Proceedings of the 34th Chinese Control Conference
July 28-30, 2015, Hangzhou, China

Fault tolerant control with switched LPV method based on hysteresis strategy and an application to a microsatellite model

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Abstract: This paper studies a class of switched Linear Parameter Varying (LPV) systems in which different control faults are modeled as different system modes according to parameter varying. Two methods are proposed based on the hysteresis method of Multiple Lyapunov function, and H_∞ state feedback and output feedback controllers are designed respectively by solving Linear Matrix Inequalities (LMIs). The method is applied to an attitude control model in a microsatellite, and simulation result verifies effectiveness of proposed methods.

Key Words: Switched LPV system, Fault-tolerant control, H_∞ controller design, Hysteresis method, Attitude system control, Microsatellite

1 Introduction

Over last ten years, switched LPV control theory has been broadly researched in LPV systems [2, 8, 10, 15, 16]. LPV theory is practical to apply linear theory to a nonlinear system or practical system such as missile [7, 11, 13], aircraft [9, 10], energy production system [1, 5, 12] and inverted pendulum [6].

Switching method makes controller design more flexible and practical [3, 7, 10]. In [7], a missile is modeled and a switched controller is designed for the missile model. The paper of [8] designs a switched LPV controller to improve the H_∞ performance and apply the method to an active magnetic bearing system. Using the similar method, an F-16 aircraft model is controlled by a switching controller with state reset [10], and fault tolerant controller is also designed with Average Dwell Time (ADT) method [9]. Fast-varying and slow-varying parameters are considered separately, and a blending method is also used to an F-16 model [3].

There are mainly two methods in switching LPV control theory. One is Hysteresis method, and the other is ADT method. Hysteresis method divides whole parameter set into subsets with adjacent sets. The adjacent sets are used to be as hysteresis regions for system switching. The ADT method guarantees decrease of switchings through constraint of different Lyapunov functions.This paper applies the Hysteresis method to realize the fault tolerant control.

This work researches H_∞ fault tolerant controller design for a system with multiple fault modes. The advantage of switched LPV system is that it can describe rapid change in a system according to parameters. It is used to describe normal mode and different fault modes for a system in this paper. Normal mode and fault modes are modeled by different subsystems in a switched LPV system, and switching between different subsystems is decided by parameter varying. With different subsystems, different sub-controllers are designed correspondingly, and they switch according to a designed switching law. Two methods are proposed for state feedback and output feedback respectively based on the Hysteresis method, and they are both concluded in LMI form.

Faults tolerant controller and switching law are designed based on LMI solving. Methods in this paper give existence conditions of corresponding multiple Lyapunov functions. The switching law of switched system is also defined as LMI solving. With solved Lyapunov functions, state feedback and output feedback H_∞ controllers can be designed.

An attitude system in a microsatellite with fault modes is simulated to verify effectiveness of methods in the paper. The attitude system is equipped with four reaction wheels, and different modes are modeled by different index parameter varying sets. When fault happens, index parameters vary into fault sets. Then system switches to corresponding fault mode, and controller switches to the fault-tolerant controller.

This paper is organized as follows: Section 2 gives preliminaries for main results; Section 3 gives H_∞ controller and switching law design; In Section 4, a simulation of an attitude system is illustrated; The conclusion is given in Section 5.

Notations are as follows: $\langle X \rangle$ is a shorthand of $X + X^T$, $\boldsymbol{I}_n$, $\boldsymbol{I}$, and $\mathbf{0}$ denote an $n \times n$-dimensional identity matrix, identity and zero matrices of appropriate dimensions. $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{S}^n$ respectively denote sets of n-dimensional real vectors, $n \times m$-dimensional real matrices and $n \times n$-dimensional symmetric real matrices, and $*$ denotes an abbreviated off-diagonal block in a symmetric matrix. $\mathsf{diag}(X_1, \cdots, X_k)$ denotes a block-diagonal matrix composed of $X_1, \cdots, X_k$.

