i2} = &\tau_i - C_i(\nu_i)\nu_i - D_i(\nu_i)\nu_i\\ &- \Delta_i(\nu_i) + \tau_{iw}(t) - M_i\dot{\alpha}_{io} + M_i\dot{\tilde{\nu}}_i.\end{aligned} \tag{70}$$

Define the second scalar function as

$$V_{i2o} = V_{i1o} + \frac{1}{2}\hat{z}_{i2}^TM_i\hat{z}_{i2}, \tag{71}$$

whose time derivative along (56) satisfies

$$\begin{aligned}\dot{V}_{i2o} &= -z_{i1}^T K_{i1o} z_{i1} - z_{i1}^T \tilde{\nu}_i + (z_{i1}^T J_i^T + \hat{z}_{i2}^T M_i K_{i1o} J_i^T)\tilde{\nu}_{ico} \\ &\quad - \mu_{io}\omega_{is} + \hat{z}_{i2}^T[\tau_i + z_{i1} - C_i(\nu_i)\nu_i \\ &\quad - D_i(\nu_i)\nu_i - \Delta_i(\nu_i) \\ &\quad + \tau_{iw}(t) - M_i(\alpha_{i1o} + \alpha_{i2o} v_{id}) + M_i \dot{\hat{\tilde{\nu}}}_i], \end{aligned} \tag{72}$$

where $\alpha_{i1o} = -K_{i1o}(\nu_i - rSz_{i1} + J_i^T \hat{\nu}_{ico}) + rSJ_i^T(\hat{\nu}_{ico} - y_{id}^{\theta_i} v_{id}) - J_i^T \dot{\hat{\nu}}_{ico}$, $\alpha_{i2o} = K_{i1o} J_i^T y_{id}^{\theta_i} + J_i^T y_{id}^{\theta_i^2} v_{id}$, $\mu_{io} = z_{i1}^T J_i{}^T y_{id}^{\theta_i} + \hat{z}_{i2}^T M_i \alpha_{i2o}$. The desired control law is chosen as

$$\tau_i = -z_{i1} - K_{i2o}\hat{z}_{i2} + f_{i3o}(\cdot), \tag{73}$$

where $f_{i3o}(\cdot) = C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + \Delta_i(\nu_i) + M_i(\alpha_{i1o} + \alpha_{i2o} v_{id}) - M_i \dot{\hat{\tilde{\nu}}}_i - \tanh(\frac{\hat{z}_{i2}^T}{\delta_i{}^T})\tau_{iwM}$. However, the desired control law defined in (73) cannot be implemented since $f_{i3o}(\cdot)$ is not available. As a consequence, let $f_{i3o}(\cdot)$ be approximated by an NN as follows:

$$f_{i3o}(\cdot) = W_{i2o}^T \sigma(\hat{\xi}_{i2o}) + \varepsilon_{i2o}, \tag{74}$$

where $\hat{\xi}_{i2o} = [1, \eta_i{}^T, \nu_i^T, \hat{\eta}_i^T, \hat{\nu}_i^T, \alpha_{i1o}^T, \alpha_{i2o}^T]^T$, $\|W_{i2o}\|_F \le W_{i2M}$, $\|\varepsilon_{i2o}\| \le \varepsilon_{i2M}$ with ε_{i2M} a positive constant. Select the control law as

$$\tau_i = -z_{i1} - K_{i2o}\hat{z}_{i2} + \hat{W}_{i2o}^T \sigma(\hat{\xi}_{i2o}), \tag{75}$$

Then, we have

$$\begin{aligned}\dot{V}_{i2o} &\le -z_{i1}^T K_{i1o} z_{i1} - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i \\ &\quad - \mu_{io}\omega_{is} + (z_{i1}^T J_i^T + \hat{z}_{i2}^T M_i K_{i1o} J_i^T)\tilde{\nu}_{ico} \\ &\quad - \hat{z}_{i2}^T[\hat{W}_{i2o}^T \sigma(\hat{\xi}_{i2o}) - W_{i2o}^T \sigma(\hat{\xi}_{i2o})] \\ &\quad + 0.2478\delta_i{}^T \tau_{iwM} + \hat{z}_{i2}^T \varepsilon_{i2o}. \end{aligned} \tag{76}$$

Define the third scalar function as

$$V_{i3o} = V_{i2o} + \frac{1}{2} tr(\tilde{W}_{i2o}^T \Gamma_W^{-1} \tilde{W}_{i2o}), \tag{77}$$

whose time derivative along (76) satisfies

$$\begin{aligned}\dot{V}_{i3o} &\le -z_{i1}^T K_{i1o} z_{i1} - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i \\ &\quad - \mu_{io}\omega_{is} + (z_{i1}^T J_i^T + \hat{z}_{i2}^T M_i K_{i1o} J_i^T)\tilde{\nu}_{ico} \\ &\quad - \hat{z}_{i2}^T[\hat{W}_{i2o}^T \sigma(\hat{\xi}_{i2o}) - W_{i2o}^T \sigma(\hat{\xi}_{i2o})] \\ &\quad + 0.2478\delta_i{}^T \tau_{iwM} + \hat{z}_{i2}^T \varepsilon_{i2o} + tr(\tilde{W}_{i2o}^T \Gamma_W^{-1} \dot{\hat{W}}_{i2o}). \end{aligned}$$

