Model Predictive Control for Space Teleoperation Systems Based on a Mixed-$H_2/H_\infty$ Approach

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Abstract: In this paper a novel model predictive control (MPC) approach is proposed based on mixed-$H_2/H_\infty$ control for space teleoperation systems with unknown, large time-varying delays and input constraints. This novel approach only measures the online delay; it does not presume the delay bounds. In addition, $H_\infty$ control has been incorporated into the original MPC, which compensates for large time-varying delays, handles input constraints, and provides the desired tracking performance. First, the space teleoperation system is built based on a type of control architecture and then converted into a discrete state-space equation. Next, a state-feedback controller is designed based on linear matrix inequality (LMI). Meanwhile, the corresponding sufficient conditions are derived for the controller, enabling the closed-loop system to be asymptotically stable and guaranteeing the prescribed mixed-$H_2/H_\infty$ performance under input constraints. The parameters of the controller are calculated online by updating the mixed-$H_2/H_\infty$ optimizing problem during each interval. Finally, comparative simulations reveal that the proposed approach can achieve better performance in terms of tracking ability than that of traditional MPC. Moreover, it can ensure the input constraints in the unknown, large time-varying delay scenario whereas the original $H_\infty$ control approach cannot guarantee performance. DOI: 10.1061/(ASCE)AS.1943-5525.0000469. © 2014 American Society of Civil Engineers.

Author keywords: Space teleoperation; Predictive control; Large time-varying delay; LMI; $H_2/H_\infty$.

Introduction

Time-varying delay is one of the major problems in bilateral teleoperation systems that urgently need to be solved. Especially in space teleoperation systems, the roundtrip time delay (RTT) can often be several seconds and even up to dozens of minutes (Burridge et al. 2009). Large time-varying delays may degrade the performance of teleoperation systems and can even cause instability in closed-loop systems (Zhou et al. 2014). Burridge et al. (2009) proposed that a system can be stabilized by means of a bilateral control approach when the time delay is less than 2 s. To some extent, the robot is supposed to have autonomous abilities to stabilize the system when the time delay is greater than 10 s. During the intermediate time delay (2–10 s), the stabilized states can be realized by bilateral control approaches or by the autonomous abilities of the robot system. Unfortunately, many existing bilateral control approaches (e.g., passive control schemes) have failed to meet the requirement in this range, which is considered a large time delay.

This paper focuses on solving these tough issues, proposing a novel MPC method based on mixed-$H_2/H_\infty$ control for bilateral space teleoperation systems in which the communication network creates large time-varying delays and inputs are constrained. A novel control architecture is also presented, within which impedance control is used to control the master robot manipulator and a MPC based on a mixed-$H_2/H_\infty$ controller is designed on the slave side in which the RTT is also measured.

The main contribution of this research is that it combines MPC and $H_\infty$ control to deal with the time-varying delay and input constraint problems in space teleoperation systems. To the best of the authors’ knowledge, it is the first application in the area of time-varying delay teleoperation. This method does not need any information about the delay in advance, even about the bounds of the delay.

Background

Many scholars have proposed control methods to conquer the time delay problems of bilateral teleoperation systems (e.g., Sheridan 1993; Arcara and Melchiorri 2002; Hokayem and Spong 2006). In this paper, two of the most important control schemes, which are based on $H_\infty$ and MPC, are reviewed.

$H_\infty$ control can guarantee system stability. At the same time, it can reduce the influence on system performance of bounded disturbance at a given level regardless of the disturbance characteristics. For this reason, $H_\infty$ control schemes have been applied in many teleoperation systems. As far as is known, they can be roughly divided into two groups. One group models time delay as a disturbance or uncertain term. For example, Leung and Francis (1995) modeled time delay as a disturbance and designed a controller using the $\mu$-synthesis method, which enables the closed-loop system to be both stabilized and robust against the delay disturbance. Considering time delay as an uncertain term, Fattouh and Sename (2003) designed an $H_\infty$ impedance controller, but this approach often leads to large conservation. Sename and