2 Preliminaries

Consider the following system:

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_{2,\sigma} F(\rho) u, \\ z &= C_1 x + D_{11} w + D_{12,\sigma} F(\rho) u, \\ y &= C_2 x + D_{21} w, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $z \in \mathbb{R}^{n_z}$ is the controlled output, and $w \in \mathbb{R}^{n_w}$ is the disturbance input, $y \in \mathbb{R}^{n_y}$ is the measurement for control, and $u \in \mathbb{R}^{n_u}$ is the control input. $\rho = [\rho_1, \cdots, \rho_{n_u}]$ is the fault parameter vector, which is in

This work is supported by China Postdoctoral Science Foundation under Grant No. 2014M551143, No. 2013M541258 and Grant by State Key Laboratory of Robotics, No.2015-Z08.

a hyper-rectangle set $\rho \in w_\rho$, and its variation rate bound is also in a hyper-rectangle set $\dot{\rho} \in \Omega_{\dot{\rho}}$. The vertices of set Ω_ρ is denoted by $\mathsf{ver}(\Omega_\rho)$.

In the system (1), $F(\rho) = \mathsf{diag}(\rho_1(t - T_{0,1}), \rho_2(t - T_{0,2}), \cdots, \rho_{n_u}(t - T_{0,n_u}))$ denotes the fault factor of the system. T_0 denotes the fault occuring time, and ρ_i is a function representing the time profile of a fault affecting the ith control input ($i = 1, \cdots, n_u$). It is assumed that ρ_i is modeled by

$$\rho_i(t - T_{0,i}) = \begin{cases} 0, & \text{if } t < T_{0,i} \\ 1 - e^{-a_i(t - T_{0,i})}, & \text{if } t \geqslant T_{0,i} \end{cases} \tag{2}$$

where the scalar $a_i > 0$ is the unknown fault-evolution rate. Small values of a_i characterize slowly developing faults (incipient faults), while large values of a_i models abrupt faults. From the form of ρ_i in equation (2), $0 \leqslant \rho_i \leqslant 1$. According to values of parameter vector, the system mode can be divided into normal-mode and fault-mode. Assume that there are n_u fault modes which correspond to n_u control inputs, and parameter set Ω_ρ is divided into subset $\Omega_{\rho,i}$ ($i = 1, 2, \cdots, n_u$) with each $\Omega_{\rho,i}$ denoting a mode. The index set $Z_{n_u} = \{0, 1, 2, \cdots, n_u\}$, and $\sigma : [0, \infty) \to Z_{n_u}$ is a switching signal decided by ρ. In the system (1), matrices $B_1, B_2, \cdots, B_{n_u}$ correspond to different fault modes respectively. When the system (1) has fault in the ith input, matrix B_σ switches to the ith fault mode from normal mode B_0.

Remark 1. *The parameter dependent matrix $F(\rho)$ presents control input faults, and the form is similar to [17]. $\rho_i \in [0, 1]$ is fault index parameter which presents condition of the ith input control. With this form, a system with fault can be described by the switched LPV system (1).*

There are switching surfaces between different parameter subsets. $\mathcal{S}_{ij}(\rho)$ is set as switching surface between two adjacent subsets $\Omega_{\rho,i}$ and $\Omega_{\rho,j}$, and it is also one directional move from $\Omega_{\rho,i}$ to $\Omega_{\rho,j}$. It is also assumed that there is overlapped subset $\Omega_{i\cap j} \neq \emptyset$ between any normal-mode set and fault-mode set.

For the system (1), the form of a state-feedback controller is

$$u = K_\sigma(\rho)x. \tag{3}$$

If the controller is an output-feedback controller, the form is as follows:

$$\begin{cases} \dot{x}_k = A_{k,\sigma}(\rho)x_k + B_{k,\sigma}(\rho)y, \\ u = C_{k,\sigma}(\rho)x_k + D_{k,\sigma}(\rho)y, \end{cases} \tag{4}$$

where $x_k \in \mathbb{R}^n$ denotes the state with $x_k = \mathbf{0}$ at $t = 0$. $A_{k,i}(\rho), B_{k,i}(\rho), C_{k,i}(\rho), D_{k,i}(\rho)$ are each subsystem's controller matrices to be designed. For the closed-loop system $G_{cl}(\rho)$, it can be expressed as

$$\begin{cases} \dot{x}_{cl} = A_{cl,\sigma}(\rho)x_{cl,\sigma} + B_{cl,\sigma}(\rho)w, \\ u = C_{cl,\sigma}(\rho)x_{cl,\sigma} + D_{k,\sigma}(\rho)w, \end{cases} \tag{5}$$

where x_{cl} denotes the state of the closed-loop switched LPV system.