Design the NN adaptive law

$$\dot{\hat{W}}_{i2o} = \Gamma_W[-\sigma(\hat{\xi}_{i2o})\hat{z}_{i2}^T - k_W \hat{W}_{i2o}]. \tag{78}$$

Then, we obtain

$$\begin{aligned}\dot{V}_{i3o} &\le -z_{i1}^T K_{i1o} z_{i1} - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i \\ &\quad - \mu_{io}\omega_{is} + (z_{i1}^T J_i^T + \hat{z}_{i2}^T M_i K_{i1o} J_i^T)\tilde{\nu}_{ico} + \hat{z}_{i2}^T \varepsilon_{i2o} \\ &\quad - k_W tr(\tilde{W}_{i2o}^T \hat{W}_{i2o}) + 0.2478\delta_i{}^T \tau_{iwM}. \end{aligned} \tag{79}$$

Define the fourth scalar function as

$$V_{i4o} = V_{i3o} + \frac{1}{2}\tilde{\nu}_{ico}^T \Gamma_c^{-1} \tilde{\nu}_{ico}, \tag{80}$$

whose time derivative along (79) satisfies

$$\begin{aligned}\dot{V}_{i4o} &\le -z_{i1}^T K_{i1o} z_{i1} - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i \\ &\quad - \mu_{io}\omega_{is} + (z_{i1}^T J_i^T + \hat{z}_{i2}^T M_i K_{i1o} J_i^T)\tilde{\nu}_{ico} + \hat{z}_{i2}^T \varepsilon_{i2o} \\ &\quad - k_W tr(\tilde{W}_{i2o}^T \hat{W}_{i2o}) + 0.2478\delta_i{}^T \tau_{iwM} + \tilde{\nu}_{ico}^T \Gamma_c^{-1} \dot{\hat{\nu}}_{ico}. \end{aligned}$$

Consider the following ocean current model update law

$$\dot{\hat{\nu}}_{ico} = -\Gamma_c[(J_i z_{i1} + J_i K_{i1o} M_i \hat{z}_{i2}) + k_c \hat{\nu}_{ico}], \tag{81}$$

where $\Gamma_c, k_c \in \mathbb{R}$ are positive constants. Then, we obtain

$$\begin{aligned}\dot{V}_{i4o} &\le -z_{i1}^T K_{i1o} z_{i1} - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i \\ &\quad - \mu_{io}\omega_{is} - k_c \tilde{\nu}_{ico}^T \hat{\nu}_{ico} - k_W tr(\tilde{W}_{i2o}^T \hat{W}_{i2o}) \\ &\quad + 0.2478\delta_i{}^T \tau_{iwM} + \hat{z}_{i2}^T \varepsilon_{i2o}. \end{aligned} \tag{82}$$

4.3. *Cooperative path-following design*

Similar to Section 3.2, consider the fifth scalar function as

$$V_o = \frac{1}{2}\theta^T L\theta + \frac{1}{2}\gamma^T\gamma + \sum_{i=1}^{n} V_{i4o}. \tag{83}$$

Its time derivative along (82) and (34) is given by

$$\begin{aligned}\dot{V}_o &\le -\omega_s^T \mathcal{K}_1 \omega_s - \gamma^T \mathcal{K}_2 \gamma + \sum_{i=1}^{n}[-z_{i1}^T K_{i1o} z_{i1} \\ &\quad - \hat{z}_{i2}^T K_{i2o}\hat{z}_{i2} - z_{i1}^T \tilde{\nu}_i - k_c \tilde{\nu}_{ico}^T \hat{\nu}_{ico} - k_W tr(\tilde{W}_{i2o}^T \hat{W}_{i2o}) \\ &\quad + 0.2478\delta_i{}^T \tau_{iwM} + \hat{z}_{i2o}^T \varepsilon_{i2o}]. \end{aligned} \tag{84}$$

4.4. *Stability analysis*

Theorem 3: *Consider a network of marine surface vehicles with the vehicles dynamics given in* (2) *and* (3)*. Suppose the communication network is undirected and connected. Design the control law* (75)*, NN adaptive law* (78)*, ocean current model update law* (81)*, NN observer* ((55) *and* (56))*, NN adaptive law* (63)*, and decentralised synchronisation* *control law* (34). *Select the control parameters such that*

$$\lambda_{\min}(K_{i1o}) > \frac{1}{2}, \lambda_{\min}(K_{i2o}) > \frac{1}{2} + \frac{1}{2k_{i1o}},$$
$$k_{i3o} > \frac{5}{4}, \frac{k_c}{2} > \frac{1}{2}, k_W > \ell + \frac{1}{2}.$$

Then, for bounded initial conditions, all signals in the closed-loop system are SGUUB, and the path-following error $y_i - y_{id}$*, along-path speed tracking error* $\dot{\theta}_i - \upsilon_{id}$*, and path variable coordination error* $\theta_i - \theta_j$ *satisfy*

$$\lim_{t\to\infty} \|y_i - y_{id}\| \le \epsilon_{i1o}, \quad (85)$$

$$\lim_{t\to\infty} \|\dot{\theta}_i - \upsilon_{id}\| \le \epsilon_{i2o}, \quad (86)$$

$$\lim_{t\to\infty} \|\theta_i - \theta_j\| \le \epsilon_{i3o}, \quad (87)$$

where $\epsilon_{i1o}, \epsilon_{i2o}, \epsilon_{i3o} \in \mathbb{R}$ *are some small constants.*

