Research on Real-time Simulation of Ekman Flow in Deep Ocean Based on Vortex *

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Abstract

Virtual simulation technology has become one of the core technologies in deep ocean research and exploration. In order to guarantee the reliability of underwater operating simulation, it is of vital importance to consider the physical influence on machinery equipments from complex ocean currents. In this paper, a dynamics computational model of Ekman flow with physical characteristics of the ocean operation area is designed to support real-time ocean simulation. And based on hydrodynamics algorithms and physical modeling techniques of Vortex, a software module for ocean dynamics simulation is developed. The simulation results indicate that the module can meet the requirements for both high dynamics accuracy and real-time performance.

Keywords: Ekman Flow; Vortex Physics Engine; Physical Modeling; Real-time Virtual Simulation

1 Introduction

Recent years, with an increasing number of offshore oil and gas exploration programs, virtual reality technology has been widely used in the simulation of ocean environments and complex operations of underwater machinery equipments and submersibles [1]. The laying down and recycling operations of underwater machinery equipments and submersibles like ROV and AUV run through the entire process of ocean exploration simulation. During these operations, the movement of equipments will be varied according to the disturbing forces of ocean currents. The key of simulation design is to build a dynamics flow model and relevant dynamics models of underwater equipments.

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2 Analysis of Ocean Currents Simulation

2.1 Feature Analysis of the Ocean Fields

Commonly, the features of the simulated ocean fields can be summed up as follows: (1) Depth of each ocean field ranges from 500 meters to 1500 meters, be far away from the coast and the ocean field can be regarded as an infinite deep ocean field compared with the limited operation area. (2) Its surface is almost horizontal and each part of the ocean shares similar density. (3) States of ocean currents are almost fixed within a single underwater operation time, ignoring changes of the wind velocity, and wind-driven currents in operation areas are deemed to be formed by steady wind within a certain time. In such ocean fields, most of the hydrodynamics effects are generated by the Ekman flow. So the hydrodynamics effects can be simplified as the regular changes of flow velocity and flow direction with the ocean depth, and the variation rules are described in the Ekman flow model.

2.2 Theories about Ekman Flow

The Ekman flow model is a mathematics model of ocean current under steady wind [2], and its hydrodynamic equations [3] can be written as Eq. (1).

\[
\begin{cases}
2w \sin(\varphi \cdot v) + \frac{1}{\rho} \left( \mu_z \frac{\partial^2 \mu}{\partial z^2} \right) = 0 \\
-2w \sin(\varphi \cdot \mu) + \frac{1}{\rho} \left( \mu_z \frac{\partial^2 v}{\partial z^2} \right) = 0 \\
-\frac{1}{\rho} \left( \mu_z \frac{\partial \rho}{\partial z} \right) - g = 0
\end{cases}
\]

(1)

In Eq. (1), \( p \) is the barometric pressure and \( \rho \) is the ocean density.

According to boundary conditions of \( z = 0 \) and \( z = -\infty \) in infinite deep flow model, the velocity solutions of flow hydrodynamics equations are obtained as:

\[
U_0 = \frac{T_y}{\sqrt{2\rho \mu_z w \sin(\varphi)}}
\]

(2)

\[
U = U_0 e^{(\frac{\varphi}{\rho})z+i(45^\circ+\frac{\pi}{2} \cdot z)}
\]

(3)

In Eq. (2, 3), \( U \) means flow velocity and its unit is \( m/s \), \( w \) is the earth’s rotation angular velocity, \( \varphi \) is the geographical latitude of the current ocean field, \( T_y \) is the shear stress generated by the wind and the sea surface, its unit is \( N \).

Eq. (2) expresses the flow velocity of the flow surface, and Eq. (3) reflects the relation between layer velocity and ocean depth. We can draw the curve of Ekman spiral [4] using Eq. (2, 3).