Fattouh (2007) also used a mixed sensitivity method and \( \mu \)-analysis and a synthesis toolbox to design an \( \mathcal{H}_\infty \) controller for bilateral teleoperation systems with an uncertain environment model and time delays. Alfi and Farrokhi (2008b) considered the stability of bilateral teleoperation systems under time delay uncertainties and modeled mismatches based on a simple system structure (Alfi and Farrokhi 2008a); the sufficient conditions of stability for both cases were proposed. Based on this structure, Khoosravi et al. (2013) presented an LMI-based robust control method for bilateral teleoperation systems with model mismatch. In this method, a mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) controller is designed to deal with uncertainty in the time delay and the measurement noise of the reflected force. The main idea behind \( \mathcal{H}_2/\mathcal{H}_\infty \) control is that the system not only should be robust but also should meet other performance standards. The robust performance is measured by the \( \mathcal{H}_\infty \) norm, and the rest of the system’s performance is measured by the \( \mathcal{H}_2 \) norm. Alfi et al. (2014) employed \( \mu \)-synthesis based on particle swarm optimization to design controllers for bilateral teleoperation systems.

The other group of \( \mathcal{H}_\infty \) control schemes uses \( \mathcal{H}_\infty \) controller design methods for time delay systems. Sadeghi et al. (2008) proposed a linear matrix inequality (LMI) approach to design robust \( \mathcal{H}_\infty \) and \( \mathcal{L}_1 \) controllers for a bilateral teleoperation system. This method does not consider force tracking, and the controller designed for the slave side is open loop. Yang et al. (2013) proposed a robust \( \mathcal{H}_\infty \) control method for space teleoperation systems to deal with large time-varying delays and the uncertainty of the environmental model parameters. Du (2013) considered the controller design problem for bilateral teleoperation systems with asymmetric time-varying delays and converted this problem into an \( \mathcal{H}_\infty \) state-feedback controller design problem for a system with multiple-input time delays.

The \( \mathcal{H}_\infty \) control methods just mentioned need to know or assume the delay bounds. If the assumed bounds are larger than their real value, the results should be conservative. However, if the assumed bounds are smaller, the results cannot achieve the required performance and can even lead to system instability.

Model predictive control is another method that is widely used in teleoperation systems because it has the advantages of compensating large time delays and handling input or output constraints. Sheng and Spong (2004) proposed a modified MPC method to deal with the time delay and input or output constraint problems of teleoperation systems. It can achieve better robust performance for bilateral teleoperation systems in the presence of uncertain transmission time delays, but it is suitable only for time delays that fluctuate in relatively small ranges. Sirouspour and Shahdi (2006) proposed a multimodel predictive controller to enhance teleoperation transparency in the presence of a known constant delay. Slama et al. (2005) applied generalized predictive control (GPC) to bilateral teleoperation systems with communication delay and force feedback. Their delayed GPC (DGPC) internal model uses a \( \Pi \)-freeness algebraic property of the mechanical delayed system for the slave side to track the master-side reference position. Later Slama et al. (2007) proposed a bilateral GPC (BGPC) model for teleoperation systems with both communication delays and slave force feedback. They considered the case where the reference trajectory was not known a priori. However, they looked only at constant or relative small time-varying delays and neglected large ones. Based on the Slama et al. (2007) method, Yang et al. (2012) proposed a GPC strategy based on a state-space model for space teleoperation systems with large time-varying delays, but it is difficult to prove the system’s stability and robustness is not considered. \( \mathcal{H}_\infty \) control and MPC each have their advantages and disadvantages. In the area of control theory, many researchers have proposed different schemes to combine their advantages. Tadmor (1992) applied the linear-system \( \mathcal{H}_\infty \) control theory to moving-horizon control and proposed zero-terminal-constraint \( \mathcal{H}_\infty \) control for continuous linear time-varying systems with outside disturbances. Since Kothare et al. (1996) first proposed using LMI to solve robust-constraint MPC problems, researchers have presented various moving-horizon \( \mathcal{H}_\infty \) control schemes (Kim and Kwon 2002; Chen and Scherer 2004; Kim 2003) and MPC methods based on mixed \( \mathcal{H}_2/\mathcal{H}_\infty \) for different systems. Orukpe et al. (2007) extended the method proposed by Kothare et al. (1996) to constrained linear discrete-time-invariant systems by introducing a mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) approach that guarantees robustness. In Orukpe (2011), a less conservative MPC control based on a mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) control scheme was presented and the issue of uncertainty in the model was considered; however, time delay systems were not addressed. None of the approaches just discussed has been used in teleoperation systems.