To achieve a switched controller, different sub-controllers for different modes and switching law should be designed to guarantee performance of the closed-loop system (5). With the system (1) in which faults can be presented by parameter ρ, different sub-controllers can be designed in different subset. The following is H_∞ performance problem which is to be solved by switched controller design:

Problem 2.1. *Consider the system* (5) *with different modes. For a given positive number γ_∞, find a switched controller $K_\sigma(\rho)$ which makes the closed-loop system (5) asymptotically stable and satisfies (6) for all normal mode and fault modes.*

$$\sup_{w \in \mathcal{L}_2, w \neq 0} \frac{\|z\|_2}{\|w\|_2} \leqslant \gamma_\infty. \tag{6}$$

The following is a lemma which is used in Section 3.

Lemma 2.2. *For given positive scalar $\gamma_{\infty,i}$, if there exist positive parameter dependent matrices $X_{cl,i}(\rho) \in \mathbb{S}^n$ such that LMIs (7) and (8) hold for all combinations of $(\rho, \dot{\rho}) \in \Omega_\rho \times \Omega_{\dot{\rho}}$ for the ith subsystem ($i = 1, \cdots, N$), and the LMI (9) holds on the switching surface $\mathcal{S}_{ij}$, then the closed-loop switched LPV system (5) is asymptotically stable and satisfies (6) with $\gamma_\infty = \max\{\gamma_{\infty,i}\}$ for all ρ.*

$$X_{cl,i}(\rho) > 0, \tag{7}$$

$$\begin{bmatrix} \left\{ \begin{matrix} \langle A_{cl,i}(\rho)X_{cl,i}(\rho) \rangle \\ -\dot{X}_{cl,i}(\rho, \dot{\rho}) \end{matrix} \right\} & * & B_{cl,i}(\rho) \\ C_{cl,i}(\rho)X_{cl,i}(\rho) & -\gamma_{\infty,i}\boldsymbol{I}_{n_z} & D_{cl,i}(\rho) \\ * & * & -\gamma_{\infty,i}\boldsymbol{I}_{n_w} \end{bmatrix} < 0, \tag{8}$$

$$X_{cl,i}(\rho) \geq X_{cl,j}(\rho). \tag{9}$$

3 Fault - tolerant H_∞ Controller Design

In this section, two methods are proposed to design state-feedback controller and output-feedback controller respectively. It is assumed that there are different sub-controllers for normal mode and fault modes correspondingly. The following methods solves different Lyapunov functions for different modes respectively, then design controllers according to different mode. The method also designs switching law for different sub-controllers to realize performance of the closed-loop system.

3.1 State-feedback Controller Design

In this subsection, a theorem is proposed to achieve a switched state-feedback controller. Based on Lemma 2.2, LMI constraints are proposed in the following theorem.

Theorem 3.1. *Consider the switched LPV system (1). Suppose that there exist positive matrices $X_i(\rho)$ and general matrices $\mathscr{K}_i(\rho)$, scalars $\gamma_{\infty,i}$ such that following LMIs*

$$\begin{bmatrix} \left\{ \begin{matrix} \langle AX_i(\rho) + \\ B_{2,i}F(\rho)\mathscr{K}_i(\rho) \rangle \\ -\dot{X}_i(\rho) \end{matrix} \right\} & * & B_1 \\ \left\{ \begin{matrix} C_1X_i(\rho) + \\ D_{12}F(\rho)\mathscr{K}_i(\rho) \end{matrix} \right\} & -\gamma_{\infty,i}\boldsymbol{I}_{n_z} & D_{11} \\ * & * & -\gamma_{\infty,i}\boldsymbol{I}_{n_w} \end{bmatrix} < 0 \tag{10}$$

hold for all combination $\rho \in \Omega_{\rho,i} \times \Omega_{\dot{\rho}}$, and LMIs

$$X_i(\rho) \geq X_j(\rho), \tag{11}$$

hold on each switching surface $\mathcal{S}_{ij}(\rho)$, then there exists a switched LPV controller of the form (3) that makes the closed-loop system (5) asymptotically stable and satisfies

