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Reliability of Vibration Transfer Path Systems

Wei Zhao[1, a*], Ping Chen[2, b] and Yimin Zhang[3, c]

[1,3]School of Mechanical Engineering and Automation, Northeastern University, Shenyang, China

[2]Shenyang Institute of Automation Chinese Academy of Sciences

[a] weizhao@mail.neu.edu.cn, [b] chping@sia.cn, [c] zhangymneu@sohu.com

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Abstract: Based on the matrix calculus, the generalized second moment technique and the stochastic finite element theory, an effective approach for the transfer reliability of vibration transfer path systems was presented. The transfer reliability of vibration transfer path systems with uncertain path parameters including mass and stiffness was analyzed theoretically and computed numerically, and the correlated mathematical expressions were obtained. Thus, it provides the theoretical foundation for the dynamic design of vibration systems in practical project, so that most uncertain factors can be considered to solve the random problems for vibration transfer path systems.

Introduction

In practical engineering structure analysis and design process, there are inevitably some uncertain factors in the system itself or the external environment. For the uncertainties from statistical factors, the variables with the certain probability distribution lead to the randomness of the whole structures and systems, which can't be ignored. Especially, in system reliability research, it is necessary that the uncertain information is processed correctly to ensure the structure security.

Currently, the research on the reliability problems for vibration transfer path system can proceed in two ways. On the one hand, it can be studied when the overrun of path transfer force or rate is looked as the measure indicator. This kind of research is based on the system responses and depends on the random response analysis for the random systems. On the other hand, the study can be finished according to the structural failure coming from the resonance or potential resonance, which exists widely in the various structures with shock, transient or random loads. When the system is resonant, the dynamic stress is so large and the life is so short that the situation is more dangerous [1, 2].

Vibration Transfer Path System Model

For the vibration transfer path system model shown in Fig.1, there are three kinds of physical parameters including stiffness, damping and mass. m_{p1}, m_{p2}, m_{p3} are respectively path mass of three transfer paths. Due to the existence of central mass, the stiffness and damping of three paths can be divided into two parts: the stiffness parameters near the vibration source are expressed as k_{sp1}, k_{sp2}, k_{sp3}, the corresponding damping parameters are expressed as c_{sp1}, c_{sp2}, c_{sp3}; the stiffness parameters near the receiver are expressed as k_{rp1}, k_{rp2}, k_{rp3}, the corresponding damping parameters are expressed as c_{rp1}, c_{rp2}, c_{rp3}.The source mass is m_s,the stiffness for the rigidity end of the vibration source is k_s,the damping is c_s.The mass of receiver is m_r, the stiffness for the rigidity end of the receiver is k_r,the damping is c_r,where only single excitation is considered. The vibration source is excited by the vertical force $F_0 \sin \omega t$, so that the transfer paths and receiver have linear motion freedom. Applying Newton's law, the vibration differential equation is given

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F}(t) \tag{1}$$

Where

$$\boldsymbol{M} = \mathrm{diag}\left(m_s \; m_{p1} \; m_{p2} \; m_{p3} \; m_r\right)$$

$$\boldsymbol{C} = \begin{bmatrix} c_s + c_{sp1} + c_{sp2} + c_{sp3} & -c_{sp1} & -c_{sp2} & -c_{sp3} & 0 \\ -c_{sp1} & c_{sp1} + c_{rp1} & 0 & 0 & -c_{rp1} \\ -c_{sp2} & 0 & c_{sp2} + c_{rp2} & 0 & -c_{rp2} \\ -c_{sp3} & 0 & 0 & c_{sp3} + c_{rp3} & -c_{rp3} \\ 0 & -c_{rp1} & -c_{rp2} & -c_{rp3} & c_r + c_{rp1} + c_{rp2} + c_{rp3} \end{bmatrix}$$

$$\boldsymbol{K} = \begin{bmatrix} k_s + k_{sp1} + k_{sp2} + k_{sp3} & -k_{sp1} & -k_{sp2} & -k_{sp3} & 0 \\ -k_{sp1} & k_{sp1} + k_{rp1} & 0 & 0 & -k_{rp1} \\ -k_{sp2} & 0 & k_{sp2} + k_{rp2} & 0 & -k_{rp2} \\ -k_{sp3} & 0 & 0 & k_{sp3} + k_{rp3} & -k_{rp3} \\ 0 & -k_{rp1} & -k_{rp2} & -k_{rp3} & k_r + k_{rp1} + k_{rp2} + k_{rp3} \end{bmatrix}$$

$$\boldsymbol{F}(t) = \{F_0 \sin(\omega t) 0\,0\,0\,0\}^{\mathrm{T}}, \quad \boldsymbol{x}(t) = \{x_s \; x_{p1} \; x_{p2} \; x_{p3} \; x_r\}^{\mathrm{T}} \tag{2}$$

