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Signal difference-based deadband H_∞ control approach for networked control systems with limited resources

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Abstract: This paper investigates a signal difference-based deadband H_∞ control approach for networked control systems (NCSs) with limited resources. The effects of variable network-induced delays, sampling intervals and data transmitting deadbands are considered simultaneously and the model of the NCS is presented. A Lyapunov functional is adopted, which makes full use of the network characteristic information including the bounds of network delay (BND), the bounds of sampling interval (BSI) and the bounds of transmission deadband (BTD). In the meanwhile, the new H_∞ performance analysis and controller design conditions for the NCSs are proposed, which describe the relationship of BND, BSI, BTD and the system's performance. Three examples are used to illustrate the advantages of the proposed methods. The results have shown that the proposed method not only effectively reduces the data traffic, but also guarantees the system asymptotically stable and achieves the prescribed H_∞ disturbance attenuation level.

Keywords: networked control system (NCS), H_∞ control, variable network-induced delay, variable sampling interval, transmission deadband.

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1. Introduction

Networked control systems (NCSs) are such control systems where the control loop is closed via communication networks [1]. NCSs have great advantages, for example, low cost, reduced weight and power requirements, simple installation and maintenance, and high reliability [2,3]. However, NCSs also introduce many new challenges in control system design such as time-varying network-induced delays, limited network resources or data packet congestion, and these disadvantages might be main resources for influencing the stability and the performance degradation of the NCSs [4–7]. Recently, some works have discussed the control problem of NCSs with delays and dropout packets [8–12]. However, the effect of sampling intervals has not been investigated in the aforementioned papers. In NCSs, the limitation of the network resources is primarily caused by the limited bandwidth of the communication network, then the limited bandwidth of the network produces a situation, where a smaller sampling period may not result in a better system performance. A small sampling period will produce too many sensing data in the network channel, and this will bring about the overloading and congestion of the network, and result in more data packet dropouts and longer delays, and thus degrade the system performance [1,13]. Presently, Wang and Yang proposed the H_∞ controller design approach for the NCSs by using an active-varying sampling period method. Cloostermal et al. [15] presented a discrete-time model for the NCSs that incorporates all network phenomena: time-varying sampling intervals, packet dropouts and time-varying delays, and gave stabilizing controller design. To the best of the author's knowledge, the control problem of NCSs with time-varying sampling has not been fully investigated and still remains challenging.

On the other hand, a large amount of data packets transmit in the limited network bandwidth, so, the probability of congestion increases, which may lead to higher transmission delay and packet loss [16]. Therefore, some scheduling methods have been proposed in the past years such as try-once-discard (TOD) protocol and round-robin (RR) protocol. Both protocols allow only one output to be transmitted via the channel at a time. Walsh et al. [17] used the TOD protocol to deal with the packets scheduling and obtained the maximum allowable transfer interval. Xu et al. [18] applied a conventional RR scheduling to deal with the

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communication constraints for NCSs, wherein the packets were transmitted in a fixed order. Liu et al. [19] investigated the exponential stability and the induced L2-gain of NCSs that were subject to time-varying transmission intervals, time-varying transmission delays and communication constraints, wherein the scheduling of sensor information towards the controller is ruled by the classical RR protocol. However, in the works mentioned above, it can be found that the total data information over the network channel has not been reduced at each sampling instant, and the influences of the amount of data transmission on system performance were not considered. Transmission data losses and communication delays strongly affect the stability and performance of the closed-loop systems. The most effective way to improve system performance is to reduce network transmission [20]. In [21] and [22], the transmission data in networked control systems were reduced by applying deadband control. In these communication networks, the data packets were sent over only if the signal value changed more than a given threshold. These results have showed that the deadband control could reduce effectively the amount of the transmission data. Recently, Zhao et al. [23] proposed a packet-based deadband control approach for internet-based NCSs, but the proposed approach only reduced the transmission of control input signals, and it did not consider the influences of the transmission of state signals in NCSs. However, for the NCSs with variable network-induced delays, variable sampling intervals and signal transmission deadbands, fewer results are available to study the consequences of all these phenomena in a common framework. It is therefore necessary to conduct an analysis on the network-induced delay, sampling interval and transmission deadband, and study how much these effects work on the overall performance of NCS.

In contrast with the above previous works, this paper explores a signal difference-based deadband H_∞ control approach to decrease the data transmission in the NCSs. By using the input delay approach, the effects of network-induced delays, sampling intervals and transmission deadbands are included in the NCS model. By employing a Lyapunov functional which takes full advantages of the information of network-induced delays, sampling intervals and transmission deadbands, the condition of H_∞ controller design is given. The obtained condition is dependent on the bounds of sampling interval (BSI), the bounds of network delay (BND) and the bounds of transmission deadband (BTD). Finally, numerical examples are provided to validate the proposed control strategy.

2. Problem formulation

The NCSs with two transmission deadbands can be described as in Fig. 1.

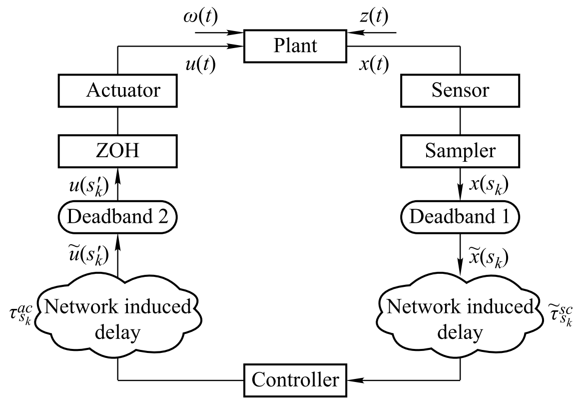

Fig. 1 Structure of the NCS with two transmission deadbands

In the NCSs, transmission deadband 1 is set up in the side of the sensor to controller and transmission deadband 2 is set up in the one of the controller to the actuaor. The plant is a continuous-time system whose state-space representation is given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{B}_\omega\boldsymbol{\omega}(t) \tag{1}$$

$$\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{2}$$

where $\boldsymbol{x}(t) \in \mathbf{R}^n$ is the system state, $\boldsymbol{\omega}(t) \in \mathbf{R}^m$ is the disturbance, $\boldsymbol{u}(t) \in \mathbf{R}^p$ is the control input, and $\boldsymbol{z}(t) \in \mathbf{R}^q$ is the controlled output. $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{B}_\omega, \boldsymbol{C}, \boldsymbol{D}$ are some constant matrices of appropriate dimensions.

In this paper, both the state and the control input signals are processed by transmission deadbands before they are sent to the controller and the actuator respectively. At the same time, it is assumed that the sensor and the sampler are clock-driven, while the controller, the zero-order holder, and the actuator are event-driven. The implementation of deadband control will lead to network data transmission reduction while maintaining acceptable system performance. In Fig. 1, we denote the input signal of transmission deadband 1 as $\boldsymbol{x}$ and its output signal as $\tilde{\boldsymbol{x}}$. At the same time, the input signal and the output signal of transmission deadband 2 are $\tilde{\boldsymbol{u}}$ and $\boldsymbol{u}$, respectively. The sampling instants are denoted as s_k $(k = 0, 1, \ldots, \infty)$. A node with the transmission deadband compares the previous value $\tilde{\boldsymbol{x}}(s_k)$ to the most recent value $\boldsymbol{x}(s_{k+1})$. The data packets are sent over the communication network only, if the signal value changes more than a given threshold. Thus, transmission deadband 1 can be described as

$$\tilde{x}_i(s_{k+1}) = \begin{cases} x_i(s_{k+1}), & |\tilde{x}_i(s_k) - x_i(s_{k+1})| > \delta_{1i}\,|x_i(s_{k+1})| \\ \tilde{x}_i(s_k), & |\tilde{x}_i(s_k) - x_i(s_{k+1})| \leqslant \delta_{1i}\,|x_i(s_{k+1})| \end{cases} \tag{3}$$

where $i = 1, 2, \ldots, n$, and $\delta_{1i}\,|x_i(s_{k+1})|$ is the threshold of the deadband. δ_{1i} is the weighting of the deadband threshold.

Remark 1 If the absolute value of the difference between $x_i(s_{k+1})$ and $\tilde{x}_i(s_k)$ is within the threshold of the deadband, then no update data are sent to the network. It can be seen that δ_{1i} inflects the size of the deadband, and as δ_{1i} increases, the node broadcasts fewer messages.