2.3 Feature Analysis of Ekman Flow Simulation Objects

According to velocity solutions of flow hydrodynamics equations, we can find out characteristics of flow dynamics simulation. First, in terms of velocity, it is known that the flow velocity of sea
surface is proportional to the wind shear stress according to Eq. (2, 3), and the variation of flow velocity matches the e negative exponential decreasing trend as the ocean depth increases. When \( z = -h \),

\[
U_{-h} = U_0 e^{-\pi} \approx 0.043U_0
\]  

(4)

The flow velocity is only 1/23 of the surface velocity [5]. Flow velocity is even weaker in deeper ocean. Secondly, in terms of flow direction, according to the index factor \( e^{(45^\circ + \frac{\pi}{2} - z)} \) in Eq. (3), it is known that the angle between the direction of flow and wind is 45°, and the flow velocity changes gradually with the increase of the depth, when \( z = -h \), the flow direction is opposite to the flow direction of the ocean surface. Coriolis forces determine that the variation trend of flow direction differs in the northern and southern hemispheres, i.e., clockwise in the northern hemisphere and anticlockwise in the southern hemisphere.

According to the analysis above, it is known: (1) Flow layers with similar depths have the same hydrodynamic characteristics in a certain depth scope; (2) The hydrodynamics equations of flows can be described as the relation between flow velocity and depth in the vertical direction.

Therefore, it is reasonable to create the Ekman flow dynamics models through dividing flow layers vertically and assume that only one Ekman flow object exists in a dynamics space. An Ekman flow object is composed of several Ekman layer objects, so the thickness of the Ekman layer depends on both the thickness of the Ekman flow and the number of layers vertically divided into. Each Ekman layer object is infinite wide in horizontal direction, and when the ocean depth is beyond \( h \), hydrostatic models are adopted instead.

3 The Ekman Flow Real-time Simulation Module

3.1 Modeling with Vortex Fluid

A critical issue to simulate ocean currents is to create flow objects with dynamics simulation effects. We used Vortex, a real-time dynamics simulation platform [6,7], to create hydrodynamics models of Ekman flow, realizing the hydrodynamics simulation of single flow layer.

The VxPlanarFluidState in Vortex engine possesses features illustrated below.

1. In terms of models, the fluid model is an ocean field model which is infinite wide in horizontal direction and vertically filled with acting force in the whole ocean.

2. The fluid model has no collision entity, but fluid density, fluid viscosity and acceleration of gravity will apply forces on rigid bodies in contact. So buoyancy, drag force, added mass force calculating and Magnus approximate calculating should be done by the dynamics engine.

3. The fluid state is only determined by its dynamic configuration parameters and has nothing to do with reactive forces from rigid bodies [8].

There is a special mechanism to deal with multi-layered ocean field in Vortex. Fig. 1 gives an introduction to this mechanism.

The contact between rigid bodies and single fluid layer is divided into three conditions: no contact, partial immersion and total immersion. As the condition of no contact calls for no discussion, partial immersion and total immersion of rigid bodies are shown in Fig. 1 (a).
The total immersive contact between dynamics objects and multiple fluid layers is shown in Fig. 1 (b) and (c). There are two principles in Vortex to provide enough real-time performance: (1) A rigid body only interacts with a single fluid layer. (2) The fluid layer which interacts with the rigid body is the lowest one of which submerges the rigid body.

### 3.2 Design of the Ekman Flow Simulation Module

Before applying wind-driven current hydrodynamic equations in flow simulation, two questions have to be considered. First, fluid objects with reasonable layer velocity must be created by the simulation module in the dynamics space, which can be realized using VxPlanarFluidState. Second, the layer velocity of Ekman flow should be calculated according to the surface wind velocity and direction, while computing methods are needed to find out the right configuration parameters of fluid objects.

#### 3.2.1 Numerical Calculation

Calculations made by the Ekman flow module can be divided into two steps. One is to calculate the parameters of dynamic effects on rigid bodies from the fluid objects, and this calculation is done by Vortex engine. Another is to calculate the configuration parameters of fluid objects. This calculation is done off-line, and the computing process is shown in Fig. 2.