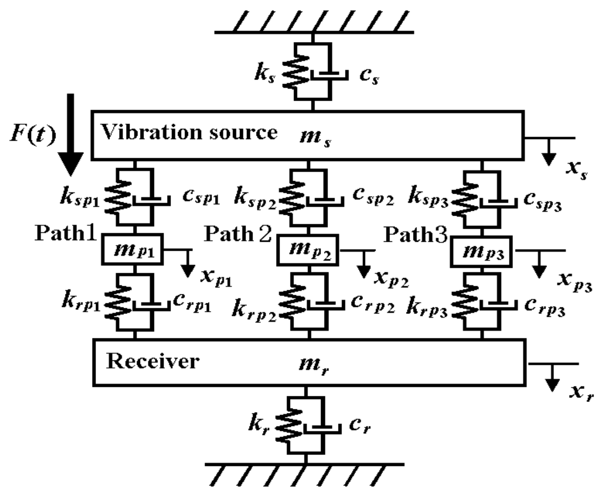

Fig. 1 Vibration transfer path system model

The vibration transfer path system with path mass parameter is equivalent to double vibration isolation devices, which demands better vibration isolation performance [3-5].

Reliability Analysis of Transfer System

According to the interference theory of reliability, the resonance state function of vibration transfer path systems is defined as

$$g_{ij}(p_j,\omega_i)=\left|p_j-\omega_i\right| \quad (i=1,2,\cdots,n;\ j=1,2,\cdots,m) \tag{3}$$

Where p_j is the j-th excitation frequency of vibration transfer path systems; ω_i is the i- order natural frequency.

According to the state function based on the difference between the excitation frequency p_j and the natural frequency ω_i, the two kinds of states of vibration transfer path system can be obtained

$$\begin{cases} g_{ij}(p_j,\omega_i)=\left|p_j-\omega_i\right|\le\gamma & \text{Failed state} \\ g_{ij}(p_j,\omega_i)=\left|p_j-\omega_i\right|>\gamma & \text{Security state} \end{cases} \tag{4}$$

Where γ is the specific range. It generally takes 5% -15% of natural frequency according to experiences. Limit state equation $g_{ij}(p_j,\omega_i)=\left|p_j-\omega_i\right|=\gamma$ is a n-dimensional surface defined as limit state surface

Let $G_{ij}=p_j-\omega_i$, then the mean and variance of the function G_{ij} are respectively

$$u_{G_{ij}}=\mathrm{E}(G_{ij})=u_{p_j}-u_{\omega_i} \tag{5}$$

$$\sigma^2_{G_{ij}}=\mathrm{Var}(G_{ij})=\sigma^2_{p_j}+\sigma^2_{\omega_i} \tag{6}$$

The quasi- resonance failure probability of vibration transfer path system is

$$P_f^{ij}=\int_{-\gamma_i\le G_{ij}\le\gamma_i} f_{\boldsymbol{b}}(\boldsymbol{b})d\boldsymbol{b} \tag{7}$$

Where $f_{\boldsymbol{b}}(\boldsymbol{b})$ is the joint probability density of the basic random parameter vector $\boldsymbol{b}=(b_1,b_2,\ldots,b_s)^T$.

Assumed that the random parameters of vibration transfer path systems are normal distribution, which is convenient for the reliability analysis. When excitation frequency and natural frequency are normal distribution and independent, the quasi-failure indicators of system are

$$\beta_1^{ij}=\frac{\mathrm{E}(\gamma_i-G_{ij})}{\sqrt{\mathrm{Var}(\gamma_i-G_{ij})}} \tag{8}$$

$$\beta_2^{ij}=\frac{\mathrm{E}(-\gamma_i-G_{ij})}{\sqrt{\mathrm{Var}(-\gamma_i-G_{ij})}} \tag{9}$$

The quasi-failure probability of system is

$$P_f^{ij} = \Phi(\beta_1^{ij}) - \Phi(\beta_2^{ij}) = \Phi\left(\frac{\gamma_i - \mu_{Gij}}{\sigma_{G_{ij}}}\right) - \Phi\left(\frac{-\gamma_i - \mu_{G_{ij}}}{\sigma_{G_{ij}}}\right) \tag{10}$$

Where Φ(·) is standard normal distribution function.