Proof: Consider the following Lyapunov function candidate

$$V_{sum} = V_o + \sum_{i=1}^{n} V_{io}. \quad (88)$$

Differentiating (88) along (65) and (84), using Young's inequalities, we obtain

$$\begin{aligned}\dot{V}_{sum} \le &-\lambda_{\min}(\mathcal{K}_1)\|\omega_s\|^2 - \lambda_{\min}(\mathcal{K}_2)\|\gamma\|^2 \\ &+ \sum_{i=1}^{n}\Bigg\{-\left[\lambda_{\min}(K_{i1o}) - \frac{1}{2}\right]\|z_{i1}\|^2 \\ &-\left[\lambda_{\min}(K_{i2o}) - \frac{1}{2}\right]\|\hat{z}_{i2}\|^2 \\ &-\left[k_{i1o}\lambda_{\min}(K_{i2o}) - \frac{k_{i1o}}{2} - \frac{1}{2}\right]\|\tilde{\eta}_i\|^2 \\ &-\left(2k_{i3o} - \frac{5}{2}\right)\|\tilde{\nu}_i\|^2 \\ &-\frac{k_c}{2}\|\tilde{\nu}_{ico}\|^2 - \left(\frac{k_c}{2} - \frac{1}{2}\right)\|\tilde{\nu}_{ic}\|^2 - \frac{k_W}{2}\left\|\tilde{W}_{i2o}\right\|_F^2 \\ &-\left(k_W - \ell - \frac{1}{2}\right)\left\|\tilde{W}_{i1o}\right\|_F^2 + H_{io}\Bigg\}, \end{aligned} \quad (89)$$

where $H_{io} = 0.2478\delta_i{}^T\tau_{iwM} + \frac{k_W}{2}\|W_{i2o}\|_F^2 + \frac{k_c}{2}\|\nu_{ico}\|^2 + \frac{1}{2}\|\varepsilon_{i2M}\|^2 + \Pi_i$. Note that, either $\|\omega_s\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(\mathcal{K}_1)}}$, or $\|\gamma\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(\mathcal{K}_2)}}$, or $\|z_{i1}\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(K_{i10}) - \frac{1}{2}}}$, or $\|\hat{z}_{i2}\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(K_{i2o}) - \frac{1}{2}}}$, or $\|\tilde{\eta}_i\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{k_{i1o}\lambda_{\min}(K_{i2o}) - \frac{k_{i1o}}{2} - \frac{1}{2}}}$, or $\|\tilde{\nu}_i\| > \sqrt{\frac{\sum_{i=1}^n H_{io}}{2k_{i3o} - \frac{5}{2}}}$, or $\|\tilde{\nu}_{ico}\| > \sqrt{\frac{2\sum_{i=1}^n H_{io}}{k_c}}$, or $\|\tilde{\nu}_{ic}\| > \sqrt{\frac{2\sum_{i=1}^n H_{io}}{k_c - 1}}$, or $\left\|\tilde{W}_{i2o}\right\|_F > \sqrt{\frac{2\sum_{i=1}^n H_{io}}{k_W}}$, or $\left\|\tilde{W}_{i1o}\right\|_F > \sqrt{\frac{2\sum_{i=1}^n H_{io}}{k_W - \ell - \frac{1}{2}}}$ renders $\dot{V}_{sum} < 0$. This proves that all signals in the closed-loop system are SGUUB. Similar to Section 3.3, we have

$$\lim_{t\to\infty} \|y_i - y_{id}\| \le \epsilon_{i1o}, \quad (90)$$

$$\lim_{t\to\infty} \|\dot{\theta}_i - \upsilon_{id}\| \le \epsilon_{i2o}, \quad (91)$$

$$\lim_{t\to\infty} \|\theta_i - \theta_j\| \le \epsilon_{i3o}, \quad (92)$$

where $\epsilon_{i1o} = \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(K_{i1o}) - \frac{1}{2}}}$, $\epsilon_{i2o} = \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(\mathcal{K}_1)}}$, $\epsilon_{i3o} = \sqrt{\frac{\lambda_{\max}(P)}{\lambda_2(L)}}[\sqrt{\epsilon_{i1o}^2(y_{idM}^{\theta_i})^2 + \lambda_{\max}(K_{i1o})\lambda_{\max}(M_i)\sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(K_{i2o}) - \frac{1}{2}}}(y_{idM}^{\theta_i})^2 + \upsilon_{id}\lambda_{\max}(M_i)\sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(K_{i2o}) - \frac{1}{2}}}(y_{idM}^{\theta_i^2})^2} + \lambda_{\max}(\mathcal{K}_1)(\sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(\mathcal{K}_1)}} + \sqrt{\frac{\sum_{i=1}^n H_{io}}{\lambda_{\min}(\mathcal{K}_2)}})]$.

This concludes the proof.

Remark 4: In contrast to the previous works (Almeida et al., 2010; Chen & Tian, 2012; Ghabcheloo et al., 2006; Ihle et al., 2007; Wang et al., 2013), the observer-based cooperative path-following scheme developed in this paper does not require the velocities of the vehicles. It is the first trial for cooperative path-following problem of multiple marine surface vehicles especially with the case the vehicle dynamics are totally unknown.