Implementing deadbands to reduce network traffic produces uncertainty in the state of the NCSs (1) and (2). Define $\Delta_1(s_{k+1}) = \mathrm{diag}\{\Delta_{11}(s_{k+1}), \Delta_{12}(s_{k+1}), \ldots, \Delta_{1n}(s_{k+1})\}$, $\Delta_{1i}(s_{k+1}) \in [-\delta_{1i}, \delta_{1i}]$, then (3) can be included in the following equality:

$$\tilde{\boldsymbol{x}}(s_{k+1}) = \boldsymbol{x}(s_{k+1}) + \Delta_1(s_{k+1})\boldsymbol{x}(s_{k+1}). \tag{4}$$

In the following, we denote $\tau_k = \tau_{s_k}^{sc} + \tau_{s_k}^{ca}$, where τ_k is the network delay from the sampler to the actuator, $\tau_{s_k}^{sc}$ is the delay from the sampler to the controller and $\tau_{s_k}^{ca}$ is the delay from the controller to the actuator. Network communication delay τ_k is naturally assumed as $\tau_m \leqslant \tau_k \leqslant \tau_M$, where τ_m and τ_M denote the lower bound of the network delay and the upper bound of the network delay, respectively.

The sampling interval $T_k = s_{k+1} - s_k$ is time-varying and its lower bound and upper bound are known:

$$0 < T_m \leqslant T_k \leqslant T_M \tag{5}$$

where T_m and T_M denote the lower bound of the sampling interval and the upper bound of the sampling interval, respectively.

In this paper, we consider the case of $s_k + \tau_k < s_{k+1}$, that is, the network delay is less than one sampling interval. Then at the instant s_k, we have

$$\tilde{\boldsymbol{u}}(s_k + \tau_{s_k}^{sc} + \tau_{s_k}^{ca}) = \boldsymbol{K}\tilde{\boldsymbol{x}}(s_k) \tag{6}$$

where $\boldsymbol{K}$ is the state-feedback gain to be determined later. For convenience, denote $s_k{}' = s_k + \tau_{s_k}^{sc} + \tau_{s_k}^{ca}$.

In the following, the relation between the input signal $\tilde{u}$ and the output signal u in the transmission deadband 2 can be described as

$$u_j(s'_{k+1}) = \begin{cases} \tilde{u}_j(s'_{k+1}), & \left|u_j(s'_k) - \tilde{u}_j(s'_{k+1})\right| > \delta_{2j}\left|\tilde{u}_j(s'_{k+1})\right| \\ u_j(s'_k), & \left|u_j(s'_k) - \tilde{u}_j(s'_{k+1})\right| \leqslant \delta_{2j}\left|\tilde{u}_j(s'_{k+1})\right| \end{cases} \tag{7}$$

where $j = 1, 2, \ldots, m$, and $\delta_{2j}\left|u_j(s'_{k+1})\right|$ is the threshold of deadband 2. δ_{2j} is the weighting of the deadband threshold.

Define $\Delta_2(s'_{k+1}) = \mathrm{diag}\{\Delta_{21}(s'_{k+1}), \Delta_{22}(s'_{k+1}), \ldots, \Delta_{2m}(s'_{k+1})\}$, $\Delta_{2j}(s'_{k+1}) \in [-\delta_{2j}, \delta_{2j}]$, then (7) can be included in

$$\boldsymbol{u}(s'_{k+1}) = \tilde{\boldsymbol{u}}(s'_{k+1}) + \Delta_2(s'_{k+1})\tilde{\boldsymbol{u}}(s'_{k+1}). \tag{8}$$

Combining (4) and (8), the control input signal can be written as

$$\boldsymbol{u}(t) = (\boldsymbol{I} + \Delta_2(s'_k))\boldsymbol{K}(\boldsymbol{I} + \Delta_1(s_k))\boldsymbol{x}(s_k)$$
$$s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1} \tag{9}$$

where $\boldsymbol{I}$ denotes the identity matrix of appropriate dimensions. $|\Delta_{1i}(s_k)| \leqslant \delta_{1i}$ and $|\Delta_{2j}(s'_k)| \leqslant \delta_{2j}$. For the sake of simplicity, it is assumed that $\delta_{1i} = \delta_1$ and $\delta_{2j} = \delta_2$. Then, it is not difficult to get $\Delta_{1_i}(s_k) \in [-\delta_1, \delta_1]$ and $\Delta_{2_j}(s'_k) \in [-\delta_2, \delta_2]$. In the following, for the sake of simplicity, we denote $\Delta_1(s_k)$ and $\Delta_2(s'_k)$ as Δ_1 and Δ_2, respectively, and we call δ_1 and δ_2 as the BTD. It is clear that in the same network conditions, with the value of BTD increasing, fewer data are sent in the network channel.

Modeling of continuous-time systems with discrete-time control inputs was investigated in [24]. The digital control law may be represented as follows by using the input delay approach:

$$\boldsymbol{u}(t) = (\boldsymbol{I} + \Delta_2)\boldsymbol{K}(\boldsymbol{I} + \Delta_1)(\boldsymbol{x}(t - (t - s_k)))$$
$$s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1}. \tag{10}$$

Denote $d(t) = t - s_k$, the piecewise-constant control law (10) can be represented as a continuous-time controller with a time-varying piecewise continuous delay:

$$\boldsymbol{u}(t) = (\boldsymbol{I} + \Delta_2)\boldsymbol{K}(\boldsymbol{I} + \Delta_1)\boldsymbol{x}(t - d(t))$$
$$s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1} \tag{11}$$

where $d(t) = t - s_k$ is piecewise linear with the derivative $\dot{d}(t) = 1$ for $t \neq s_k + \tau_k$.

At the same time, we have

$$\tau_m \leqslant d(t) \leqslant T_M + \tau_M, \ s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1}. \tag{12}$$

Since the network communication delay is smaller than the sampling interval, we obtain $\tau_m \leqslant \tau_M \leqslant T_m \leqslant T_M$. Let $\tau_M/T_m \leqslant \rho \leqslant 1$, then we can write (12) as

$$\tau_m \leqslant d(t) \leqslant T_M + \frac{\tau_M + \rho T_m}{2} = \overline{\eta}. \tag{13}$$

We call $\overline{\eta}$ the maximum allowable equivalent delay bound (MAEDB)[11]. It is clear that in the same network conditions, the larger the value of MAEDB is, the larger the allowable networked communication delays and sampling intervals are in NCSs.

Substituting the controller (11) into NCSs (1) and (2), we can obtain the closed-loop networked control system:

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}(\boldsymbol{K} + \Delta(\boldsymbol{K}))\boldsymbol{x}(t - d(t)) + \boldsymbol{B}_\omega\boldsymbol{\omega}(t),$$
$$s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1} \tag{14}$$

$$\boldsymbol{z}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}(\boldsymbol{K} + \Delta(\boldsymbol{K}))\boldsymbol{x}(t - d(t)),$$
$$s_k + \tau_k \leqslant t < s_{k+1} + \tau_{k+1} \tag{15}$$

where $\Delta(\boldsymbol{K}) = \Delta_2\boldsymbol{K} + \boldsymbol{K}\Delta_1 + \Delta_2\boldsymbol{K}\Delta_1$.

Our objective is to find a state-feedback controller $\boldsymbol{K}$, which stabilizes systems (1) and (2) with $\boldsymbol{\omega}(t) = 0$ and makes them satisfy

$$J = \int_0^\infty (\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t) - \gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t))\mathrm{d}t < 0, \tag{16}$$

for $x(0) = 0$ and all $\boldsymbol{\omega}(t) \neq 0$. The scalar γ is a prescribed positive scalar and indicates an H_∞ disturbance attenuation level.

Remark 2 By the input delay approach, we give a general framework for the linear matrix inequality (LMI) approach to derive the H_∞ performance analysis condition for the NCSs with network delays, variable sampling intervals and signal transmission deadbands. It should be noted that the delay $d(t)$ is piecewise linear with $\dot{d}(t) = 1$ for $t \neq s_k + \delta_k$, and it integrates the information of network communication delays and sampling intervals.

3. Main results

3.1 Robust H_∞ performance analysis for the NCSs with transmission deadbands

Given the bounds of network delays τ_m and τ_M, the bounds of sampling intervals T_m and T_M, and transmission deadband bounds δ_1 and δ_2, assuming that the matrices $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$, $\boldsymbol{D}$, $\boldsymbol{B}_\omega$ and the control gain $\boldsymbol{K}$ in systems (14) and (15) are known, we will formulate the conditions under which the closed-loop networked control system with signal transmission deadbands is robustly stable with an H_∞ disturbance attenuation level γ.