The first step is to read the configuration parameters and configure the ocean environment, then figure out key characteristic parameters such as \( T_y \), \( \mu_z \) and \( h \), the values can be calculated using empirical formulas [9,10] as follows in Eq. (5)

\[
\begin{align*}
T_y &= 0.0008 \rho_a W^2 \\
\mu_z &= 4.3W^2 \\
h &= \frac{7.6W}{\sqrt{\sin \varphi}}
\end{align*}
\]

This empirical formulas meets the precision requirements in engineering and provides better computing performance. Then we can import \( T_y \), \( \mu_z \) and \( h \) into Eq. (2, 3), combining with the layer number and the altitude of the flow layer, and figure out the flow velocity \( |U_i| \) of each flow layer and the angle between flow direction and the polar axis \( \theta_i \) in the local polar coordinate.
Till now, we get the configuration parameters of each fluid object. However, $|U_i|$ and $\theta_i$ need to be converted according to a certain flow layer into configuration parameters which can be used directly.

Fig. 2: Calculation process of the Ekman flow simulation module

The conversion mainly contains two processes. The first one is to project the vector on rectangular coordinates, as the modulus of the vector is $|U_i|$, and the angle between the vector and the X axis is $\theta_w + \theta_i$. The projection on the X axis is the horizontal vector component $U_{xi}$, and the projection on the Y axis is the horizontal vector component $U_{yi}$, and the projection on the Z axis is the vertical vector component $U_{zi}$ and $U_{zi} = 0$. Then three vector components are converted into configuration parameters of the Vortex fluid objects.

After finishing the calculation process of key parameters, the next step is to create and configure the flow object with the parameters figured out, finally generating the Ekman flow dynamics models.

### 3.2.2 Software Module Structure

The structure of Ekman flow real-time dynamics simulation module is shown in Fig. 3. Classes can be divided into two parts. The first part contains Vortex classes, such as VxFrame, VxUniverse, VxFuidState and VxPlanarFluidState, the functions of which are to build real-time simulation framework and create dynamics space together with fluid models. The second part contains the classes used for creating Ekman flow objects.

The classes of Ekman flow objects include flow object classes and flow factory classes. Flow object classes contain WindCurrent class and its derived class WindCurrent_North_Hemisphere and WindCurrent_South_Hemisphere. Flow factory classes contain OceanCurrentFactory class and its derived classes, i.e., OceanCurrentFactory_North and OceanCurrentFactory_Sorth.
The key calculation and environment parameters of flow objects are encapsulated in WindCurrent class. Table 1 shows the key calculation parameters.

Table 1: Key calculation parameters of local ocean environment

<table>
<thead>
<tr>
<th>Name</th>
<th>Type</th>
<th>Attributes &amp; Functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ocean_depth</td>
<td>VxReal</td>
<td>Elevation of seabed, used to describe depth of the ocean</td>
</tr>
<tr>
<td>WC_name</td>
<td>VxName</td>
<td>The name of Ekman Flow object</td>
</tr>
<tr>
<td>currentlevelnum</td>
<td>int</td>
<td>Number of flow layers parameter determining simulation accuracy</td>
</tr>
<tr>
<td>Wind_v</td>
<td>VxVector3</td>
<td>Wind velocity parameter on the sea surface</td>
</tr>
<tr>
<td>Wind_density</td>
<td>VxReal</td>
<td>Density parameter of atmosphere on the sea</td>
</tr>
<tr>
<td>Latitude</td>
<td>VxReal</td>
<td>Latitude parameter of the ocean</td>
</tr>
<tr>
<td>Ocean_density</td>
<td>VxReal</td>
<td>Density parameter of the ocean</td>
</tr>
<tr>
<td>Ocean_universe</td>
<td>VxUniverse</td>
<td>The dynamics simulation space where the ocean located</td>
</tr>
</tbody>
</table>

Interfaces in class WindCurrent are mainly used to calculate flow velocities of multiple flow layers and store the velocities in terms of polar coordinates, which are used to calculate configuration parameters of Vortex fluid objects. Interfaces in its derived classes are mainly used in the conversion from calculation parameters, which determine the movement of Ekman flow, to config-
uration parameters which configure Vortex fluid objects. Meanwhile, the configuration interfaces in derived classes are applied to create and configure Ekman flow objects. The WindCurrent class has only one instance so that only one Ekman flow object in one dynamic space is promised. The two WindCurrent classes in Fig. 3 are designed respectively for different rotating characteristics in southern and northern hemispheres.

Customized interfaces are provided by the factory classes for particular objects, for example, the factory class in Fig. 3 is mainly used to customize the flow object in southern and northern hemispheres respectively.