In this paper, a novel MPC is proposed based on mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) control for bilateral space teleoperation systems that successfully combines the advantages of both MPC and \( \mathcal{H}_\infty \) control. The master, slave, and environment dynamics are transformed into a time-varying discrete state-space equation, based on which a state feedback controller is designed. Then, based on LMI, sufficient conditions are derived such that the controller enables the closed-loop system to be asymptotically stable and guarantees a prescribed mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) performance. At each sampling time, the LMI-optimizing problems are updated with the system’s instantaneous states and time delays, and the controller parameters are calculated online. In other words, this method changes \( \mathcal{H}_2/\mathcal{H}_\infty \) performance online according to the current time delay to ensure that the system is stable and satisfies input constraints. Last, some comparative simulations are carried out to verify the effectiveness of the method.

This paper is organized as follows. The next section describes the space teleoperation architecture and formulates the system state equation and the mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) problem. In the following section sufficient conditions for the existence of the robust MPC controller are derived. The third section presents comparative simulation results on space teleoperation systems. The final section presents conclusions.

Problem Formulation

The proposed space teleoperation architecture is shown in Fig. 1. An impedance controller is designed on the master side to control the master robot manipulator, and a MPC based on mixed-\( \mathcal{H}_2/\mathcal{H}_\infty \) control is designed on the slave side. A human operator on the ground applies a force \( f_s(t) \) on the joystick, and the master controller calculates the control force \( f_m(t) \) according to the reference impedance and the delayed slave force feedback \( f_{ed}(t) \)—that is, the react force between slave manipulator and environment. Accordingly, the joystick gives the reference position \( x_m(t) \) and velocity \( v_m(t) \), which are transmitted to the space station by the communication network. Then, according to the four states—delayed reference position \( x_{md}(t) \) and velocity \( v_{md}(t) \), and slave position \( x_s(t) \) and velocity \( v_s(t) \)—a mixed \( \mathcal{H}_2/\mathcal{H}_\infty \)-based MPC controller is designed to control the slave robot manipulator in order to obtain the stability and robustness of the system under the input constraints. For simplicity, a one-degree-of-freedom (DOF) master/slave system is considered. The dynamics and the environment are expressed as follows:

\[
\begin{align*}
    m_m \ddot{x}_m(t) + b_m \dot{x}_m(t) + k_m x_m(t) &= f_m(t) + f_A(t) \\
    m_s \ddot{x}_s(t) + b_s \dot{x}_s(t) + k_s x_s(t) &= f_s(t) - f_f(t)
\end{align*}
\]
where $x, f \in \mathbb{R}$; subscripts $m, s, h$, and $e$ = master, slave, human, and environment indexes, respectively; parameters $m, b, k = \text{inertia, damping coefficient, and stiffness gain, respectively}; x = \text{position}; f_h(t) = \text{force applied by a human operator}; f_m(t) \text{ and } f_s(t) = \text{control input forces applied on the master and slave robot, respectively}; \text{and } f_c(t) = \text{contact force between the slave manipulator and the environment}.$

Impedance control, which is a widely used method in teleoperation systems, is applied to control the master robot manipulator. Assume that the desired impedance of the master is as follows:

\[
\dot{\mathbf{x}} = \mathbf{A}_m \mathbf{x}_m + \mathbf{B}_m \mathbf{u} + \mathbf{H}_m f_m(t) + \mathbf{D}_m \mathbf{w}(t - \tau_{ms})
\]

where \(f_m(t) = f_s(t - \tau_{ms}) \text{ and } x_{md}(t) = x_m(t - \tau_{ms}).\)

Combining Eqs. (1\text{a}) and (2), easily yields the master control law as follows:

\[
f_m(t) = \left( \frac{m}{\mathbf{m}_m} \mathbf{b}_m \right) \mathbf{x}_m(t) + \left( \frac{k_m}{\mathbf{m}_m} \dot{\mathbf{x}}_m(t) \right) + \left( \frac{m}{\mathbf{m}_m} - 1 \right) f_h(t) - \frac{m}{\mathbf{m}_m} f_{ed}(t)
\]

The combination of Eqs. (1b) and (1c) yields:

\[
m_k \ddot{x}_s(t) + (b_s + b_e) \dot{x}_s(t) + (k_s + k_e) x_s(t) = f_s(t)
\]