Theorem 1 Consider the NCS in Fig. 1. For given scalars τ_m, τ_M, T_m, T_M $(0 \leqslant \tau_m \leqslant \tau_M \leqslant T_m \leqslant T_M)$, $\delta_1 > 0, \delta_2 > 0, 0 \leqslant \alpha < 1, \tau_M/T_m \leqslant \rho \leqslant 1$, and a matrix $\boldsymbol{K}$, the closed-loop NCSs (14) and (15) are asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$, $\boldsymbol{Q}_i = \boldsymbol{Q}_i{}^{\mathrm{T}} > 0$ $(i = 1, 2, 3)$, $\boldsymbol{U}_j = \boldsymbol{U}_j{}^{\mathrm{T}} > 0$ $(j = 1, 2)$, and $\boldsymbol{N}_l, \boldsymbol{T}_l, \boldsymbol{M}_l, \boldsymbol{E}_l$ $(l = 1, 2, 3, 4, 5)$ of appropriate deimensions, such that

$$\boldsymbol{\Gamma} = \begin{bmatrix} \boldsymbol{\Gamma}_1 & \boldsymbol{\Gamma}_2 & \mathcal{B}^{\mathrm{T}}\boldsymbol{U}_1 & \mathcal{B}^{\mathrm{T}}\boldsymbol{U}_2 & \mathcal{C}^{\mathrm{T}} \\ & \boldsymbol{\Gamma}_3 & 0 & 0 & 0 \\ & * & -\dfrac{1}{\sigma}\boldsymbol{U}_1 & 0 & 0 \\ & * & * & -\dfrac{1}{\bar{\sigma}}\boldsymbol{U}_2 & 0 \\ & * & * & * & -\boldsymbol{I} \end{bmatrix} < 0$$

$$\boldsymbol{\Gamma}_1 = \begin{bmatrix} \boldsymbol{\Gamma}_{11} & \boldsymbol{\Gamma}_{12} & \boldsymbol{\Gamma}_{13} & \boldsymbol{\Gamma}_{14} & \boldsymbol{\Gamma}_{15} & \boldsymbol{P}\boldsymbol{B}_w \\ & \boldsymbol{\Gamma}_{22} & \boldsymbol{\Gamma}_{23} & \boldsymbol{\Gamma}_{24} & \boldsymbol{\Gamma}_{25} & 0 \\ & * & \boldsymbol{\Gamma}_{33} & \boldsymbol{\Gamma}_{34} & \boldsymbol{\Gamma}_{35} & 0 \\ & * & * & \boldsymbol{\Gamma}_{44} & \boldsymbol{\Gamma}_{45} & 0 \\ & * & * & * & \boldsymbol{\Gamma}_{55} & 0 \\ & * & * & * & * & -\gamma^2\boldsymbol{I} \end{bmatrix}$$

$$\boldsymbol{\Gamma}_2 = \begin{bmatrix} \boldsymbol{N}_1 & \boldsymbol{T}_1 & \boldsymbol{M}_1 & \boldsymbol{E}_1 \\ \boldsymbol{N}_2 & \boldsymbol{T}_2 & \boldsymbol{M}_2 & \boldsymbol{E}_2 \\ \boldsymbol{N}_3 & \boldsymbol{T}_3 & \boldsymbol{M}_3 & \boldsymbol{E}_3 \\ \boldsymbol{N}_4 & \boldsymbol{T}_4 & \boldsymbol{M}_4 & \boldsymbol{E}_4 \\ \boldsymbol{N}_5 & \boldsymbol{T}_5 & \boldsymbol{M}_5 & \boldsymbol{E}_5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boldsymbol{\Gamma}_{11} = \boldsymbol{P}\boldsymbol{A} + \boldsymbol{A}^{\mathrm{T}}\boldsymbol{P} + \sum_{i=1}^{3}\boldsymbol{Q}_i + \boldsymbol{N}_1 + \boldsymbol{N}_1^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{12} = \boldsymbol{P}\boldsymbol{B}(\boldsymbol{K} + \Delta(k)) + \boldsymbol{N}_2^{\mathrm{T}} - \boldsymbol{T}_1 + \boldsymbol{M}_1 - \boldsymbol{E}_1$$

$$\boldsymbol{\Gamma}_{13} = \boldsymbol{E}_1 + \boldsymbol{N}_3^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{14} = -\boldsymbol{M}_1 + \boldsymbol{N}_4^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{15} = \boldsymbol{T}_1 - \boldsymbol{N}_1 + \boldsymbol{N}_5^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{22} = \boldsymbol{M}_2 + \boldsymbol{M}_2^{\mathrm{T}} - \boldsymbol{T}_2 - \boldsymbol{T}_2^{\mathrm{T}} - \boldsymbol{E}_2 - \boldsymbol{E}_2^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{23} = \boldsymbol{E}_2 + \boldsymbol{M}_3^{\mathrm{T}} - \boldsymbol{T}_3^{\mathrm{T}} - \boldsymbol{E}_3^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{24} = -\boldsymbol{M}_2 + \boldsymbol{M}_4^{\mathrm{T}} - \boldsymbol{T}_4^{\mathrm{T}} - \boldsymbol{E}_4^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{25} = \boldsymbol{T}_2 - \boldsymbol{N}_2 + \boldsymbol{M}_5^{\mathrm{T}} - \boldsymbol{T}_5^{\mathrm{T}} - \boldsymbol{E}_5^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{33} = -\boldsymbol{Q}_1 + \boldsymbol{E}_3 + \boldsymbol{E}_3^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{34} = -\boldsymbol{M}_3 + \boldsymbol{E}_4^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{35} = \boldsymbol{T}_3 - \boldsymbol{N}_3 + \boldsymbol{E}_5^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{44} = -\boldsymbol{Q}_2 - \boldsymbol{M}_4 - \boldsymbol{M}_4^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{45} = \boldsymbol{T}_4 - \boldsymbol{N}_4 - \boldsymbol{M}_5^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_{55} = -(1-\alpha)\boldsymbol{Q}_3 + \boldsymbol{T}_5 - \boldsymbol{N}_5 + \boldsymbol{T}_5^{\mathrm{T}} - \boldsymbol{N}_5^{\mathrm{T}}$$

$$\boldsymbol{\Gamma}_3 = \mathrm{diag}\left[-\frac{1}{\alpha\bar{\sigma}}\boldsymbol{U}_2 \quad -\frac{1}{\bar{\sigma} - \alpha\underline{\sigma}}\boldsymbol{U}_2 \quad -\frac{1}{\sigma}\boldsymbol{U}_1 \quad -\frac{1}{\sigma}\boldsymbol{U}_1\right]$$

$$\mathcal{B} = [\boldsymbol{A} \quad \boldsymbol{B}(\boldsymbol{K} + \Delta(\boldsymbol{K})) \quad 0 \quad 0 \quad 0 \quad \boldsymbol{B}_\omega]$$

$$\mathcal{C} = [\boldsymbol{C} \quad \boldsymbol{D}(\boldsymbol{K} + \Delta(\boldsymbol{K})) \quad 0 \quad 0 \quad 0 \quad 0]$$

$$\sigma = T_M + \frac{\tau_M + \rho T_m}{2} - \tau_m$$

$$\bar{\sigma} = T_M + \frac{\tau_M + \rho T_m}{2}, \quad \underline{\sigma} = \tau_m \tag{17}$$

Proof Consider the following piecewise Lyapunov functional:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t) + V_6(t) \tag{18}$$

where

$$V_1(t) = \boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{P}\boldsymbol{x}(t)$$

$$V_2(t) = \int_{t-\tau_m}^{t} \boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{Q}_1\boldsymbol{x}(s)\mathrm{d}s$$

$$V_3(t) = \int_{t-(T_M + \frac{\tau_M + \rho T_m}{2})}^{t} \boldsymbol{x}(s)^{\mathrm{T}}\boldsymbol{Q}_2\boldsymbol{x}(s)\mathrm{d}s$$

$$V_4(t)=\int_{t-\alpha d(t)}^{t}\boldsymbol{x}(s)^{\mathrm T}\boldsymbol{Q}_3\boldsymbol{x}(s)\mathrm{d}s$$

$$V_5(t)=\int_{-(T_M+\frac{\tau_M+\rho T_m}{2})}^{-\tau_m}\int_{t+\beta}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s\mathrm{d}\beta,$$

$$V_6(t)=\int_{t-d(t)}^{t}(T_M+\frac{\tau_M+\rho T_m}{2}-t+s)\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s,$$

$\boldsymbol{P}=\boldsymbol{P}^{\mathrm T}>0$, $\boldsymbol{Q}_m={\boldsymbol{Q}_m}^{\mathrm T}>0$ $(m=1,2,3)$, $\boldsymbol{Z}_j={\boldsymbol{Z}_j}^{\mathrm T}>0$ $(j=1,2)$.