(6), and the fault tolerant switched LPV state-feedback controller is in the form of

$$K_i(\rho) = \mathscr{K}_i(\rho) X_i^{-1}(\rho). \tag{12}$$

Proof. For a state-feedback controller, matrices in system (5) are:

$$\begin{aligned} &A_{cl,i}(\rho) = A + B_{2,i} F(\rho) K_i(\rho), \\ &B_{cl} = B_1, \\ &C_{cl,i}(\rho) = C_1 + D_{12,i} F(\rho) K_i(\rho), \\ &D_{cl} = D_{11}. \end{aligned}$$

Substitude above matrices into LMI (8) and set $X_{cl,i}(\rho) = X_i(\rho)$, $\mathscr{K}_i(\rho) = K_i(\rho) X_i(\rho)$, then LMI (10) can be achieved. LMI (11) is direct deduce of LMI (9) in Lemma 2.2. Proof is finished. □

Remark 2. *The Gridding method is needed to solve LMIs (10) and (11). Considering parameter dependent controller $K(\rho)$, gridding points should be chosen in parameter set, and constraint LMIs on all points to guarantee to guarantee them hold in whole parameter set.*

3.2 Output-feedback Controller Design

When all the state can not be achieved, an output-feedback controller is more practical. In this subsection, a theorem is proposed to design a parameter dependent dynamic output-feedback controller for the system (1).

Theorem 3.2. *Consider the switched LPV system (1). Suppose that there exist positive matrices $X_i(\rho)$, $Z_i(\rho) \in \mathbb{S}^n$, general matrices $\mathscr{A}_{k,i}(\rho) \in \mathbb{R}^{n\times n}$, $\mathscr{B}_{k,i}(\rho) \in \mathbb{R}^{n\times n_u}$, $\mathscr{C}_{k,i}(\rho) \in \mathbb{R}^{n_u\times n}$, $\mathscr{D}_{k,i}(\rho) \in \mathbb{R}^{n_u\times n_y}$ and scalars $\gamma_{\infty,i}$ such that following LMIs*

$$\begin{bmatrix} X_i(\rho) & \boldsymbol{I}_n \\ \boldsymbol{I}_n & Z_i(\rho) \end{bmatrix} > 0, \tag{13}$$

$$\begin{bmatrix} \Upsilon_i & \begin{bmatrix} \begin{bmatrix} \left\{ \begin{matrix} C_1 X_i(\rho) + D_{12,i} \\ \times F(\rho) \mathscr{C}_{k,i}(\rho) \end{matrix} \right\} \end{bmatrix}^T \\ \begin{bmatrix} \left\{ \begin{matrix} C_1 + D_{12,i} F(\rho) \\ \times \mathscr{D}_{k,i}(\rho) C_2 \end{matrix} \right\} \end{bmatrix}^T \end{bmatrix} & \begin{bmatrix} B_1 + B_{2,i} F(\rho) \mathscr{D}_{k,i}(\rho) D_{21} \\ Z_i(\rho) B_1 + \mathscr{B}_{k,i}(\rho) D_{21} \end{bmatrix} \\ * & -\gamma_{\infty,i} \boldsymbol{I}_{n_z} & D_{11} + D_{12,i} F(\rho) \mathscr{D}_{k,i}(\rho) D_{21} \\ * & * & -\gamma_{\infty,i} \boldsymbol{I}_{n_w} \end{bmatrix} < 0 \tag{14}$$

hold for all combination $(\rho, \dot{\rho}) \in \Omega_{\rho,i} \times \Omega_{\dot{\rho},i}$, and LMIs

$$X_i(\rho) \geq X_j(\rho), \tag{15}$$

$$X_i(\rho) - Z_i^{-1}(\rho) \geq X_j(\rho) - Z_j^{-1}(\rho) \tag{16}$$

hold on each switching surface $\mathcal{S}_{ij}(\rho) \times \Omega_\delta$, then there exists a switched LPV controller of the form (4) makes the system (5) asymptotically stable and satisfies (6), where

$$\Upsilon_i = \left\langle \begin{bmatrix} \left\{ \begin{matrix} AX_i(\rho) + \\ B_{2,i} \mathscr{C}_{k,i}(\rho) \\ -\dot{X}(\rho) \end{matrix} \right\} & \left\{ \begin{matrix} A_i(\rho) + B_{2,i} F(\rho) \\ \times \mathscr{D}_{k,i}(\rho) C_2 \end{matrix} \right\} \\ \mathscr{A}_{k,i}(\rho) & Z_i(\rho) A(\rho) + \mathscr{B}_{k,i}(\rho) C_2 \end{bmatrix} \right\rangle,$$