4 Results of the Ekman Flow Simulation

The flow velocity numerical line charts of simulated Ekman flow are drawn on condition of a flow layer which has been longtime affected by west wind with a velocity of 20 m/s at latitude of 20 degrees in the southern and northern hemispheres. Considering that the ocean environment must be suitable for ocean exploration operations, in practice, no operations should be carried on above 8 sea state (fresh gale on the sea, wind velocity 20 m/s) [11], and the depth of frictional influence $h$ reaches the maximum at 8 sea state. So, the wind velocity of 20 m/s is chosen to do the numerical simulation. Fig. 4 shows the line chart of flow layer velocity vector in 3-D space, while Fig. 5 shows in 2-D plane.

![3-D line chart of Ekman flow layer velocity vector](image1)

(a) In the northern hemisphere  (b) In the southern hemisphere

Fig. 4: 3-D line chart of Ekman flow layer velocity vector vertex

![2-D projection of 3-D line chart of flow layer velocity vector](image2)

(a) In the northern hemisphere  (b) In the southern hemisphere

Fig. 5: 2-D projection of 3-D line chart of flow layer velocity vector vertex
In Fig. 4, the Z axis stands for depth of the flow layer, while the values of the X axis and Y axis reflect the projections of flow velocity vector in the horizontal plane, and the unit is m/s. When the depth of flow layer equals to $z$, the velocity vector of this flow layer is the vector from $(0, 0, z)$ to the corresponding vertex in the line chart. Fig. 4 (b) indicates that the velocity of flow models created by the Ekman flow simulation module matches the e negative exponential decreasing trend described in Eq. (3), so the velocity can be very small even negligible when the depth reaches $-h$.

In Fig. 5, the angular variation rule of the flow velocity is shown. The Y axis is towards the North and the X axis is towards the East. The direction of flow velocity shows a clockwise/anticlockwise trend as the depth increasing in the Northern/Southern Hemisphere, beginning with 45 degrees deviated from the wind direction. When the depth reaches $-h$, the flow direction becomes reversed compared with that on the sea surface. Fig. 4 and 5 are based on the condition that the flow is divided into 10 flow layers.

Fig. 6 shows the numerical characteristics of the velocity vector when the number of flow layers increases in a particular ocean environment. The layer numbers are 10, 20 and 30 respectively.

For Ekman flows in the same environment, the precision of flow objects increases as the flow being divided into smaller layers, however, the increase of layer objects also means greater computation burden on the simulation system. For the ocean fields affected by the fresh gale of 20 m/s in the long time, 30 layer objects is precise enough for the simulation of deep water lowering operations as its vertex line is already quite close to the Ekman spiral. During the ROV lowering simulation under the condition of 30 layer objects, we got a frame rate result around 21 fps, which can provide good real-time performance, hence it is reasonable to set the max layer number to be 30.

5 Discussions and Conclusions

We presented an Ekman flow real-time simulation module, used in the ocean exploration simulation system with high physical reality, in accord with rational Ekman flow mathematical properties on Vortex dynamics simulation platform. It provides the simulation of Ekman flow models under different wind scales in southern and northern hemispheres and achieves realistic simulation results while dynamics models in the simulation will be affected by the disturbing forces from Ekman flow models when moving vertically. Since the real-time performance is decreasing as
increasing the number of layer objects, less layer objects should be created on the premise of
dynamic reality when simulating ocean environment.

However there are potential improvements of Ekman flow simulation:

(1) Since the Ekman flow dynamics models are created based on the hydrodynamics equations,
it leads to a simplified realization but ignoring the influences of water factors such as water
temperature and salinity. Although the simulation results match the trend of hydrodynamics,
hardly can they match the measured values exactly.

(2) The Ekman flow dynamics module divides the ocean field into multi flow layers with the same
depth. However, for flow layers in deeper ocean, the difference of flow velocity between two
neighboring layers is very little, so simulation efficiency can be improved by decreasing the
number of flow layers as the depth of the ocean increases. If layers are divided by equal flow
velocity variation, there will be more flow layers near the sea surface. In this case, a rigid body
could go through several flow layers, resulting in the reduction of simulation authenticity. So,
better dividing algorithms are needed in the future.

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