Let $x_1 = x_s, x_2 = \dot{x}_s$; the slave dynamics are transformed into the following state-space equation:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-k_s / m_s & -b_s / m_s
\end{bmatrix}
\begin{bmatrix}
x_1(t) \\
x_2(t)
\end{bmatrix}
\quad +
\begin{bmatrix}
0 \\
1 / m_s
\end{bmatrix}
\mathbf{u}(t)
\]

where $u(t) = f_s(t)$. Let $x_1(t) = x_m(t - \tau_{ms}), x_2(t) = x_m(t - \tau_{ms})$; the master dynamics are transformed into the following state-space equation:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-k_m / m_m & -b_m / m_m
\end{bmatrix}
\begin{bmatrix}
x_1(t - \tau) \\
x_2(t - \tau)
\end{bmatrix}
\quad +
\begin{bmatrix}
0 \\
1 / m_m
\end{bmatrix}
\mathbf{u}(t - \tau)
\]

where $\tau(t) = \tau_{ms}(t) + \tau_{sm}(t)$ and $|\tau(t)| < 1; w(t) = f_h(t), f_h(t) = \text{human operating force}.$

Because human energy is limited, $f_h(t)$—that is, $w(t)$—satisfies the bounded-energy condition

\[
\int_{t=0}^{\infty} \|w(t)\|^2 dt \leq \alpha
\]

Let

\[
x(t) = \begin{bmatrix} x_1^T(t) & x_2^T(t) & x_3^T(t) & x_4^T(t) \end{bmatrix}^T
\]

Thus, the entire system state-space equation is as follows:

\[
x(t) = \mathbf{A}x(t) + \mathbf{A}_1 x(t - \tau) + \mathbf{B}_u + \mathbf{B}_w w(t - \tau_{ms})
\]

where

\[
\begin{array}{c}
\mathbf{A} = \\
\mathbf{A}_1 = \\
\mathbf{B} = \\
\mathbf{B}_w =
\end{array}
\]

Eq. (7) is discretized, and the objective control output $z(k)$ is defined in Eq. (8):

\[
x(k + 1) = \mathbf{A}_d x(k) + \mathbf{A}_{d1} x[k - d(k)] + \mathbf{B}_d u(k) + \mathbf{B}_{dw} w[k - d_m(k)]
\]

\[
z(k) = \left[ \begin{array}{c} \mathbf{C} x(k) \end{array} \right]^T + \mathbf{qu}(k)^T
\]

where

\[
\begin{array}{c}
z(k) = \{ x_m[k - d_m(k)] - \alpha_p x_s(k) \}^T; \\
\mathbf{C} = \begin{bmatrix} -\alpha_p & 0 & 1 & 0 \end{bmatrix}; \\
\mathbf{d}_m(k) = \frac{\tau_{ms}(t)}{T_s}
\end{array}
\]

$T_s$ = sample period; and $q$ = the weighting scalar.
Consider the following state-feedback controller:
\[ u(k) = Kx(k) \]  
(9)

Applying this controller to Eq. (8) yields the closed-loop system as
\[ x(k + 1) = (A_d + B_dK)x(k) + A_dx[k - d(k)] + B_dw[k - d_m(k)] \]
\[ z(k) = \begin{bmatrix} C \\ qK \end{bmatrix} x(k) \]
(10)

The aim is to design the state-feedback controller in Eq. (9) such that, for all addressed unknown delays, the following constraints are satisfied:
- When outside disturbance \( w(k) = 0 \), the closed-loop system in Eq. (10) is asymptotically stable.
- For a given \( \gamma_2 > 0 \), the controlled output \( z(k) \) satisfies the \( H_2 \) constraint:
\[ \|z\|_2^2 = \sum_{k=0}^{\infty} z_k^2 < \gamma_2 \]
(11)

- For a given \( \gamma_\infty > 0 \), the controlled output \( z(k) \) satisfies Eq. (12) under the zero-initial condition:
\[ J = \sum_{k=0}^{\infty} [z^T(k)[z(k) - \gamma_\infty^2 w^T(k - d_m(k))] \leq 0, \]
\[ \forall w(k - d_m(k)) \neq 0 \]
(12)

- For a given \( u_{\text{max}} > 0 \), the inputs satisfy
\[ u^T(k)u(k) \leq u_{\text{max}}^2, \quad \forall k \]
(13)