Obviously, $V(t)$ is discontinuous. Note that $V(t)$ is continuous and positive except the instants $s_k+\delta_k$. Therefore, we consider the following two cases:

(i) For $s_k+\tau_k<t<s_{k+1}+\tau_{k+1}$, calculating the derivative of $V(t)$ with respect to t along the solutions of the system and using the Leibniz-Newton formula, yield that

$$\begin{aligned}
&\dot V(t)+\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t)=\\
&2\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{P}\dot{\boldsymbol{x}}(t)+\sum_{i=1}^{3}\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Q}_i\boldsymbol{x}(t)-\\
&\boldsymbol{x}^{\mathrm T}(t-\tau_m)\boldsymbol{Q}_1\boldsymbol{x}(t-\tau_m)-\\
&\boldsymbol{x}^{\mathrm T}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))\boldsymbol{Q}_2\boldsymbol{x}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))-\\
&(1-\alpha)\boldsymbol{x}^{\mathrm T}(t-\alpha d(t))\boldsymbol{Q}_3\boldsymbol{x}(t-\alpha d(t))+\\
&\dot{\boldsymbol{x}}^{\mathrm T}(t)[(T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m)\boldsymbol{U}_1+\\
&(T_M+\frac{\tau_M+\rho T_m}{2})\boldsymbol{U}_2]\dot{\boldsymbol{x}}(t)-\\
&\int_{t-(T_M+\frac{\tau_M+\rho T_m}{2})}^{t-d(t)}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\tau_m}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s-\\
&\int_{t-\alpha d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s-\int_{t-d(t)}^{t-\alpha d(t)}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s+\\
&\boldsymbol{z}^{\mathrm T}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t)\leqslant\\
&2\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{P}(\boldsymbol{A}x(t)+\boldsymbol{B}(\boldsymbol{K}+\Delta(\boldsymbol{K}))\boldsymbol{x}(t-d(t))+\\
&\boldsymbol{B}_\omega\boldsymbol{\omega}(t))+\sum_{i=1}^{3}\boldsymbol{x}^{\mathrm T}(t)\boldsymbol{Q}_i\boldsymbol{x}(t)-\\
&\boldsymbol{x}^{\mathrm T}(t-\tau_m)\boldsymbol{Q}_1\boldsymbol{x}(t-\tau_m)-\\
&\boldsymbol{x}^{\mathrm T}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))\boldsymbol{Q}_2\boldsymbol{x}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))-\\
&(1-\alpha)\boldsymbol{x}^{\mathrm T}(t-\alpha d(t))\boldsymbol{Q}_3\boldsymbol{x}(t-\alpha d(t))+\\
&(\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}(\boldsymbol{K}+\Delta(\boldsymbol{K}))\boldsymbol{x}(t-d(t))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t))^{\mathrm T}\cdot\\
&[(T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m)\boldsymbol{U}_1+(T_M+\frac{\tau_M+\rho T_m}{2})\boldsymbol{U}_2]\cdot\\
&(\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}(\boldsymbol{K}+\Delta(\boldsymbol{K}))\boldsymbol{x}(t-d(t))+\boldsymbol{B}_\omega\boldsymbol{\omega}(t))-\\
&\int_{t-(T_M+\frac{\tau_M+\rho T_m}{2})}^{t-d(t)}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\tau_m}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\dot{\boldsymbol{x}}(s)\mathrm{d}s-\\
&\int_{t-\alpha d(t)}^{t}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s-\int_{t-d(t)}^{t-\alpha d(t)}\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\dot{\boldsymbol{x}}(s)\mathrm{d}s+\\
&2\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{M}[\boldsymbol{x}(t-d(t))-\boldsymbol{x}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))-\\
&\int_{t-(T_M+\frac{\tau_M+\rho T_m}{2})}^{t-d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s]+\\
&2\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{E}[\boldsymbol{x}(t-\tau_m)-\boldsymbol{x}(t-d(t))-\int_{t-d(t)}^{t-\tau_m}\dot{\boldsymbol{x}}(s)\mathrm{d}s]+\\
&2\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{N}[\boldsymbol{x}(t)-\boldsymbol{x}(t-\alpha d(t))-\int_{t-\alpha d(t)}^{t}\dot{\boldsymbol{x}}(s)\mathrm{d}s]+\\
&2\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{T}[\boldsymbol{x}(t-\alpha d(t))-\boldsymbol{x}(t-d(t))-\\
&\int_{t-d(t)}^{t-\alpha d(t)}\dot{\boldsymbol{x}}(s)\mathrm{d}s]+\\
&(\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}(\boldsymbol{K}+\Delta(\boldsymbol{K}))\boldsymbol{x}(t-d(t)))^{\mathrm T}\cdot\\
&(\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}(\boldsymbol{K}+\Delta(\boldsymbol{K}))\boldsymbol{x}(t-d(t)))-\gamma^2\boldsymbol{\omega}^{\mathrm T}(t)\boldsymbol{\omega}(t)\leqslant\\
&\boldsymbol{\zeta}^{\mathrm T}(t)[\boldsymbol{\Gamma}_1+(T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m)\boldsymbol{M}{\boldsymbol{U}_1}^{-1}\boldsymbol{M}^{\mathrm T}+\\
&(T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m)\boldsymbol{E}\boldsymbol{U}_1^{-1}\boldsymbol{E}^{\mathrm T}+\\
&\alpha(T_M+\frac{\tau_M+\rho T_m}{2})\boldsymbol{N}\boldsymbol{U}_2^{-1}\boldsymbol{N}^{\mathrm T}+\\
&((T_M+\frac{\tau_M+\rho T_m}{2})-\alpha\tau_m)\boldsymbol{T}\boldsymbol{U}_2^{-1}\boldsymbol{T}^{\mathrm T}+\\
&\mathcal{B}^{\mathrm T}((T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m)\boldsymbol{U}_1+\\
&(T_M+\frac{\tau_M+\rho T_m}{2})\boldsymbol{U}_2)\mathcal{B}+\mathcal{C}^{\mathrm T}\mathcal{C}]\boldsymbol{\zeta}(t)-\\
&\int_{t-(T_M+\frac{\tau_M+\rho T_m}{2})}^{t-d(t)}\boldsymbol{L}_1{\boldsymbol{U}_1}^{-1}{\boldsymbol{L}_1}^{\mathrm T}\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\tau_m}\boldsymbol{L}_2{\boldsymbol{U}_1}^{-1}{\boldsymbol{L}_2}^{\mathrm T}\mathrm{d}s-\int_{t-\alpha d(t)}^{t}\boldsymbol{L}_3{\boldsymbol{U}_2}^{-1}{\boldsymbol{L}_3}^{\mathrm T}\mathrm{d}s-\\
&\int_{t-d(t)}^{t-\alpha d(t)}\boldsymbol{L}_4{\boldsymbol{U}_2}^{-1}{\boldsymbol{L}_4}^{\mathrm T}\mathrm{d}s
\end{aligned}$$

where

$$\begin{aligned}
\boldsymbol{L}_1&=\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{M}+\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\\
\boldsymbol{L}_2&=\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{E}+\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_1\\
\boldsymbol{L}_3&=\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{N}+\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2\\
\boldsymbol{L}_4&=\boldsymbol{\zeta}^{\mathrm T}(t)\boldsymbol{T}+\dot{\boldsymbol{x}}^{\mathrm T}(s)\boldsymbol{U}_2
\end{aligned}$$

$$\boldsymbol{\zeta}(t)=[\boldsymbol{x}^{\mathrm{T}}(t)\ \boldsymbol{x}^{\mathrm{T}}(t-d(t))\ \boldsymbol{x}^{\mathrm{T}}(t-\tau_m)\cdot$$
$$\boldsymbol{x}^{\mathrm{T}}(t-(T_M+\frac{\tau_M+\rho T_m}{2}))\ \boldsymbol{x}^{\mathrm{T}}(t-\alpha d(t))\ \boldsymbol{\omega}^{\mathrm{T}}(t)]^{\mathrm{T}}.$$

By the Schur complements, combining (17) to obtain $\dot{V}(t)+\boldsymbol{z}^{\mathrm{T}}(t)\boldsymbol{z}(t)-\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(t)\boldsymbol{\omega}(t)<0$ for $t_k<t<t_{k+1}$ (In the following, we denote $t_k=s_k+\tau_k$ for $k=0,1,2,3,\ldots$). On the other hand, it is clear that $\dot{V}(t)<0$ when $\boldsymbol{\omega}(t)=0$. This implies that system (14) with $\boldsymbol{\omega}=0$ is asymptotically stable for $t_k<t<t_{k+1}$.

(ii) It is noted that $d(t)$ is discontinuous at the instants $s_k+\tau_k$. There exist: for $t=s_k+\tau_k$, $d(t)=\tau_k$, $\forall k\in\mathbf{N}$; for $t=(s_k+\tau_k)^-$, $d(t)=s_k+\tau_k-s_{k-1}$, $\forall k\in\mathbf{N}$.

