

## Multivariable Decoupling Control Based on TC Control in the Diving and Floating Process of AUV

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**Abstract.** The diving plane motion of autonomous underwater vehicles (AUVs) is a complex multivariable nonlinear system with pitch-heave coupling. Tornambe's controllers (TCs) can online estimate the dynamic uncertainty regardless of the type of disturbance. The TCs for depth and pitch angle are adopted to achieve multivariable decoupling control by introducing virtual control inputs. The methodology can obtain fast and non-overshoot control of depth and pitch angle in the diving and floating process. Simulation results demonstrated the effectiveness of the proposed control scheme.

### 1. Introduction

This paper describes a solution to multivariable decoupling control for vertical velocity and pitch angle of autonomous underwater vehicle in the diving plane. Usually in the diving and floating process of AUV, the desired pitch angle and depth are set as certain constants in a certain time. C. Silvestre and A. Pascoal adopted the methodology of nonlinear gain-scheduling control to design a controller for depth. The controllers implemented have proven extremely reliable over a long series of missions<sup>[1]</sup>. Lionel Lapierre proposed a diving-control design based on Lyapunov theory and back-stepping techniques. Then using adaptive and switching schemes, the control system is able to meet the required robustness<sup>[2]</sup>. Bessa W M et al. designed a depth regulator is based on the sliding mode control strategy and enhanced by an adaptive fuzzy algorithm for uncertainty/disturbance compensation<sup>[3]</sup>.

In this paper the decoupling control for AUV's vertical velocity and pitch angle is carried out by TCs. The TC can estimate internal and external uncertainty and make accurate compensation. Simulation results showed that the AUV can regulate vertical velocity and pitch angle with fast response time and small overshoot by the proposed control scheme.

### 2. TC decoupling control

Tornambe proposed a nonlinear robust decentralized controller (TC)<sup>[4]</sup> that doesn't require accurate model and has simple structure and is easily implemented.

Consider a class of multi-input multi-output (MIMO) uncertain nonlinear system, the state equation is

$$\begin{cases} \dot{x}_1 = f_1(\mathbf{x}) + b_{11}u_1 + \cdots + b_{1m}u_m \\ \vdots \\ \dot{x}_m = f_m(\mathbf{x}) + b_{m1}u_1 + \cdots + b_{mm}u_m \\ y_i = x_i \quad (1 \leq i \leq m), \end{cases} \quad \mathbf{B} = \begin{bmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{m1} & \cdots & b_{mm} \end{bmatrix}, \quad (1)$$

where  $\mathbf{x} = (x_1, x_2, \dots, x_m)^T \in \mathbf{R}^m$ ,  $\mathbf{u} = (u_1, u_2, \dots, u_m)^T \in \mathbf{R}^m$ ,  $m$  is the dimension of state vector.

*Assumptions.*

(a.1) The state variables  $\mathbf{x} = (x_1, x_2, \dots, x_m)^T \in \mathbf{R}^m$  are observable.

(a.2) The magnification coefficient  $b_{ij}(\mathbf{x}, t) (1 \leq i, j \leq m)$  is the function of state variables and time. The magnification coefficient matrix is reversible, that is  $\mathbf{B}^{-1}$  exists.

The virtual control inputs that are  $U = Bu$  are introduced. The equation (1) can be changed as

$$\dot{x} = f(x) + U, y = x, \tag{2}$$

where  $U = (U_1, U_2, \dots, U_m)^T \in R^m$ ,  $f = (f_1, f_2, \dots, f_m)^T \in R^m$ .

The relationship between input and output of the  $i$ -th channel is as follows

$$\dot{x}_i = f_i(x) + U_i, y_i = x_i. \tag{3}$$

The virtual control input  $U_i$  and output  $y_i$  of every channel is the relationship of single input and single output. So the total decoupling control of the  $i$ -th output  $y_i$  and virtual control input  $U_i$  can be gotten. The practical output  $y_i$  can fast and accurately tracking the reference  $r_i$  by TC. Under the assumption (a.2), the actual control inputs can be given as  $u = B^{-1}U$ . The total decoupling control progress is as shown in Figure 1.