**Controller Design**

**Theorem 1:** For a given scalar \( \gamma_\infty > 0 \) or \( \gamma_2 > 0 \), the teleoperation system in Eq. (10) is asymptotically stable and satisfies \( H_2/H_\infty \) performance under the constrained input in Eq. (13) if there exist positive-definite matrices, \( X = X^T > 0 \) and \( W = W^T > 0 \), and scalar \( U \) such that
\[
\begin{bmatrix}
-X & * & * & * & * & * \\
0 & -W & * & * & * & * \\
0 & 0 & -\gamma_2^2 \gamma_\infty^2 I & * & * & * \\
A_dX + B_dY & A_dI & -X & * & * & * \\
CX & 0 & 0 & 0 & -W & * \\
qY & 0 & 0 & 0 & 0 & -\gamma_2^2 I \\
\end{bmatrix} \leq 0
\]
(14)

\[
\begin{bmatrix}
-1 & * & * & * & * & * \\
\gamma_2^2 \alpha & -\gamma_2^2 \gamma_\infty^2 \alpha & * & * & * & * \\
x_k & 0 & -X & * & * & * \\
x_{k-1} & 0 & 0 & -W & * & * \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & * \\
x_{k-d} & 0 & 0 & 0 & 0 & -W \\
\end{bmatrix} \leq 0
\]
(15)

**Proof:** Let \( X = \gamma_2^2 \gamma_\infty^2 P^{-1} \), \( Y = \gamma_2^2 \gamma_\infty^2 W^{-1} \); then Eq. (14) can be rewritten as
\[
\begin{bmatrix}
-P & * & * & * & * & * \\
0 & -R & * & * & * & * \\
0 & 0 & -\gamma_2^2 I & * & * & * \\
A_d + B_dK & A_dI & -P^{-1} & * & * & * \\
I & 0 & 0 & 0 & -R^{-1} & * \\
C & 0 & 0 & 0 & 0 & -I \\
qK & 0 & 0 & 0 & 0 & -I \\
\end{bmatrix} \leq 0
\]
(18)

Define a Lyapunov function for the system [Eq. (10)] as
\[
V(k) = x^T(k)Px(k) + \sum_{m=1}^{d(k)} x^T(k-m)Rx(k-m)
\]
(19)

For \( |\dot{x}(t)| < 1 \), \( d(k + 1) = d(k) \) for the current time delay \( d(k) \). Therefore
\[
\Delta V(k) = V(k + 1) - V(k) = x^T(k + 1)Px(k + 1) + x^T(k)(-P + R)x(k) - x^T(k - d)Rx(k - d)
\]
\[= \xi(k)^T A \xi(k) - z^T(k)z(k) + \gamma_\infty^2 w^T(k)w(k) \]
(20)

where
\[
A = \begin{bmatrix}
A_d^T PA_{cl} - P + R + C^T C & * & * & * & * & * \\
A_d^T PA_{cl} & A_d^T PA_{dl} - R & * & * & * & * \\
B_d^T PA_{cl} & B_d^T PA_{dl} & B_d^T PB_d & * & * & * \\
\end{bmatrix}
\]

When \( w(k) = 0 \), by Schur complement, Eq. (18) implies that \( A < 0 \), and for \( z^T(k)z(k) \geq 0 \) there is
\[
\Delta V(k) = V(k + 1) - V(k) < 0
\]
(21)

Therefore, the closed-loop system [Eq. (10)] is asymptotically stable.

For a stable system, \( \lim_{k \rightarrow \infty} x(k) = 0 \). Adding Eq. (20) from \( k = 0 \) to \( \infty \) achieves Eq. (22)
\[
-V(0) = \sum_{k=0}^{\infty} \xi(k)^T A \xi(k) - \sum_{k=0}^{\infty} \xi(k)^T z(k) - \gamma_\infty^2 w^T(k)w(k) \]
(22)

Therefore, under the zero initial condition—that is, \( x(0) = x(-1) = \cdots = x(-d) = 0 \), the following obtains:
\[
\sum_{k=0}^{\infty} \xi(k)^T z(k) - \gamma_\infty^2 w^T(k)w(k) = \sum_{k=0}^{\infty} \xi(k)^T A \xi(k) \geq 0
\]
(23)