Note that the value of the state $x(t)$ before and after the instants $s_k+\tau_k$ remains unchanged (since $\boldsymbol{x}(t)$ is continuous). Then, we have $V_i((s_k+\tau_k)^-)=V_i(s_k+\tau_k)$ $(i=1,2,3,5)$ in Lyapunov functional (19). Moreover, for $V_4(t)$ and $V_6(t)$, there exist $V_4((s_k+\tau_k)^-)\geqslant V_4(t_k)$ and $V_6((s_k+\tau_k)^-)\geqslant V_6(s_k+\tau_k)$. Thus, $V((s_k+\tau_k)^-)\geqslant V(s_k+\tau_k)$ for $k=0,1,2,3,\ldots$. For $t\in[t_k,\ t_{k+1})$, we have

$$V(t)-V(t_k)\leqslant\int_{t_k}^{t}-\boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{z}(s)+\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{\omega}(s)\mathrm{d}s.$$

Since $\lim\limits_{k\to\infty}t_k=\infty$, we have $\bigcup\limits_{k=0}^{\infty}[t_k\ \ t_{k+1})=[t_0,\ \infty)$. It follows that

$$V(t)-V(t_0)\leqslant\int_{t_0}^{t}-\boldsymbol{z}^{\mathrm{T}}(s)z(s)+\gamma^2\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{\omega}(s)\mathrm{d}s.$$

Let $t\to\infty$,

$$\int_{t_0}^{\infty}\boldsymbol{z}^{\mathrm{T}}(s)\boldsymbol{z}(s)\mathrm{d}s\leqslant\gamma^2\int_{t_0}^{\infty}\boldsymbol{\omega}^{\mathrm{T}}(s)\boldsymbol{\omega}(s)\mathrm{d}s,$$

which means $\|\boldsymbol{z}(t)\|_2\leqslant\gamma\|\boldsymbol{\omega}(t)\|_2$. □

Remark 3 Theorem 1 presents an H_∞ performance analysis condition of the networked control system with time-varying delays, time-varying sampling intervals and signal transmission deadbands, which is an LMI condition when the controller gain K is given. Theorem 1 describes the relationship between BND, BSI, BTD and the H_∞ disturbance attenuation level γ. Note that $d(t)$ is a piecewise-linear function and satisfies $\dot{d}(t)=1$. We choose a Lyapunov functional with discontinuities at the jump instants $(s_k+\tau_k,\ k=0,1,2,3\ldots)$ which takes full consideration of the characteristic information of the system state $\boldsymbol{x}(t)$ and the derivative of delay term $d(t)$. In the process of calculating the derivative of $V(t)$, we use the delay partitioning approach and a positive scalar $0<\alpha<1$ to divide the interval $[t-d(t),t]$ in term $\int_{t-d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{U}_2\boldsymbol{x}(s)\mathrm{d}s$ into two subintervals, that is, $[t-d(t),t-\alpha d(t)]$ and $[t-\alpha d(t),t]$. This can make full use of more information of the state. Moreover, in the proceedings of eliminating the integral terms such as $\int_{t-\alpha d(t)}^{t}\boldsymbol{x}^{\mathrm{T}}(s)\boldsymbol{U}_2\boldsymbol{x}(s)\mathrm{d}s$, we use the Leibniz-Newton formula and introduce some free weighting matrices to express the links between the terms in the formula. This has the potential to yield less conservative results.

Remark 4 The networked control systems (14) and (15) without transmission deadbands naturally become the following NCSs subject to time-varying delays and time-varying sampling intervals:

$$\dot{\boldsymbol{x}}(t)=\boldsymbol{A}\boldsymbol{x}(t)+\boldsymbol{B}\boldsymbol{K}\boldsymbol{x}(t-d(t)),\quad s_k+\tau_k\leqslant t<s_{k+1}+\tau_{k+1}\tag{19}$$

$$\boldsymbol{z}(t)=\boldsymbol{C}\boldsymbol{x}(t)+\boldsymbol{D}\boldsymbol{K}\boldsymbol{x}(t-d(t)),\quad s_k+\tau_k\leqslant t<s_{k+1}+\tau_{k+1}\tag{20}$$

where $d(t)$ satisfies (12), and the bounds of the input delay $d(t)$ also depends on the bounds of delays τ_m and τ_M, and the bounds of sampling intervals T_m and T_M. Combining the Lyapunov functional (19) (setting $\Delta(\boldsymbol{K})=0$) with Theorem 1, we can derive the robust H_∞ performance analysis condition for NCSs (20) and (21).

3.2 Robust H_∞ controller design for the NCSs with transmission deadbands

This section is devoted to solving the problem of H_∞ controller design for NCSs (14) and (15). The following theorem presents conditions for the existence of the desired controller.

Theorem 2 Consider the NCS in Fig. 1. For given scalars τ_m, τ_M, T_m, T_M $(0\leqslant\tau_m\leqslant\tau_M\leqslant T_m\leqslant T_M)$, $\delta_1>0$, $\delta_2>0$, $0\leqslant\alpha<1$, $\tau_M/T_m\leqslant\rho\leqslant1$, the closed-loop NCSs (14) and (15) are asymptotically stable with an H_∞ disturbance attenuation level γ if there exist matrices $\boldsymbol{X}=\boldsymbol{X}^{\mathrm{T}}>0$, $\tilde{\boldsymbol{Q}}_i=\tilde{\boldsymbol{Q}}_i^{\mathrm{T}}>0$ $(i=1,2,3)$, $\tilde{\boldsymbol{U}}_j=\tilde{\boldsymbol{U}}_j^{\mathrm{T}}>0$ $(j=1,2)$, $\tilde{\boldsymbol{N}}_l$, $\tilde{\boldsymbol{T}}_l$, $\tilde{\boldsymbol{M}}_l$, $\tilde{\boldsymbol{E}}_l$, $(l=1,2,3,4,5)$, and $\boldsymbol{Y}$ of appropriate dimensions, such that

$$\begin{bmatrix}\boldsymbol{\varXi} & \tilde{\boldsymbol{B}} & \tilde{\boldsymbol{Y}}\\ & -\dfrac{1}{\varepsilon_a}\boldsymbol{I} & 0\\ & * & -\dfrac{1}{\varepsilon_b}\boldsymbol{I}\end{bmatrix}<0\tag{21}$$

where

$$\boldsymbol{\varXi}=\begin{bmatrix}\boldsymbol{\varXi}_1 & \boldsymbol{\varXi}_2 & \boldsymbol{A}_L & \boldsymbol{A}_L & \boldsymbol{C}_L\\ & \boldsymbol{\varXi}_3 & 0 & 0 & 0\\ & * & -\dfrac{1}{\bar{\sigma}}(\tilde{\boldsymbol{U}}_1-2\boldsymbol{X}) & 0 & 0\\ & * & * & -\dfrac{1}{\sigma}(\tilde{\boldsymbol{U}}_2-2\boldsymbol{X}) & 0\\ & * & * & * & -\boldsymbol{I}\end{bmatrix}\tag{22}$$

$$\boldsymbol{\Xi}_1=\begin{bmatrix}\boldsymbol{\Xi}_{11} & \boldsymbol{\Xi}_{12} & \boldsymbol{\Xi}_{13} & \boldsymbol{\Xi}_{14} & \boldsymbol{\Xi}_{15} & \boldsymbol{B}_\omega\\ & \boldsymbol{\Xi}_{22} & \boldsymbol{\Xi}_{23} & \boldsymbol{\Xi}_{24} & \boldsymbol{\Xi}_{25} & 0\\ & * & \boldsymbol{\Xi}_{33} & \boldsymbol{\Xi}_{34} & \boldsymbol{\Xi}_{35} & 0\\ & * & * & \boldsymbol{\Xi}_{44} & \boldsymbol{\Xi}_{45} & 0\\ & * & * & * & \boldsymbol{\Xi}_{55} & 0\\ & * & * & * & * & -\gamma^2\boldsymbol{I}\end{bmatrix}$$

$$\boldsymbol{\Xi}_2=\begin{bmatrix}\tilde{\boldsymbol{N}}_1 & \tilde{\boldsymbol{T}}_1 & \tilde{\boldsymbol{M}}_1 & \tilde{\boldsymbol{E}}_1\\ \tilde{\boldsymbol{N}}_2 & \tilde{\boldsymbol{T}}_2 & \tilde{\boldsymbol{M}}_2 & \tilde{\boldsymbol{E}}_2\\ \tilde{\boldsymbol{N}}_3 & \tilde{\boldsymbol{T}}_3 & \tilde{\boldsymbol{M}}_3 & \tilde{\boldsymbol{E}}_3\\ \tilde{\boldsymbol{N}}_4 & \tilde{\boldsymbol{T}}_4 & \tilde{\boldsymbol{M}}_4 & \tilde{\boldsymbol{E}}_4\\ \tilde{\boldsymbol{N}}_5 & \tilde{\boldsymbol{T}}_5 & \tilde{\boldsymbol{M}}_5 & \tilde{\boldsymbol{E}}_5\\ 0 & 0 & 0 & 0\end{bmatrix}$$

$$\boldsymbol{\Xi}_{11}=\boldsymbol{AX}+\boldsymbol{XA}^{\mathrm{T}}+\sum_{i=1}^{3}\tilde{\boldsymbol{Q}}_i+\tilde{\boldsymbol{N}}_1+\tilde{\boldsymbol{N}}_1^{\mathrm{T}}+\tilde{\boldsymbol{L}}_1$$

$$\boldsymbol{\Xi}_{12}=\boldsymbol{BY}+\tilde{\boldsymbol{N}}_2^{\mathrm{T}}-\tilde{\boldsymbol{T}}_1+\tilde{\boldsymbol{M}}_1-\tilde{\boldsymbol{E}}_1$$