and the switched LPV output-feedback controller is in the form of

$$\left\{ \begin{aligned} A_{k,i}(\rho) &= Z_i^{-1}(\rho) \left[Z_i(\rho) A(\rho) X_i(\rho) \right. \\ &\quad + Z_i(\rho) B_{2,i} F(\rho) \mathscr{C}_{k,i}(\rho) - \mathscr{A}_{k,i}(\rho) - \\ &\quad \left. \left(Z_i(\rho) B_{2,i} F(\rho) \mathscr{D}_{k,i}(\rho) - \mathscr{B}_{k,i}(\rho) \right) C_2 X_i(\rho) \right] Y_i^{-1}(\rho), \\ B_{k,i}(\rho) &= Z_i^{-1}(\rho) \left(Z_i(\rho) B_{2,i} F(\rho) \mathscr{D}_{k,i}(\rho) - \mathscr{B}_{k,i}(\rho) \right), \\ C_{k,i}(\rho) &= \left(\mathscr{C}_{k,i}(\rho) - \mathscr{D}_{k,i}(\rho) C_2 X_i(\rho) \right) Y_i^{-1}(\rho), \\ D_{k,i}(\rho) &= \mathscr{D}_{k,i}(\rho) \end{aligned} \right. \tag{17}$$

with $Y_i(\rho)$ being defined as $Y_i(\rho) = X_i(\rho) - Z_i^{-1}(\rho)$.

Proof. Substituting the controller (4) into the closed-loop system (5), and applying Lemma 2.2, the following LMI can be achieved:

$$\begin{bmatrix} H_i[1,1] & H_i[1,2] & \begin{bmatrix} \left\{ \begin{matrix} B_1 + B_{2,i} \times \\ F(\rho) \mathscr{D}_{k,i}(\rho) D_{21} \end{matrix} \right\} \\ B_{k,i}(\rho) D_{21} \end{bmatrix} \\ * & -\gamma_{\infty,i} \boldsymbol{I}_{n_z} & \left\{ \begin{matrix} D_{11} + D_{12,i} \times \\ F(\rho) \mathscr{D}_{k,i}(\rho) D_{21} \end{matrix} \right\} \\ * & * & -\gamma_{\infty,i} \boldsymbol{I}_{n_w} \end{bmatrix} < 0, \tag{18}$$

where

$$H_i[1,1] = \left\langle \begin{bmatrix} AX_i(\rho) + B_{2,i} F(\rho) \mathscr{C}_{k,i}(\rho) - \dot{X}(\rho) & A + B_{2,i} F(\rho) \mathscr{D}_{k,i}(\rho) C_2 \\ \mathscr{A}_{[2,1]}(\rho) & Z_i(\rho) A + \mathscr{B}_{k,i}(\rho) C_2 + \frac{dZ_i^{-1}(\rho)}{dt} \end{bmatrix} \right\rangle,$$

$$H_i[1,2] = \begin{bmatrix} \left[C_1 X_i(\rho) + D_{12,i} F(\rho) \mathscr{C}_{k,i}(\rho) \right]^T \\ \left\{ \begin{matrix} \left[C_1 Y_i(\rho) + D_{12,i} F(\rho) \left(C_{k,i}(\rho) \right. \right. \\ \left. \left. + \mathscr{D}_{k,i}(\rho) C_2 \right) Y_i(\rho) \right]^T \end{matrix} \right\} \end{bmatrix},$$

$$\begin{aligned} \mathscr{A}_{[2,1]}(\rho) &= A_{k,i}(\rho) Y_i(\rho) + Y_i(\rho) A^T + B_{k,i}(\rho) C_2 X_i(\rho) \\ &\quad + Y_i(\rho) \left(C_{k,i}^T(\rho) + C_2^T D_{k,i}^T(\rho) \right) F(\rho) B_{2,i}^T, \\ \mathscr{B}_{k,i}(\rho) &= Z_i(\rho) B_{2,i} F(\rho) D_{k,i}(\rho) - Z_i(\rho) B_{k,i}(\rho) \\ \mathscr{C}_{k,i}(\rho) &= C_{k,i}(\rho) Y_i(\rho) + D_{k,i}(\rho) C_2 X_i(\rho), \\ \mathscr{D}_{k,i}(\rho) &= D_{k,i}(\rho), \end{aligned}$$

and Lyapunov function matrix as

$$X_{cl,i}(\rho) = \begin{bmatrix} X_i(\rho) & Y_i(\rho) \\ * & Y_i(\rho) \end{bmatrix}. \tag{19}$$