5. Simulation example

In this section, an example is given for each design to show the efficacy of the proposed control method. In our simulation study, we consider the model of Cybership II, a 1:70 scale supply vessel replica built in a marine control laboratory in the Norwegian University of Science and Technology. The model parameters are obtained from Skjetne, Fossen, and Kokotovic (2005). Without loss of generality, the time-varying disturbances are introduced (Tee & Ge, 2006). $\tau_{iw}(t) = [0.3\cos(t), 0.7\sin(2t)\cos(0.7t), 0]^T$ represents the unknown disturbance from the environment usually induced by wind and wave.

5.1. *State feedback controller case*

Consider a group of three vehicles with a communication network that induces a graph with the Laplacian matrix $L = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$. The desired paths are three circles. The initial conditions of the vehicles are chosen such that they will not collide. The initial velocities are $\nu_i(0) = [0 \text{ m/s}, 0 \text{ m/s}, 0 \text{ rad/s}]^T$. The initial position and heading are chosen

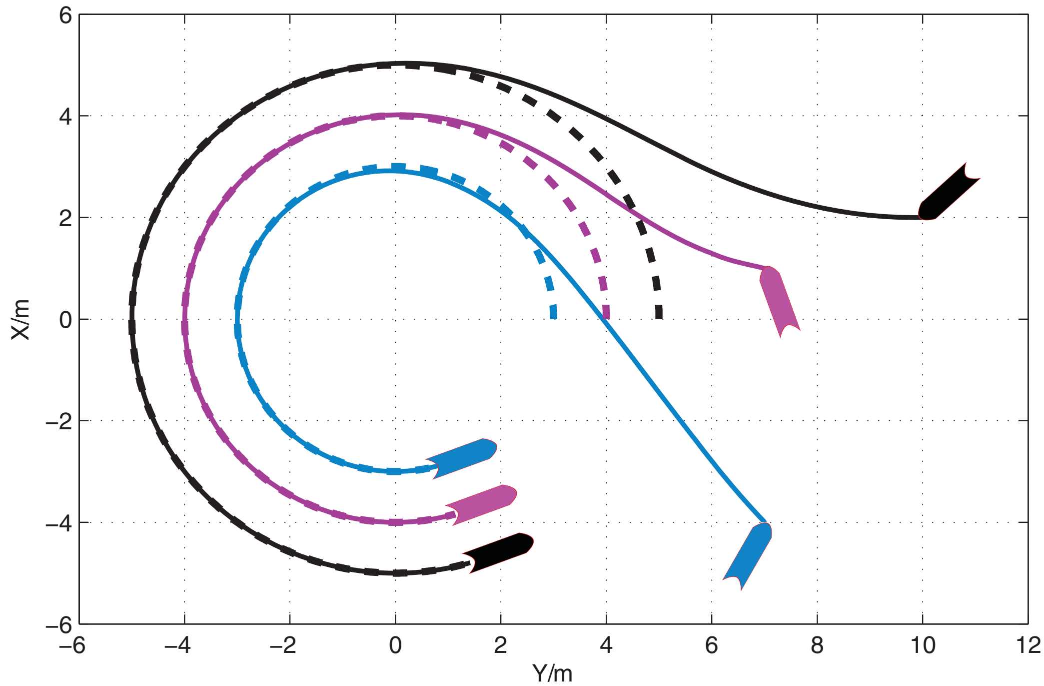

Figure 2. Desired and actual vehicles paths.

as $\eta_1(0) = [7 \text{ m}, -4 \text{ m}, \pi/3 \text{ rad}]^T$, $\eta_2(0) = [8 \text{ m}, -2 \text{ m}, 2\pi/3 \text{ rad}]^T$, $\eta_3(0) = [9 \text{ m}, 0 \text{ m}, 5\pi/4 \text{ rad}]^T$. The NN adaptive law parameters are chosen as $k_W = 100$, $\Gamma_W = 0.1$. The desired speed assignment is $v_{id} = 0.1$ m/s. The ocean current is $\nu_{ic} = [0.1 \text{ m/s}, -0.1 \text{ m/s}, 0]^T$ and update law parameters of $\hat{\nu}_{ic}$ is chosen as $k_c = 20$, $\Gamma_c = 1$. The controller gains are selected as $K_{i1} = diag[0.1, 0.5, 0.25]$, $K_{i2} = diag[6, 20, 14]$, $\mathcal{K}_1 = diag[1, 1, 1]$, $\mathcal{K}_2 = diag[10, 10, 10]$.

Linearly parameterised NN online approximators are used, in which $\hat{W}_i \in \mathbb{R}^{8\times 3}$ are the NN adaptive weights. $\xi_i = [1, \nu_i^T, \alpha_{i1}^T, \alpha_{i2}^T]^T \in \mathbb{R}^{10}$ are the input signals to the NN approximators. The NN adaptive weights are trained