$$\boldsymbol{\Xi}_{13}=\tilde{\boldsymbol{E}}_1+\tilde{\boldsymbol{N}}_3^{\mathrm{T}},\quad \boldsymbol{\Xi}_{14}=-\tilde{\boldsymbol{M}}_1+\tilde{\boldsymbol{N}}_4^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{15}=\tilde{\boldsymbol{T}}_1-\tilde{\boldsymbol{N}}_1+\tilde{\boldsymbol{N}}_5^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{22}=\tilde{\boldsymbol{M}}_2+\tilde{\boldsymbol{M}}_2^{\mathrm{T}}-\tilde{\boldsymbol{T}}_2-\tilde{\boldsymbol{T}}_2^{\mathrm{T}}-\tilde{\boldsymbol{E}}_2-\tilde{\boldsymbol{E}}_2^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{23}=\tilde{\boldsymbol{E}}_2,\quad \boldsymbol{\Xi}_{24}=-\tilde{\boldsymbol{M}}_2,\quad \boldsymbol{\Xi}_{25}=\tilde{\boldsymbol{T}}_2-\tilde{\boldsymbol{N}}_2$$

$$\boldsymbol{\Xi}_{33}=-\tilde{\boldsymbol{Q}}_1+\tilde{\boldsymbol{E}}_3+\tilde{\boldsymbol{E}}_3^{\mathrm{T}},\quad \boldsymbol{\Xi}_{34}=-\tilde{\boldsymbol{M}}_3+\tilde{\boldsymbol{E}}_4^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{35}=\tilde{\boldsymbol{T}}_3-\tilde{\boldsymbol{N}}_3+\tilde{\boldsymbol{E}}_5^{\mathrm{T}},\quad \boldsymbol{\Xi}_{44}=-\tilde{\boldsymbol{Q}}_2-\tilde{\boldsymbol{M}}_4-\tilde{\boldsymbol{M}}_4^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{45}=\tilde{\boldsymbol{T}}_4-\tilde{\boldsymbol{N}}_4-\tilde{\boldsymbol{M}}_5^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_{55}=-(1-\alpha)\boldsymbol{Q}_3+\tilde{\boldsymbol{T}}_5-\tilde{\boldsymbol{N}}_5+\tilde{\boldsymbol{T}}_5^{\mathrm{T}}-\tilde{\boldsymbol{N}}_5^{\mathrm{T}}$$

$$\boldsymbol{A}_L=\begin{bmatrix}\boldsymbol{AX}^{\mathrm{T}} & \boldsymbol{BY} & 0 & 0 & 0 & \boldsymbol{B}_\omega\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{C}_L=\begin{bmatrix}\boldsymbol{CX}^{\mathrm{T}} & \boldsymbol{DY} & 0 & 0 & 0 & \boldsymbol{B}_\omega\end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{\Xi}_3=\mathrm{diag}\left[-\frac{1}{\alpha\bar{\sigma}}\tilde{\boldsymbol{U}}_2\quad -\frac{1}{\bar{\sigma}-\alpha\underline{\sigma}}\tilde{\boldsymbol{U}}_2\quad -\frac{1}{\sigma}\tilde{\boldsymbol{U}}_1\quad -\frac{1}{\sigma}\tilde{U}_1\right]$$

$$\sigma=T_M+\frac{\tau_M+\rho T_m}{2}-\tau_m$$

$$\bar{\sigma}=T_M+\frac{\tau_M+\rho T_m}{2},\quad \underline{\sigma}=\tau_m$$

$$\tilde{\boldsymbol{B}}=[\boldsymbol{B}^{\mathrm{T}}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \boldsymbol{B}^{\mathrm{T}}\ \boldsymbol{B}^{\mathrm{T}}\ \boldsymbol{D}^{\mathrm{T}}]^{\mathrm{T}}$$

$$\tilde{\boldsymbol{Y}}=\begin{bmatrix}0 & \boldsymbol{Y} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^{\mathrm{T}}$$

$$\varepsilon_b=\varepsilon_1+\varepsilon_2+\varepsilon_3\delta_1^2,\quad \varepsilon_a=\frac{1}{\varepsilon_1}\delta_2^2+\frac{1}{\varepsilon_2}\delta_1^2+\frac{1}{\varepsilon_3}\delta_2^2 \tag{23}$$

In this case, the state-feedback gain is given by

$$\boldsymbol{K}=\boldsymbol{YX}^{-1}.$$

Proof Define

$$\boldsymbol{G}_B=[\boldsymbol{B}^{\mathrm{T}}\boldsymbol{P}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{U}_1\ \boldsymbol{B}^{\mathrm{T}}\boldsymbol{U}_2\ \boldsymbol{D}^{\mathrm{T}}]$$

$$\boldsymbol{G}_K=[0\ \boldsymbol{K}\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$$

$$\tilde{\boldsymbol{\Gamma}}=\begin{bmatrix}\tilde{\boldsymbol{\Gamma}}_1 & \boldsymbol{\Gamma}_2 & \bar{\mathcal{B}}^{\mathrm{T}}\boldsymbol{U}_1 & \bar{\mathcal{B}}^{\mathrm{T}}\boldsymbol{U}_2 & \bar{\mathcal{C}}^{\mathrm{T}}\\ & \boldsymbol{\Gamma}_3 & 0 & 0 & 0\\ & * & -\frac{1}{\sigma}\boldsymbol{U}_1 & 0 & 0\\ & * & * & -\frac{1}{\bar{\sigma}}\boldsymbol{U}_2 & 0\\ & * & * & * & -\boldsymbol{I}\end{bmatrix}<0$$

$$\tilde{\boldsymbol{\Gamma}}_1=\begin{bmatrix}\boldsymbol{\Gamma}_{11} & \tilde{\boldsymbol{\Gamma}}_{12} & \boldsymbol{\Gamma}_{13} & \boldsymbol{\Gamma}_{14} & \boldsymbol{\Gamma}_{15} & \boldsymbol{PB}_w\\ & \boldsymbol{\Gamma}_{22} & \boldsymbol{\Gamma}_{23} & \boldsymbol{\Gamma}_{24} & \boldsymbol{\Gamma}_{25} & 0\\ & * & \boldsymbol{\Gamma}_{33} & \boldsymbol{\Gamma}_{34} & \boldsymbol{\Gamma}_{35} & 0\\ & * & * & \boldsymbol{\Gamma}_{44} & \boldsymbol{\Gamma}_{45} & 0\\ & * & * & * & \boldsymbol{\Gamma}_{55} & 0\\ & * & * & * & * & -\gamma^2\boldsymbol{I}\end{bmatrix}$$

$$\tilde{\boldsymbol{\Gamma}}_{12}=\boldsymbol{PBK}$$

$$\bar{\mathcal{B}}=\begin{bmatrix}\boldsymbol{A} & \boldsymbol{BK} & 0 & 0 & 0 & \boldsymbol{B}_\omega\end{bmatrix}$$

$$\bar{\mathcal{C}}=\begin{bmatrix}\boldsymbol{C} & \boldsymbol{DK} & 0 & 0 & 0 & 0\end{bmatrix}. \tag{24}$$

From (17), we have

$$\boldsymbol{\Gamma}=\tilde{\boldsymbol{\Gamma}}+\boldsymbol{G}_K{}^{\mathrm{T}}\Delta_2\boldsymbol{G}_B+\boldsymbol{G}_B{}^{\mathrm{T}}\Delta_2\boldsymbol{G}_K+\Delta_1\boldsymbol{G}_K{}^{\mathrm{T}}\boldsymbol{G}_B+\boldsymbol{G}_B{}^{\mathrm{T}}\boldsymbol{G}_K\Delta_1+\Delta_1\boldsymbol{G}_K{}^{\mathrm{T}}\Delta_2\boldsymbol{G}_B+\boldsymbol{G}_B{}^{\mathrm{T}}\Delta_2\boldsymbol{G}_K\Delta_1\leqslant$$
$$\tilde{\boldsymbol{\Gamma}}+\varepsilon_1\boldsymbol{G}_K{}^{\mathrm{T}}\boldsymbol{G}_K+\frac{1}{\varepsilon_1}\Delta_2{}^2\boldsymbol{G}_B{}^{\mathrm{T}}\boldsymbol{G}_B+\varepsilon_2\boldsymbol{G}_K{}^{\mathrm{T}}\boldsymbol{G}_K+\frac{1}{\varepsilon_2}\Delta_1{}^2\boldsymbol{G}_B{}^{\mathrm{T}}\boldsymbol{G}_B+\varepsilon_3\Delta_1{}^2\boldsymbol{G}_K{}^{\mathrm{T}}\boldsymbol{G}_K+\frac{1}{\varepsilon_3}\Delta_2{}^2\boldsymbol{G}_B{}^{\mathrm{T}}\boldsymbol{G}_B\leqslant$$
$$\tilde{\boldsymbol{\Gamma}}+(\varepsilon_1+\varepsilon_2+\varepsilon_3\delta_1^2)\boldsymbol{G}_K{}^{\mathrm{T}}\boldsymbol{G}_K+\left(\frac{1}{\varepsilon_1}\delta_2^2+\frac{1}{\varepsilon_2}\delta_1^2+\frac{1}{\varepsilon_3}\delta_2^2\right)\boldsymbol{G}_B{}^{\mathrm{T}}\boldsymbol{G}_B,$$

$$\begin{bmatrix}\tilde{\boldsymbol{\Gamma}} & \boldsymbol{G}_B^{\mathrm{T}} & \boldsymbol{G}_K^{\mathrm{T}}\\ & -\frac{1}{\varepsilon_a}\boldsymbol{I} & 0\\ & * & -\frac{1}{\varepsilon_b}\boldsymbol{I}\end{bmatrix}<0. \tag{25}$$

It can be shown that if (17) holds, there exists $\boldsymbol{\Gamma}<0$, that is, $\dot{V}(t)<0$ for all $s_k+\tau_k<t<s_{k+1}+\tau_{k+1}$.