For the vibration transfer path system, if any one of the excitation frequencies is close to the natural frequency, then the system will produce the resonance. Thus, the whole system is considered to be in the failure or quasi- failure state. Consequently, when the transmission reliability is analyzed based on the relations of excitation frequencies and a natural frequency, the random system is a series system [6-8]. The system quasi-failure probability is

$$P_f = 1 - \prod_{i=1}^{n}\prod_{j=1}^{m}(1 - P_f^{ij}) \tag{11}$$

The reliability is

$$R = 1 - P_f = \prod_{i=1}^{n}\prod_{j=1}^{m}(1 - P_f^{ij}) \tag{12}$$

Numerical Example

Consider the five-degree-of-freedom vibration transfer path system model in Fig.1. The vibration source: m_s = 0.7724 kg, c_s = 0.5N·s/m, k_s = 100 N/m; the receiver: m_r = 1.0556 kg, c_r= 1N·s/m, k_r = 1800 N/m; the excitation magnitude F_0 = 10 N. The random path is represented by the random parameter vector $\mathbf{b}_{15\times1}=(m_{p1}\ m_{p2}\ m_{p3}\ c_{sp1}\ c_{sp2}\ c_{sp3}\ c_{rp1}\ c_{rp2}\ c_{rp3}\ k_{sp1}\ k_{sp2}\ k_{sp3}\ k_{rp1}\ k_{rp2}\ k_{rp3})^T$. The random mass, damping and spring of the transfer paths are all normal distribution with a variance equal to 0.05, and the mean values of these parameters are m_{p1}=0.9 kg, m_{p2}=0.5 kg, m_{p3}=0.6 kg, c_{sp1}=c_{rp1}=1.0N·s/m, c_{sp2}=c_{rp2}=1.5N·s/m, c_{sp3}=c_{rp3}=0.5N·s/m, k_{sp1}=k_{rp1}=900N/m, k_{sp2}=k_{rp2}=450 N/m, k_{sp3}=k_{rp3}=600 N/m respectively.

The means of natural frequencies are respectively

$$\omega_1 = 8.4552,\quad \omega_2 = 42.9305,\quad \omega_3 = 44.7214,\quad \omega_4 = 47.7453,\quad \omega_5 = 65.4045$$

When γ takes 5% and 10% of the mean natural frequency, the transfer reliability with excitation frequency can be calculated by formula (12). The corresponding curve is shown in Fig.2.

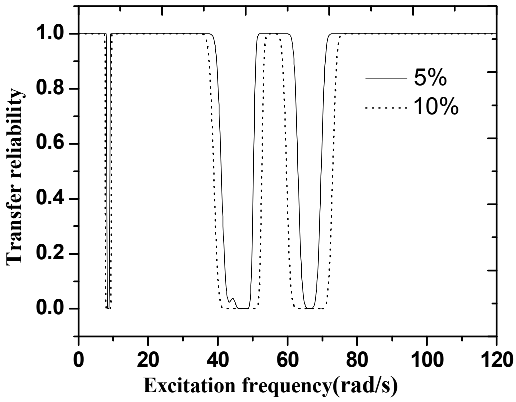

Fig. 2 The transfer reliability with exciting frequency

According to the curve, when excitation frequency is far away from the natural frequencies of vibration transfer path system, the reliability value is close to 1 and the system is security. The corresponding frequencies constitute the safety working range. As the excitation frequency is close to the natural frequency of the system, the reliability gradually reduces until it reaches zero. At this point, the system is in resonance failure or quasi-resonance failure state, which is dangerous and should be avoided in actual project. The dangerous range reduces as the value of γ reduces. It is also known that the dangerous frequency range increases as the path mass is considered in the vibration transfer path system. Therefore, the path mass parameters should be maximized to prevent system failure by resonance in vibration transfer path analysis.

Conclusions

According to the relation norm that the absolute value of the difference between the natural frequency and the excitation frequency doesn't exceed the specified value, the reliability mode and transfer reliability of resonance problem are defined for the vibration transfer path systems. The transfer reliability of the system with random paths is studied using the reliability theory combined with the stochastic finite element method. Based on the provided theory, the corresponding dynamic analysis can be made to avoid the resonance, which is very important for the system optimized design.

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