Thus, Eq. (12) obtains if \( A < 0 \). By Schur complement, Eq. (18) implies that \( A < 0 \), so the closed-loop system satisfies \( H_\infty \) performance.
Eq. (22) can be rewritten as follows:

\[
\|z\|_2^2 = V(0) + \sum_{k=0}^{\infty} \xi(k)^T A_k \xi(k) + \sum_{k=0}^{\infty} \gamma_{\infty} w^T(k) w(k)
\]

\[
\leq x^T(0) P x(0) + \sum_{m=0}^{d(0)} x^T(-m) R x(-m) + \gamma_{\infty} \alpha^2 \leq \gamma_{\infty}^2
\]  

(24)

By Schur complement, Eq. (15) is equivalent to Eq. (24), so the closed-loop system satisfies \( H_2 \) performance.

Because \( A < 0 \), it follows from Eq. (20) that

\[
V(i + 1) - V(i) \leq -\xi^T(i) \xi(i) + \gamma_{\infty} \alpha w^T(i) w(i)
\]

Adding the inequality [Eq. (25)] from \( i = 0 \) to \( i = k - 1 \) yields Eq. (26) for all \( k > 0 \)

\[
V(k) \leq V(0) + \sum_{i=0}^{k-1} \xi^T(i) \xi(i) - \gamma_{\infty} \alpha w^T(i) w(i)]
\]

\[
\leq x^T(0) P x(0) + \sum_{m=0}^{d(0)} x^T(-m) R x(-m) + \gamma_{\infty} \alpha
\]  

(26)

Combining (24) and (26) gives

\[
x^T(k) P x(k) + \sum_{m=1}^{d(k)} x^T(k - m) R x(k - m) \leq \gamma_{\infty}^2
\]  

(27)

The inequality [Eq. (27)] can be rewritten as

\[
x^T(k) X^{-1} x(k) + \sum_{m=1}^{d(k)} x^T(k - m) W^{-1} x(k - m) \leq 1
\]  

(28)

Thus

\[
\Theta = \left[ x^T X^{-1} x + \sum_{m=1}^{d(k)} x^T(-m) W^{-1} x(-m) \leq 1 \right]
\]

is an invariant set. Weight matrix \( W \) is a positive set, so Eq. (29) is an invariant ellipsoid for the predicted states of the system:

\[
\varepsilon = \{ x | x^T X^{-1} x \leq 1 \}
\]  

(29)

Next, the constraint on the input in Eq. (13) can be transformed to LMI in Eq. (16) using the method in Kothare et al. (1996). The proof is omitted here because of limited space. The proof ends here.

Note 1: The MPC method based on mixed \( H_2/H_{\infty} \) for space teleoperation systems subject to constrained input and unknown time-varying delay can be described as the following LMI-based optimization problem:

\[
\text{min} \gamma_{\infty}^2 \quad X, W, Y, U \quad \text{s.t.} \ (14), (15), (16)
\]  

(30)

At sample time \( k \), for a given \( \gamma_2 \), the optimization results are \( (\gamma_{\infty}, X_\infty, W_\infty, Y_\infty, U_\infty) \) by solving Eq. (30). Then the control parameter \( K_k = Y_k X_k^{-1} \) is used to control the system until the next sample time. At each sampling time, Eq. (30) is updated with the instantaneous time delay and states and is calculated online.

Simulation Results

In this section simulations are conducted for the three cases—that is, sinusoidal human force input, 0.1-Hz square input, and 0.005-Hz square input for different master and slave parameters—to verify the effectiveness of the proposed method. In all simulations, the human operator force is modeled as (Du 2013)

\[
f_h(t) = f_h^0(t) - k_h x_m(t) - b_h k_m(t)
\]  

(31)

where \( k_h = 0.1 \text{ N/m} \) and \( b_h = 0.5 \text{ N} \cdot \text{s/m} \).