Define $J=\mathrm{diag}\{J_1,J_2,J_3\}$, $J_1=\mathrm{diag}\{\boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}, \boldsymbol{P}^{-1}\}$, $J_2=\mathrm{diag}\{\boldsymbol{U}_1^{-1},\boldsymbol{U}_2^{-1}\}$, $J_3=\mathrm{diag}\{\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$.

Pre- and post-multiplying both sides of (26) by J, and denoting $\boldsymbol{X}=\boldsymbol{P}^{-1}$, $\tilde{\boldsymbol{N}}_l=\boldsymbol{XN}_l\boldsymbol{X}$, $\tilde{\boldsymbol{T}}_l=\boldsymbol{XT}_l\boldsymbol{X}$, $\tilde{\boldsymbol{M}}_l=\boldsymbol{XM}_l\boldsymbol{X}$, $\tilde{\boldsymbol{E}}_l=\boldsymbol{XE}_l\boldsymbol{X}$ $(l=1,2,3,4,5)$, $\tilde{\boldsymbol{Q}}_i=\boldsymbol{XQ}_i\boldsymbol{X}$ $(i=1,2,3)$, $\tilde{\boldsymbol{U}}_j=\boldsymbol{XU}_j\boldsymbol{X}$ $(j=1,2)$, $\boldsymbol{Y}=\boldsymbol{KX}$. Then, we have

$$\begin{bmatrix}\tilde{\boldsymbol{\Xi}} & \tilde{\boldsymbol{B}} & \tilde{\boldsymbol{Y}}\\ & -\frac{1}{\varepsilon_a}\boldsymbol{I} & 0\\ & * & -\frac{1}{\varepsilon_b}\boldsymbol{I}\end{bmatrix}<0 \tag{26}$$

where

$$\tilde{\boldsymbol{\Xi}} = \begin{bmatrix} \boldsymbol{\Xi}_1 & \boldsymbol{\Xi}_2 & \boldsymbol{A}_L & \boldsymbol{A}_L \\ & \boldsymbol{\Xi}_3 & 0 & 0 \\ & * & -\dfrac{1}{\overline{\sigma}}\boldsymbol{U}_1{}^{-1} & 0 \\ & * & * & -\dfrac{1}{\sigma}\boldsymbol{U}_2{}^{-1} \end{bmatrix}. \quad (27)$$

The conditions in above inequality (27) are not LMI because of the nonlinear terms $\boldsymbol{U}_j^{-1}$. It is noted that $\boldsymbol{U}_j^{-1} = \boldsymbol{X}\tilde{\boldsymbol{U}}_j^{-1}\boldsymbol{X}$ $(j = 1, 2)$. In order to solve this non-convex problem, the inequalities in the following are needed.

$$(\tilde{\boldsymbol{U}}_m - \boldsymbol{X})\tilde{\boldsymbol{U}}_m^{-1}(\tilde{\boldsymbol{U}}_m - \boldsymbol{X}) \geqslant 0, \quad m = 1, 2 \quad (28)$$

which is equivalent to

$$-\boldsymbol{X}\tilde{\boldsymbol{U}}_j^{-1}\boldsymbol{X} \leqslant \tilde{\boldsymbol{U}}_j - 2\boldsymbol{X}, \quad j = 1, 2. \quad (29)$$

Then, we can obtain (22). □

Remark 5 Consider the NCSs (20) and (21) with time-varying delays and sampling intervals. From Theorem 2 (for $\Delta(K) = 0$), we can easily obtain the H_∞ controller design condition of systems (20) and (21).

4. Examples

In the following, three examples will be given to illustrate the effectiveness and applicability of the proposed approaches.

Example 1 Consider the following system [25,26]:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}\boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}\boldsymbol{u}(t) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}\boldsymbol{\omega}(t) \quad (30)$$

$$\boldsymbol{z}(t) = \begin{bmatrix} 0 & 1 \end{bmatrix}\boldsymbol{x}(t) + 0.1\boldsymbol{\omega}(t). \quad (31)$$

Consider the NCS in Fig. 1, wherein no transmission deadbands exist. Choosing the low bound of communication delays $\tau_m = 0$ s, the MAEDB $\overline{\sigma} = 0.869\,5$ s, by using Remark 5, we can achieve the minimum H_∞ disturbance attenuation level $\gamma = 0.22$, while the result in [25] and [26] is $\gamma = 1.00$. In this case, the controller is $\boldsymbol{K} = [-0.000\,0 - 4.615\,5]$. From this example, one can see that this papers provides a less conservative result than [25] and [26].

Example 2 Consider a simplified model of the inverted pendulum process [27] as

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -1.84 & 0.33 \\ 7.18 & -1.14 \end{bmatrix}\boldsymbol{x}(t) +$$

$$\begin{bmatrix} 2.43 \\ -0.42 \end{bmatrix}\boldsymbol{u}(t) + \begin{bmatrix} 1.86 \\ -0.76 \end{bmatrix}\boldsymbol{\omega}(t) \quad (32)$$

$$\boldsymbol{z}(t) = \begin{bmatrix} 0.57 & 0.78 \end{bmatrix}\boldsymbol{x}(t) + 0.56\boldsymbol{\omega}(t). \quad (33)$$

Consider the NCS in Fig. 1, wherein no transmission deadbands exist. Choosing the bounds of communication delays $\tau_m = 0.1$ s, $\tau_M = 0.15$ s, the bounds of sampling intervals $T_m = 0.18$ s and $T_M = 0.48$ s, the scalars $\alpha = 0.36$, and $\rho = 0.85$, by using Remark 5, we can achieve the minimum H_∞ disturbance attenuation level $\gamma = 7.703\,4$. In this case, one can obtain the following solution:

$$\boldsymbol{X} = \begin{bmatrix} 0.108\,0 & -0.181\,1 \\ -0.181\,1 & 1.590\,8 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} -0.011\,1 & -0.076\,9 \end{bmatrix}$$

and the state feedback controller gain is $\boldsymbol{K} = [-0.226\,9 \quad -0.074\,2]$.

Example 3 Consider the following system [28]:

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix}\boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}\boldsymbol{u}(t) + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}\boldsymbol{\omega}(t) \quad (34)$$

$$\boldsymbol{z}(t) = \begin{bmatrix} 0.1 & 0.5 \end{bmatrix}\boldsymbol{x}(t). \quad (35)$$

Case 1 Consider the NCS in Fig. 1, wherein no transmission deadbands exist. Choosing the low bound of communication delays $\tau_m = 0$ s, the MAEDB $\overline{\sigma} = 0.3$ s, by using Remark 5, we can achieve the minimum H_∞ disturbance attenuation level $\gamma = 0.051\,2$, while the result in [28] is $\gamma = 0.903\,8$. In this case, the controller is $\boldsymbol{K} = [-3.529\,9 \quad -18.439\,4]$.

Case 2 Consider the case that there are two transmission deadbands in NCSs. Choosing the bounds of the network-induced delays $\tau_m = 0$ s and $\tau_M = 0.1$ s, the bounds of sampling intervals $T_m = 0.12$ s and $T_M = 0.2$ s, deadband bounds are $\delta_1 = 0.4$ and $\delta_2 = 0.2$, and the scalars $\alpha = 0.2$, $\rho = 0.9$. By using Theorem 3, solving the LMI problem (22), one can obtain the following solution:

$$\boldsymbol{X} = \begin{bmatrix} 645.451\,2 & -103.122\,0 \\ -103.122\,0 & 17.156\,6 \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} 33.372\,4 & -14.254\,1 \end{bmatrix}.$$

Hence, the state feedback controller gain $\boldsymbol{K} = \begin{bmatrix} -2.041\,2 & -13.099\,8 \end{bmatrix}$, and the H_∞ disturbance attenuation level $\gamma = 0.656\,3$. The disturbance signal $\omega(t)$ is given as

$$\boldsymbol{\omega}(t) = \begin{cases} 2\sin t, & 5 \leqslant t \leqslant 15 \\ 0, & t < 5 \text{ or } t > 15 \end{cases}. \quad (36)$$

Fig. 2 shows the state response and the control input under deadband control, where the initial state of the NCS is $\boldsymbol{x}_0 = [0,\ 0]^{\mathrm{T}}$. It can been seen that the closed-loop NCS is asymptotically stable with the above obtained control gain. Table 1 gives the minimum H_∞ disturbance attenuation level γ for different bounds of the deadband, wherein the bounds of the network-induced delays are chosen as $\tau_m = 0$ s, $\tau_M = 0.1$ s and the bounds of sampling intervals are chosen as $T_m = 0.12$ s and $T_M = 0.2$ s.