Pre- and post-multiply $\text{diag}(\mathscr{Z}_i^T, \boldsymbol{I}_{n_z}, \boldsymbol{I}_{n_w})$ and $\text{diag}(\mathscr{Z}_i, \boldsymbol{I}_{n_z}, \boldsymbol{I}_{n_w})$ to the left hand side of LMI (18) with $\mathscr{Z}_i = \begin{bmatrix} \boldsymbol{I}_n & Z_i(\rho) \\ \boldsymbol{0} & -Z_i(\rho) \end{bmatrix}$, then the LMI (18) is transformed into LMI (14), where

$$\begin{aligned} \mathscr{A}_{k,i}(\rho) &= A^T + Z_i(\rho) A X_i(\rho) - Z_i(\rho) A_{k,i}(\rho) Y_i(\rho) \\ &\quad - C_2^T D_{k,i}^T(\rho) F(\rho) B_{2,i}^T \\ &\quad + Z_i(\rho) B_{2,i} F(\rho) C_{k,i}(\rho) Y_i(\rho) \\ &\quad + \mathscr{B}_{k,i}(\rho) C_2 X_i(\rho) + \dot{Z}_i(\rho) Z_i^{-1}(\rho). \end{aligned}$$

According to the solved matrices $X_i(\rho)$ $Z_i(\rho)$, $\mathscr{A}_{k,i}(\rho), \mathscr{B}_{k,i}(\rho), \mathscr{C}_{k,i}(\rho), \mathscr{D}_{k,i}(\rho)$ and scalars $\gamma_{\infty,i}$, the controller can be obtained by equations (17). Proof is finished. □

4 Attitude Control Simulation for a Microsatellite

In this section, Theorem 3.1 in last section is applied into a microsatellite attitude system model. The system model is equipped by four reaction wheels with redundancy one for 3-axis control [4].

The Euler's equations of motion for the microsatellite are given by

$$I_1\dot{\omega}_1 - (I_2 - I_3)\,\omega_2\omega_3 = T_1, \quad (20)$$
$$I_2\dot{\omega}_2 - (I_3 - I_1)\,\omega_3\omega_1 = T_2, \quad (21)$$
$$I_3\dot{\omega}_3 - (I_1 - I_2)\,\omega_1\omega_2 = T_3, \quad (22)$$

where I_1, I_2, I_3 are principal moments of inertia, $\omega_1, \omega_2, \omega_3$ are the body-axis components of angular velocity, and $T_i = u_i + T_{di}$ are the body-axis components of the external torques acting on the microsatellite, which contain control torques u_i and environmental disturbance torques T_{di}.

Considering the redundancy reaction wheel, u_1, u_2 and u_3 are normal three wheels and u_4 is set as redundancy control input for fault tolerance. Similar to [14], the system is simplified as the following form:

$$\begin{aligned} &I_1\ddot{\phi} - n\,(I_1 - I_2 + I_3)\,\dot{\psi} + 4n^2\,(I_2 - I_3)\,\phi \\ &\qquad = \rho_1\,(t - T_{0,1})\,u_1 + f(\sigma)\,u_4 + T_{d1}, \quad (23) \\ &I_2\ddot{\theta} + 3n^2\,(I_1 - I_3)\,\theta \\ &\qquad = \rho_2\,(t - T_{0,2})\,u_2 + f(\sigma)\,u_4 + T_{d2}, \quad (24) \\ &I_3\ddot{\psi} + n\,(I_1 - I_2 + I_3)\,\dot{\phi} + n^2\,(I_2 - I_1)\,\psi \\ &\qquad = \rho_3\,(t - T_{0,3})\,u_3 + f(\sigma)\,u_4 + T_{d3}. \quad (25) \end{aligned}$$

In Equations (23)-(25), u_i is the control torque and $T_{di} = T_{mi} + T_{ai}$ represents the sum of worst-case aerodynamic and magnetic torques. ϕ, θ, ψ are derivation of roll, pitch and yaw angles. n is the orbital rate of the microsatellite. $\rho_i \in [0, 1]$ are fault index parameter, and $f(\sigma)$ is set as follows:

$$f(\sigma) = \begin{cases} 0, & \text{if } \sigma = 0 \\ 1. & \text{if } \sigma \neq 0 \end{cases}$$

Fault signal σ represents different modes in the satellite system. In normal condition, signal $\sigma = 0$, while $\sigma = i$ when there is fault in the ith control input.