First, \( f_h^0(t) \) is assumed to be a sinusoidal wave with 1 amplitude and 1 rad/s frequency. Parameters of the master and slave manipulators are chosen to be the same as in Sadeghi et al. (2008): \( m_m = 1 \text{ kg}, \quad b_m = 1 \text{ N} \cdot \text{s/m}, \quad k_m = 1 \text{ N/m}; \quad m_s = 1 \text{ kg}, \quad b_s = 1 \text{ N} \cdot \text{s/m}, \quad k_s = 0 \text{ N/m}; \quad \) the environment parameters are \( b_e = 0.1 \text{ N} \cdot \text{s/m} \) and \( k_e = 0.1 \text{ N/m} \). During the simulation process, the control period \( T_c = 0.2 \text{ s} \); \( H_2 \) performance is chosen as \( \gamma_2 = 1 \); \( q = 0.01 \); and \( u_{\text{max}} = 7 \). The value \( \alpha = 16 \) is chosen for LMI simulation. The RTT is shown in Fig. 2. At each sampling time, the LMI-optimizing problems are solved by the instantaneous parameter and \( \gamma \) are respectively chosen as 4 s and 3 s, so that the controller enlarges \( \gamma_{\infty} \) when time delays vary sharply so as to restrict the inputting \( u \) to a desirable range and to guarantee the stability of the closed-loop system; contrariwise, \( \gamma_{\infty} \) is decreased to ensure better tracking performance when time delays change smoothly.

The \( H_{\infty} \) control method algorithm is compared with the constant \( H_{\infty} \) performance \( \gamma_{\infty} \) proposed by Yang et al. (2013), considering the input constraint [Eq. (15)] that calculates the controller parameter by solving the inequalities in Eq. (12) in Yang et al. (2013) and the input constraint inequality of Eq. (18) in this paper. The simulation parameters and RTT used are same for both methods. The maximum and minimum time delays assumed in the simulation are respectively chosen as 4 s and 3 s, so \( d_2 = 4/T_c = 20 \text{ s} \) and \( d_1 = 3/T_c = 15 \text{ s} \). This yields \( H_{\infty} \) performance \( \gamma_{\infty} = 0.1193 \) and controller parameter \( K = [-10.3484,-3.4964,9.6621,4.2934] \). During the simulation process, the controller parameter and \( H_{\infty} \) performance \( \gamma_{\infty} \) shown in Fig. 3(a) do not change, even when the time delay changes considerably. Thus, when RTT varies dramatically at simulation time 15 s–20 s, as shown in Fig. 2, the input violates the constraint, but the input of the new method satisfies the constraint, as shown in Fig. 3(d).

![Fig. 2. RTT used when the human input force is sinusoidal or square with 0.1-Hz frequency](image-url)
We also compare the proposed algorithm with the MPC method proposed by Kothare et al. (1996). The position, force, and control input curves of all three methods are shown in Figs. 3(b–d), where the broken line, dotted line, and dash-dotted line respectively denote the corresponding curves of the method in this paper, the method in Kothare et al. (1996), and the method in Yang et al. (2013). The continuous line denotes the master reference position.

Fig. 3(b), shows that the slave’s position can commendably track the master’s and no large difference exists between the proposed approach and the other approaches. However, Fig. 3(c) shows that the environmental forces $f_{ed}$ are smaller than the human forces $f_h$. The reason is that the impedance controller parameter $k_m$ is chosen as 1 N/m in the simulation. According to Eq. (2), $f_h(t) - f_{ed}(t) = k_m x_m^s(t)$ obtains when the system stabilizes at the equilibrium point $x_m^s(t)$. Thus, in this case, scaling occurs in the force tracking. In addition, it is obvious that the force curves of the MPC scheme proposed by Kothare et al. (1996) jitter more sharply than those of the other schemes. When time delay varies sharply, the proposed method and that in Kothare et al. (1996) ensure that the input satisfies the constraint, but the control input of the scheme proposed by Yang et al. (2013) violates the constraint, as shown in Fig. 3(d). In this figure, the input of the Kothare et al. (1996) method changes more than that of the other methods, which causes the force curves to jitter more sharply than the other curves. Therefore, it can be concluded that the MPC method in this paper not only ensures the position-tracking performance but also guarantees that the input satisfies the constraints under the large varying and unknown RTT, and has better performance than the MPC in Kothare et al. (1996) and the $H_\infty$ control in Yang et al. (2013).

Notably, the control input in the Kothare et al. (1996) method changes dramatically, but the control inputs in this paper and in Yang et al. (2013) change more smoothly. Moreover, the control period $T_c = 0.2$ s is long enough for implementing the input of the proposed method in practice.