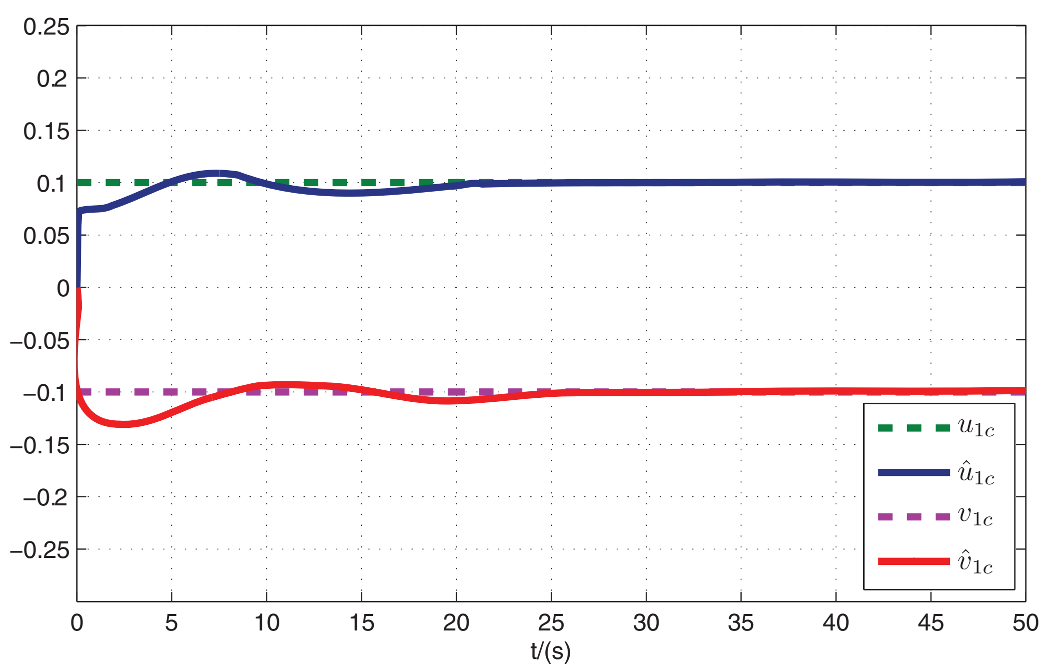

Figure 3. Constant ocean current disturbance and its estimate.

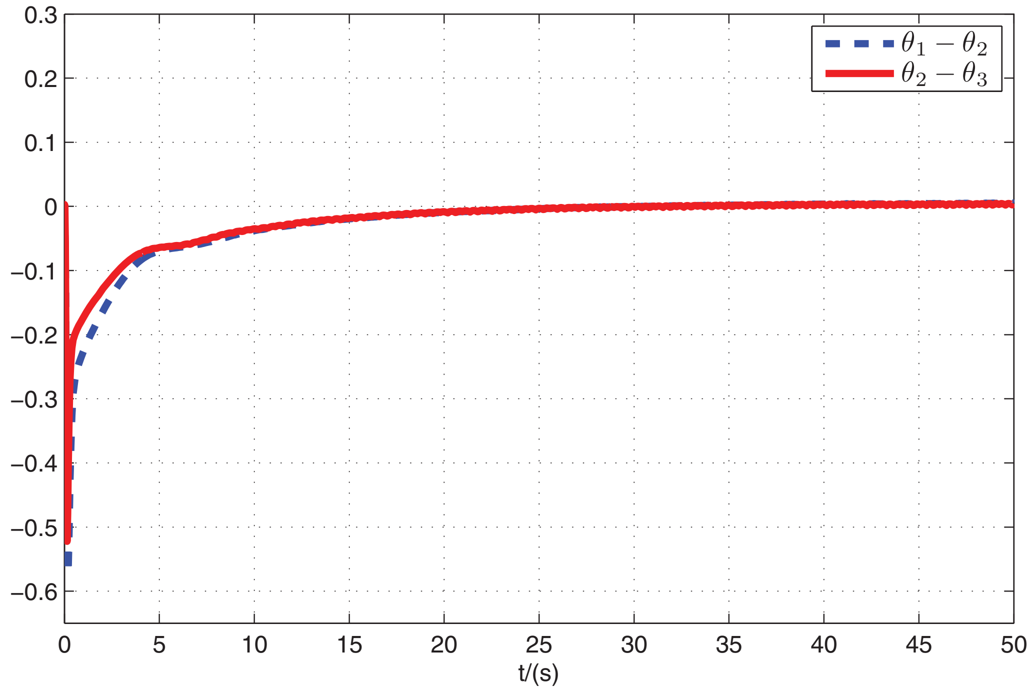

Figure 4. Path variable cooperation errors.

by the adaptive law (30). At the beginning, the NN weights are initialised with zero. Once started, the NN weights can be adjusted online to obtain a better performance.

Figure 2 shows the paths of the vehicles, where the desired paths are denoted by dotted line and the actual paths are denoted by solid line. As can be seen, with the proposed adaptive control strategy, each vehicle follows to its assigned path without the explicit knowledge of the model. Figure 3 shows the constant ocean current disturbance and its estimate of vehicle 1. The path variable cooperation errors of θ can be seen from Figure 4.