Switching surfaces between different modes set are all one-directional according to the parameter ρ. In this example, the normal set is $\rho_i \in [0.8, 1]$, and fault set is $\rho_i \in [0, 0.9]$. $\rho_i \in [0.8, 0.9]$ is the adjacent subset for hysteresis switching. The switching signal σ is dependent on the parameter ρ. In normal mode, there is no fault in the system, $\rho_i = 1$. If fault happens in the ith control input, ρ_i will decay to 0. $\rho = 0.8$ is the switching surface from normal mode to fault mode, and $\rho = 0.9$ is the reverse directional surface.

Rewrite equations (23)-(25) as the following form:

$$M\ddot{q} + D\dot{q} + Kq = G_d d + G_{u,\sigma} u,$$

where

$$\begin{aligned} &M = \mathsf{diag}(I_1, I_2, I_3), \\ &D = \begin{bmatrix} 0 & 0 & -n\,(I_1 + I_3 - I_2) \\ 0 & 0 & 0 \\ n\,(I_1 + I_3 - I_2) & 0 & 0 \end{bmatrix}, \\ &K = \mathsf{diag}(4n^2\,(I_2 - I_3), 3n^2\,(I_1 - I_3), n^2\,(I_2 - I_1)), \\ &G_d = \boldsymbol{I}_3, \\ &G_{u,0} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, G_{u,1} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \\ &G_{u,2} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}, G_{u,3} = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \end{aligned}$$

Remark 3. *Different modes in the attitude system are presented by different $G_{u,i}$. If fault happens in the system, the corresponding control input loses effectiveness, and the redundancy reaction wheel is activated. Both $B_{2,\sigma}$ and $D_{12,\sigma}$ are dependent with $G_{u,\sigma}$.*

The state is $x = \left[\phi\ \theta\ \psi\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\right]^T$, controlled output as $z = \left[\ddot{\phi}\ \ddot{\theta}\ \ddot{\psi}\ \dot{\phi}\ \dot{\theta}\ \dot{\psi}\ \phi\ \theta\ \psi\right]^T$. The state equations are as the following form:

$$\dot{x} = Ax + B_1 w + B_{2,\sigma} F(\rho)\,u, \quad (26)$$
$$z = C_1 x + D_{11} w + D_{12,\sigma} F(\rho)\,u, \quad (27)$$

where

$$\begin{aligned} &A = \begin{bmatrix} \mathbf{0} & \boldsymbol{I}_3 \\ -M^{-1}K & -M^{-1}D \end{bmatrix}, B_1 = \begin{bmatrix} \mathbf{0} \\ G_d \end{bmatrix}, \\ &B_{2,i} = \begin{bmatrix} \mathbf{0} & G_{u,i} \end{bmatrix}, C_1 = \begin{bmatrix} -M^{-1}K & -M^{-1}D \\ \mathbf{0} & \boldsymbol{I}_3 \\ \boldsymbol{I}_3 & \mathbf{0} \end{bmatrix}, \\ &D_{11} = \begin{bmatrix} G_d \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, D_{12,i} = \begin{bmatrix} G_{u,i} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}. \end{aligned}$$

Theorem 3.1 is applied to the satellite system (23)-(25). The matrices $X_i(\rho)$ are set as $X_i(\rho) = X_{i,0} + \rho X_{i,1}$, and LMIs are solved according to inequalities (10)-(11). Solved H_∞ performance index is 0.1471, and matrices $X_i(\rho)$ and $\mathscr{K}_i(\rho)$ are solved. Then, state-feedback controllers $K_i(\rho)$ are achieved.

Connecting the solved fault-tolerant controller to the plant, simulation results are as Figure 1-4. In the simulation, the initial value is $x_0 = [3\ -2\ 1\ 0\ 0\ 0]^T$, and fault occurs in the 1st reaction wheel at time $t = 3$. When $t < 3$, $u_4 = 0$, $B_{2,\sigma} = B_{2,0}$ and system is in normal mode. When fault occurs, the system switches into fault mode 1, and $B_{2,\sigma} = B_{2,1}$. At $t = 10$, the fault is removed and the 1st wheel is recovered.