To further verify the effectiveness of this method, the case is simulated when human input force is a square with 0.1-Hz frequency and the master and slave parameters are chosen as $\hat{m}_m = 2$ kg, $\hat{b}_m = 0.7N \cdot s/m$, $\hat{k}_m = 0.5N/m$, $m_s = 2$ kg, $b_s = 0.7N \cdot s/m$, $k_s = 0.5N/m$. The maximum control input $u_{max} = 5$. Other parameters, including RTT, are the same as those in the previous case. The current simulation results are shown in Figs. 4(a–c). They are $H_\infty$ performance $\gamma_\infty$, position tracking, and control input. From the figures, it can be concluded that the proposed approach can guarantee the position-tracking performance and also enable the input to satisfy constraints in the large varying and unknown RTT scenario.
When human input force is a square with 0.005-Hz frequency and RTT is as shown in Fig. 5. Other parameters are the same as when the human operating force is a square with 0.1-Hz frequency. Simulation results are shown in Figs. 6(a–d). Fig. 6(a) shows that the proposed method adjusts $H_\infty$ performance $\gamma_\infty$ when the position and time delay vary sharply to ensure input constraints. Figs. 6(b and c) indicate that the position and force-tracking performance of the method in this paper and the one in Yang et al. (2013) are better than that of the Kothare et al. (1996) scheme. The control input of the latter varies sharply, but this is not expressed in Fig. 6(d). The control inputs of all three methods satisfy the constraint because the human input force is a square with low frequency and the positions change in small ranges when time delay varies sharply. Therefore, in this case the results of the proposed method are similar to the scheme in Yang et al. (2013).

When human input force is a square with 0.005-Hz frequency and RTT is as shown in Fig. 5. Other parameters are the same as when the human operating force is a square with 0.1-Hz frequency. Simulation results are shown in Figs. 6(a–d). Fig. 6(a) shows that the proposed method adjusts $H_\infty$ performance $\gamma_\infty$ when the position and time delay vary sharply to ensure input constraints. Figs. 6(b and c) indicate that the position and force-tracking performance of the method in this paper and the one in Yang et al. (2013) are better than that of the Kothare et al. (1996) scheme. The control input of the latter varies sharply, but this is not expressed in Fig. 6(d). The control inputs of all three methods satisfy the constraint because the human input force is a square with low frequency and the positions change in small ranges when time delay varies sharply. Therefore, in this case the results of the proposed method are similar to the scheme in Yang et al. (2013).

As the RTT varies in the range of 3 s to 7 s, the proposed method fails to obtain the suboptimum solution, even by enhancing the $H_2$ performance $\gamma_2$. Thus, the case in which time delay varies on a large scale will be investigated in future work. For this case, it can

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**Fig. 5.** RTT used when the human input force is square with 0.005-Hz frequency

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**Fig. 4.** (a) $H_\infty$ performance $\gamma_\infty$ under 0.1-Hz square input force; (b) position-tracking curves under 0.1-Hz square input force; (c) control input curves under 0.1-Hz square input force
be treated as data lost or can be solved by designing some other methods.

**Conclusions**

This paper proposed a novel method for bilateral teleoperation systems in the presence of unknown, large time-varying delays and input constraints. This method does not need to know the bounds on the time delay in advance, but only needs to measure the time delay online. Moreover, at each sampling time, the $H_2/H_\infty$ performance is adaptively adjusted according to the current time delay whereas existing $H_\infty$ control approaches yield invariable performance. Therefore, the proposed approach guarantees that the teleoperation system, under circumstances of unknown time-varying delay is stable, satisfies input constraints, and has good tracking performance. Compared to existing methods, the scheme proposed in this paper has better tracking performance than that of the general MPC and, at the same time, guarantees input satisfying constraints under unknown, large time-varying delay. The $H_\infty$ control approach cannot make such a guarantee.

The proposed approach combines the MPC and $H_\infty$ control methods to deal with large time-varying delay and input constraint problems in space teleoperation systems. It incorporates $H_\infty$ control in MPC and thus exhibits better performance in terms of robustness than does general MPC; moreover, it does not need to know the upper and lower bounds that are generally necessary for existing $H_\infty$-control methods.

**Acknowledgments**

The authors thank Professor Fuwen Yang from East China University of Science and Technology, Shanghai, China, for his constructive and inspiring comments and suggestions. This work was partially supported by the Natural Science Foundation of China (No. 61203331 and 61171160), Hubei Province Key Laboratory of Systems Science in Metallurgical Process (Wuhan University of Science and Technology) (No. Z201301), and Hubei Provincial Department of Education (No. T201302).

**References**