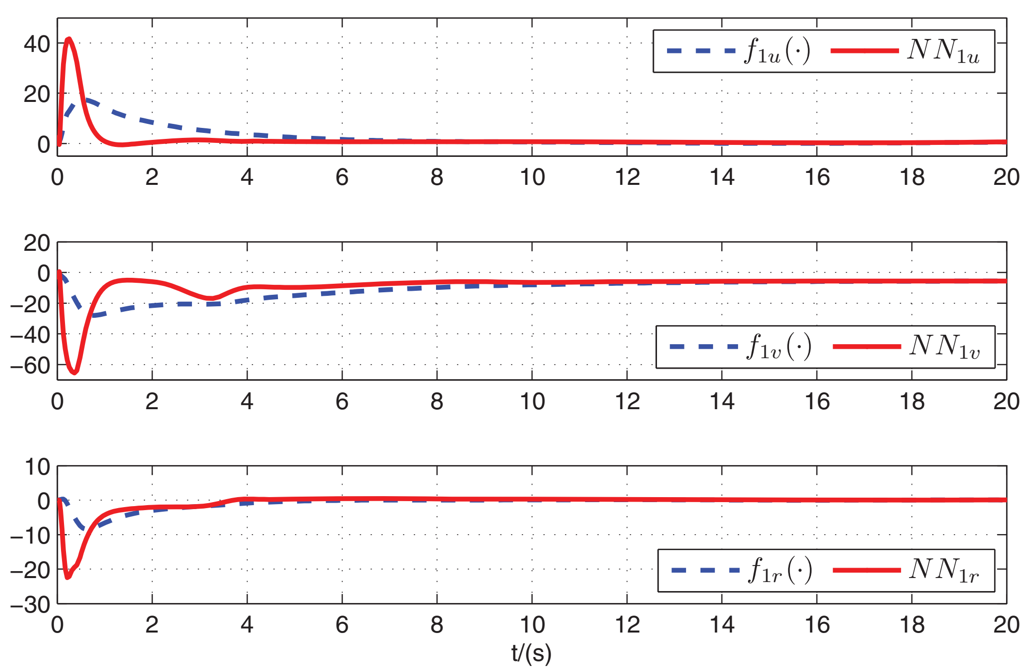

Figure 5. NN approximation performance.

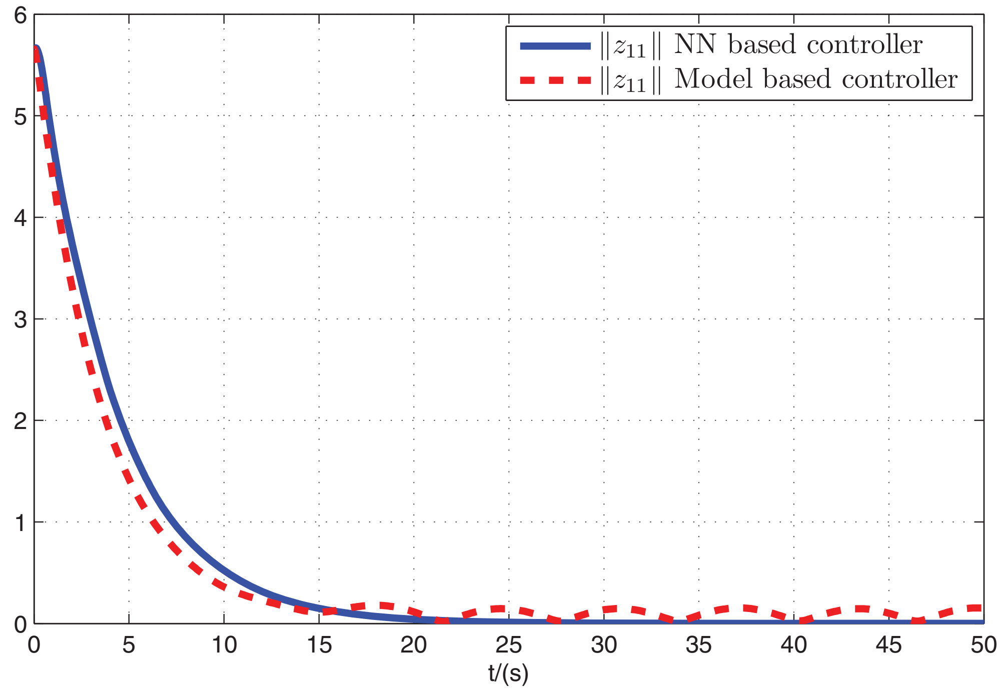

Figure 6. Comparison of tracking performance.

In order to show the NN approximation performance in a more realistic condition, the disturbance $\tau_{iw}(t)$ is reset to be an irregular wave signal such as white noise. For simulation, the magnitude of $\tau_{iw}(t)$ is limited to $[-0.5, 0.5]$ and the sampling time is set at 0.008 seconds. Figure 5 shows the approximation performance of vehicle 1, where $f_{1u}(\cdot)$, $f_{1v}(\cdot)$, and$f_{1r}(\cdot)$ are the uncertainties of the three directions, NN_{1u}, NN_{1v}, andNN_{1r} are the corresponding outputs of NN. It can be seen that the uncertainties can be efficiently compensated by NNs.