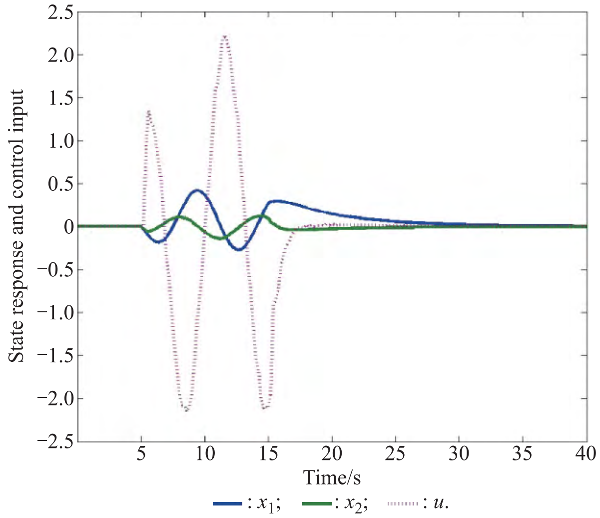

Fig. 2 State response and control input of the closed-loop NCS with two transmission deadbands

Table 1 H_∞ disturbance attenuation level γ with given $\tau_m = 0$ s, $\tau_M = 0.1$ s, $T_m = 0.12$ s and $T_M = 0.2$ s

Bounds of the deadband	γ
$\delta_1 = 0.4$ and $\delta_2 = 0.2$	0.656 3
$\delta_1 = 0.2$ and $\delta_2 = 0.1$	0.242 3

Fig. 3 shows the difference between the previous value $\tilde{x}(s_k)$ and the most recent value $\tilde{x}(s_{k+1})$ of the output signals from transmission deadband 1 for $\delta_1 = 0.2$ and $\delta_2 = 0.1$, wherein the curve output is not zero, which indicates the state signals are transmitted in the network channel. From Fig. 3, it can be found the area of zero shows the state signal of the networked systems is not sent in the channel. One can see that the amount of data transmission can be reduced by adopting the deadband control approach. Fig. 4 shows the output situation of transmission deadband 1 for $\delta_1 = 0.4$ and $\delta_2 = 0.2$. It can be seen that the quantity of data transmission can be reduced more compared to the case of $\delta_1 = 0.2$ and $\delta_2 = 0.1$.

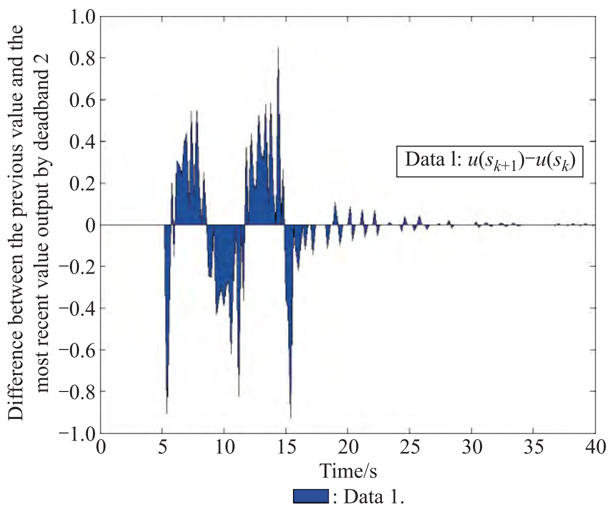

Fig. 3 Transmitting situation of the state for $\delta_1 = 0.2, \delta_2 = 0.1$

From Fig. 4, for the whole simulation time t ranging from 0 s to 40 s, the transmitting time of the state signal mainly focuses on 5 s to 25 s, so, the quantity of the data transmission is about 50% for this time interval.

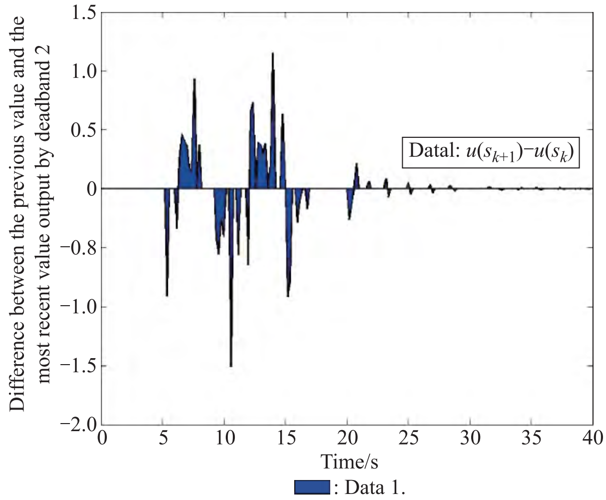

Fig. 4 Transmitting situation of the state for $\delta_1 = 0.4, \delta_2 = 0.2$

The transmitting situations of the control input signal for $\delta_1 = 0.2, \delta_2 = 0.1$ and $\delta_1 = 0.4, \delta_2 = 0.2$ have been shown in Fig. 5 and Fig. 6, respectively.

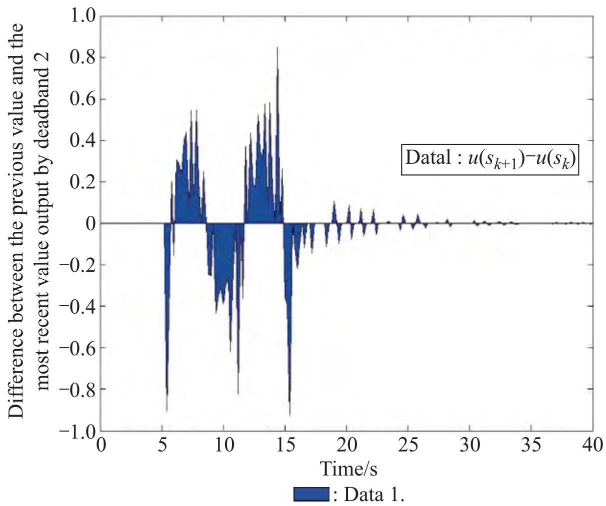

Fig. 5 Transmitting situation of the control input for $\delta_1 = 0.2$, $\delta_2 = 0.1$

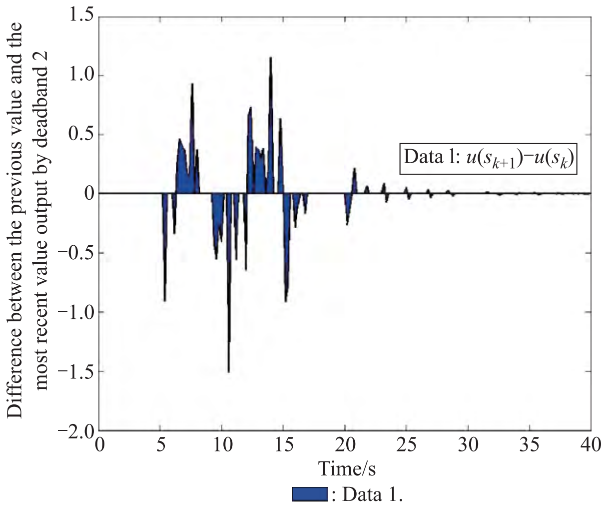

Fig. 6 Transmitting situation of the control input for $\delta_1 = 0.4$, $\delta_2 = 0.2$

Notice that the transmitting number of the control input signal for the case $\delta_1 = 0.4, \delta_2 = 0.2$ is less than the one of the case $\delta_1 = 0.2, \delta_2 = 0.1$. These have shown that the proposed signal difference-based deadband H_∞ control approach can effectively reduce the data transmission in the network channel, and the designed H_∞ controller can guarantee the system asymptotically stable and obtain a certain disturbance attenuation level.

5. Conclusions

This paper provides a signal deadband H_∞ control approach to reduce network data traffic in the view of limited network resources and high network collision in NCSs. Considering network-induced delays, sampling intervals and transmitting deadbands, the NCSs model is presented. By using a Lyapunov-functional approach, a less conservative H_∞ performance analysis condition of such NCSs is proposed. Moreover, the given deadband H_∞ controller design condition is in term of LMI. Three examples are used to indicate the effectiveness of the given methods. The results of simulative experiments show that the data traffic in the NCSs is obviously reduced, and the H_∞ control performance of the systems is guaranteed by the provided approach.

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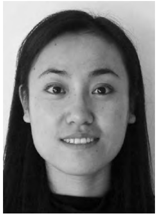

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