From Figure 1 and 3, the performance of the system is little influenced by the fault, and the switching does not decrease the performance.

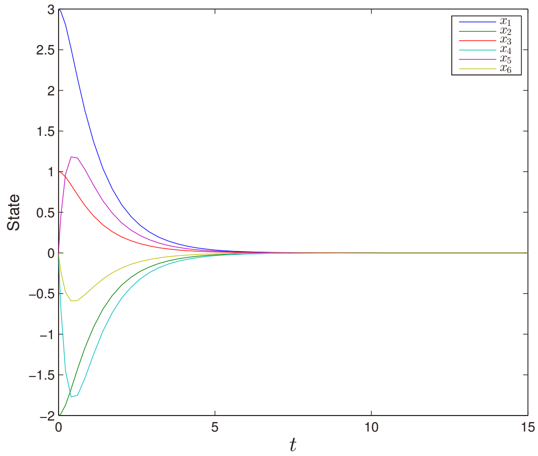

Fig. 1: State response of the closed-loop satellite system

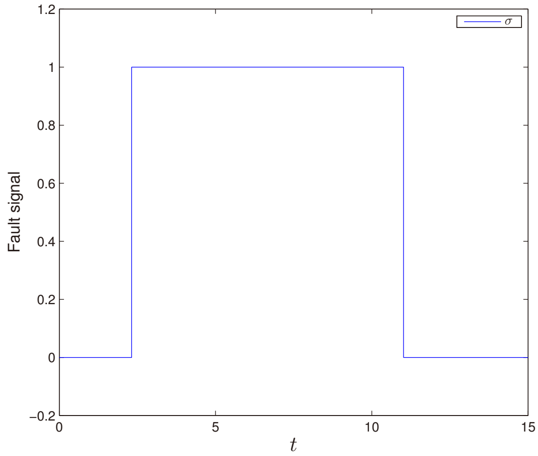

Fig. 4: Fault signal of the satellite system

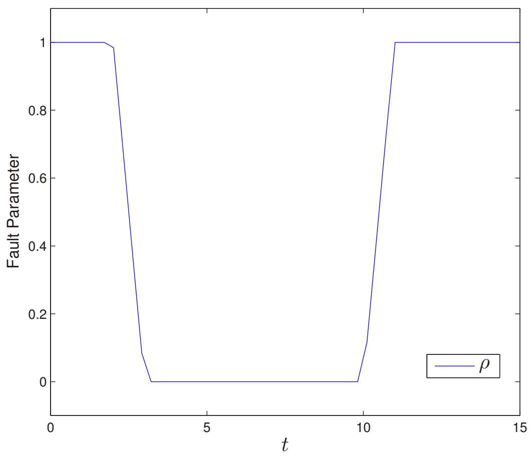

Fig. 2: Fault parameter of the satellite system

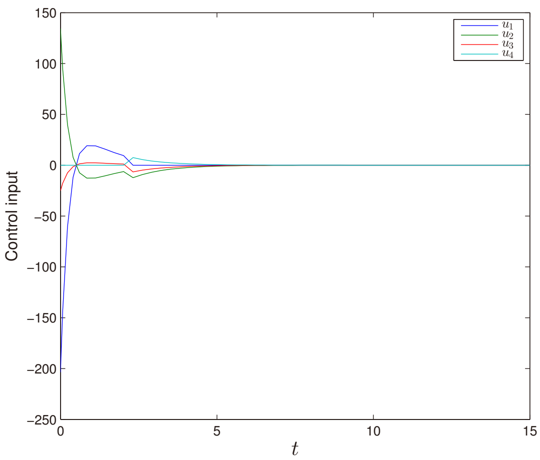

Fig. 3: Control input of the satellite system

5 Conclusion

This paper studies fault-tolerant controller design for a system with normal mode and fault modes. A system is modeled to be a switched LPV system with multiple modes which is decided by parameters. Two theorems are proposed to design state-feedback and output-feedback controller for H_∞ control. With both theorems, the result is represented as LMI form.

An attitude model in a satellite is simulated by Theorem 3.1 to verify effectiveness of the result. A fault-tolerant controller is designed and the simulation result is achieved.

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