In the next section, we compare the tracking performance of the proposed NN-based controller (25) with a

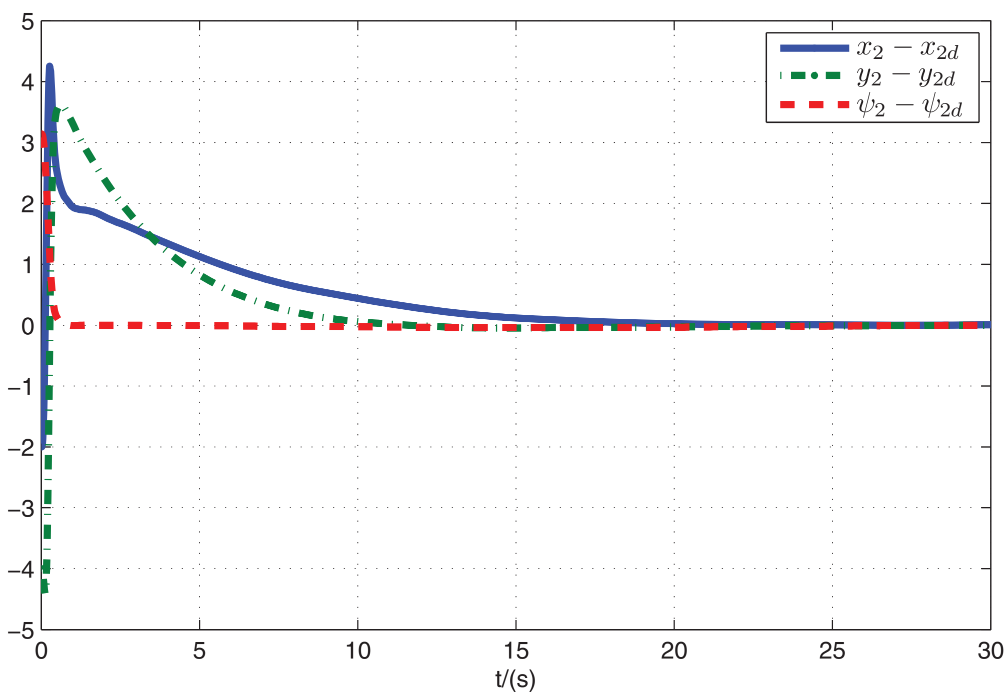

Figure 7. Path-following errors.

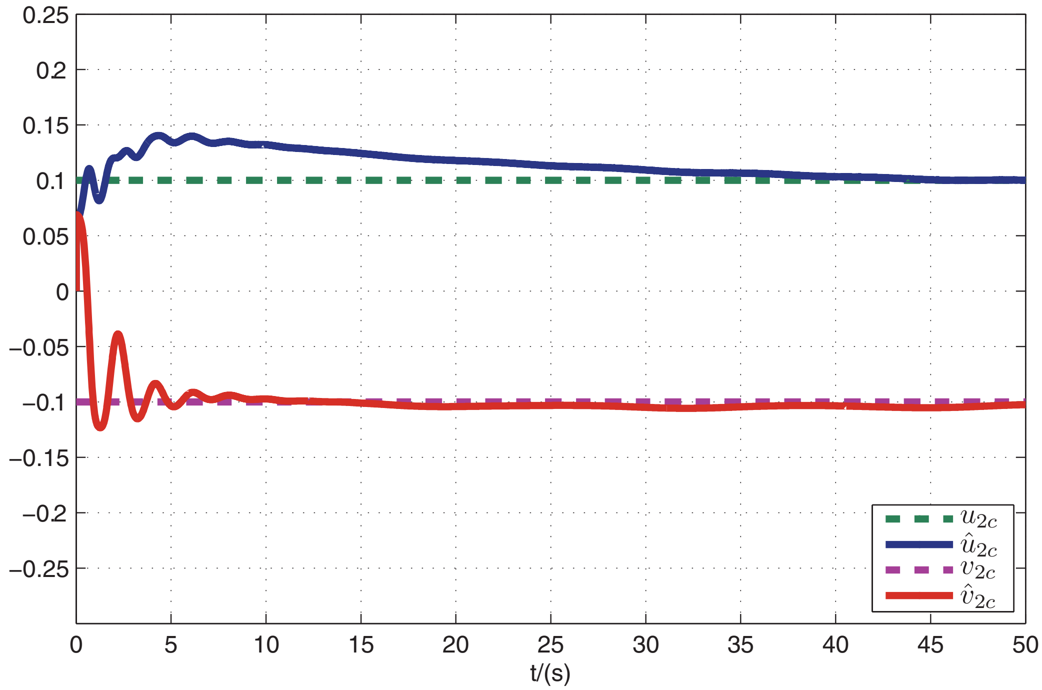

Figure 8. Constant ocean current disturbance and its estimate.

model-based nonadaptive backstepping controller:

$$\tau_i = -z_{i1} - K_{i2}z_{i2} + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + M_i(\alpha_{i1} + \alpha_{i2}v_{id}).$$

Figure 6 shows the comparison of tracking performance between the two controllers. The NN-based controller performs better than the model-based controller with smaller tracking error. The better performance is due to the fact that the bounded disturbance and unmodelled dynamics are compensated by NNs, even though no information related

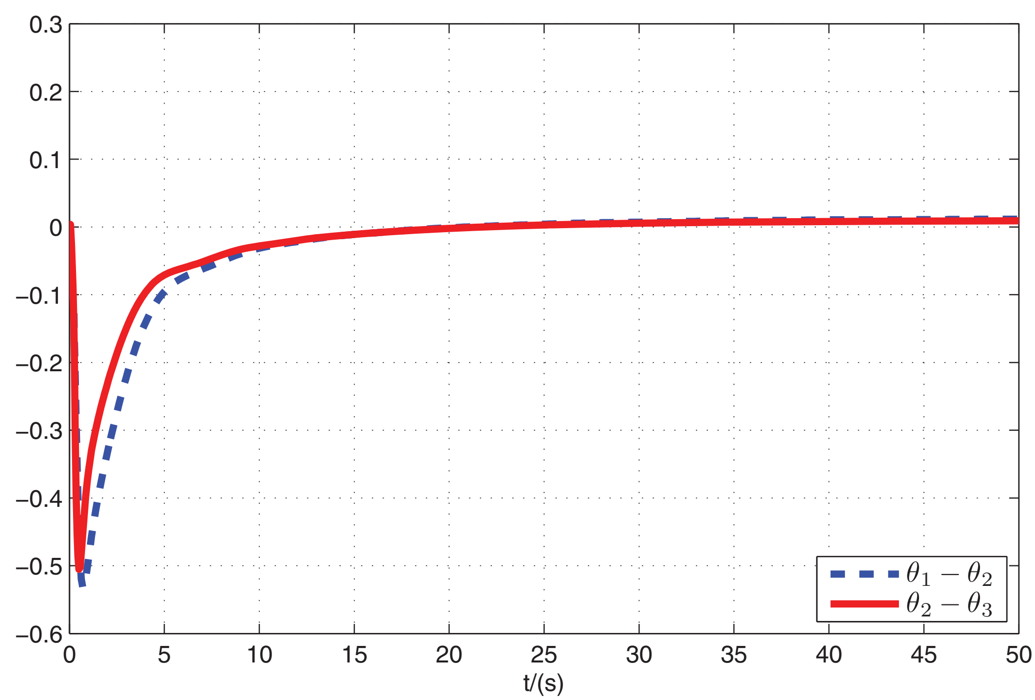

Figure 9. Path variable cooperation errors.

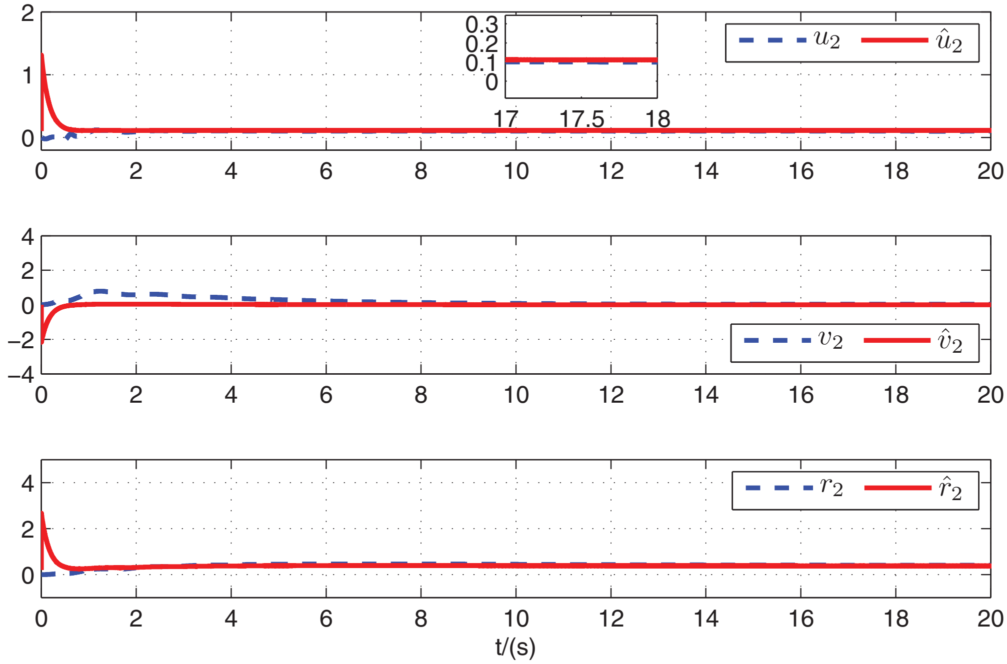

Figure 10. Velocity estimate.

to the disturbance and unmodelled dynamics is used. In contrast, model-based controller does not have any disturbance compensation due to the lack of information and results in a larger tracking error.

5.2. Output feedback controller case

In this case, the observer-based cooperative path-following controller in (55), (56) and (75) with adaptive laws (63) and (78) are applied to the control of three vehicles. The controller gains are chosen as $K_{i1o} = diag[0.25, 0.25, 2]$, $K_{i2o} = diag[30, 100, 70]$. The other parameters are chosen as the same as in state feedback case. The tracking errors of position and heading are plotted in Figure 7, where it is demonstrated that the errors are bounded to a small neighbourhood of the origin. The estimate of the ocean current converges to a value close to its actual value as shown in Figure 8. The path variable errors can be seen from Figure 9. Figure 10 shows that the velocities can be estimated precisely by the proposed observer.

6. Conclusions

This paper addresses the cooperative path-following problem of multiple marine surface vehicles subject to dynamical uncertainties and environmental disturbances. Backstepping technique, NN adaptive approach, and graph theory are brought together to design the controller. The desired cooperative behaviour is achieved by the synchronisation of along-path speed and path variables. Furthermore, an observer-based cooperative path-following scheme is developed without measuring the velocities of the vehicles. Based on Lyapunov analysis, it is proved that with the developed algorithm, all signals in the closed-loop system are SGUUB. Simulation results showed the effectiveness of the proposed control approaches .

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