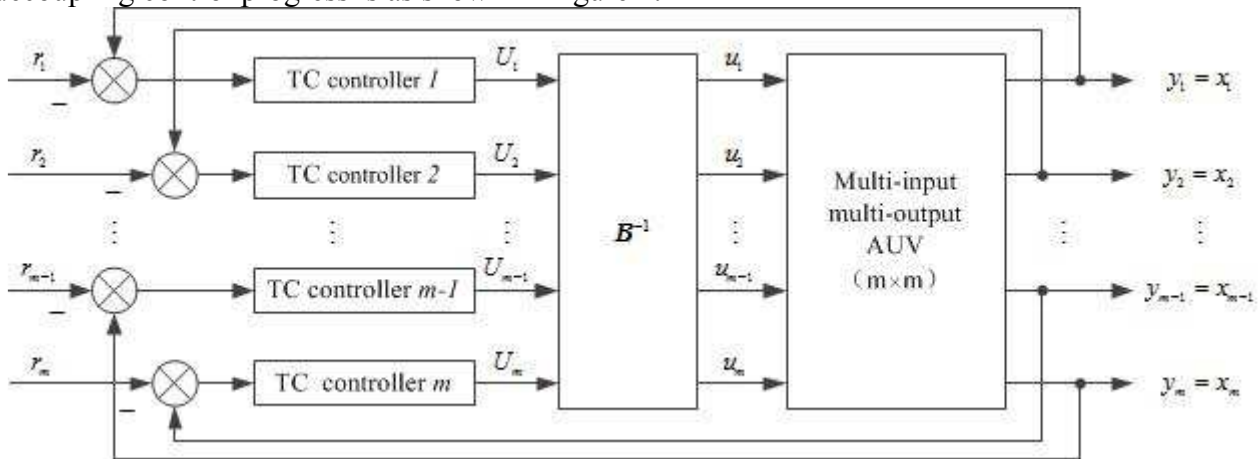


Fig.1. Multivariable decoupling control based on TC control

### 3. Multivariable decoupling control of AUV based on TC control

#### 3.1 The vertical motion model of AUV

The vertical movement equation of the submarine with the following assumptions: (1) A model for a streamlined autonomous underwater vehicle is considered that explicitly displays the coupling between vertical velocity and pitch angle. (2) The AUV vertical motion is independent on horizontal motion and the stable rolling motion is neglected. Due to the geometrical symmetry of the diving plane and horizontal plane, the location of gravity center  $x_G, y_G$  and the buoyancy center  $x_B, y_B$  are zeroes. Then the simplified diving plane model is written as

$$\dot{\theta} = q, \tag{4}$$

$$\begin{aligned} & -\left(mx_G + \frac{\rho}{2}L^4M'_w\right)\dot{w} + \left(I_y - \frac{\rho}{2}L^5M'_q\right)\dot{q} \\ & = \frac{\rho}{2}L^3\left(M'_w u_0 + M'_{w|w}|w_0|\right)w + \left[\frac{\rho}{2}L^5M'_{q|q}|q_0| + \frac{\rho}{2}L^4\left(M'_q u_0 + M'_{|w|q}|w_0|\right) - mx_G u_0 - mz_G w_0\right]q \end{aligned} \tag{5}$$

$$-z_G W \theta + \frac{\rho}{2}L^3\left(M'_* u_0^2 + M'_{|w|} u_0 |w_0| + M'_{ww} w_0^2 + M'_{\delta_s} u_0^2 \delta_s\right) - x_G W + \tau_M,$$

$$\begin{aligned} & \left(m - \frac{\rho}{2}L^3Z'_w\right)\dot{w} - \left(mx_G + \frac{\rho}{2}L^4Z'_q\right)\dot{q} \\ & = \left[\frac{\rho}{2}L^3Z'_{w|q}|q_0| + \frac{\rho}{2}L^2\left(Z'_w u_0 + Z'_{w|w}|w_0| + Z'_{ww} w_0\right)\right]w + \left(mu_0 + mz_G q_0 + \frac{\rho}{2}L^4Z'_{q|q}|q_0| + \frac{\rho}{2}L^3Z'_q u_0\right)q \end{aligned} \tag{6}$$

$$+ \frac{\rho}{2}L^2\left(Z'_* u_0^2 + Z'_{|w|} u_0 |w_0| + Z'_{\delta_s} u_0^2 \delta_s\right) + \tau_Z.$$

The equations (4), (5) and (6) can be rewritten in the state space as follows

$$\begin{bmatrix} m - Z_{\dot{w}} & -mx_G - Z_{\dot{q}} & 0 \\ -mx_G - M_{\dot{w}} & I_y - M_{\dot{q}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} Z_w & Z_q + mu_0 & 0 \\ M_w & M_q - mx_G u_0 & -z_G W \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + B \begin{bmatrix} \tau_Z \\ \tau_M \end{bmatrix} + R, \quad (7)$$

$$\text{where } H = \begin{bmatrix} m - Z_{\dot{w}} & -mx_G - Z_{\dot{q}} & 0 \\ -mx_G - M_{\dot{w}} & I_y - M_{\dot{q}} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad P = \begin{bmatrix} Z_w & Z_q + mu_0 & 0 \\ M_w & M_q - mx_G u_0 & -z_G W \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

$$R = [d_w \quad d_q \quad 0]^T, \quad Z_{\dot{w}} = \frac{\rho}{2} L^3 Z'_{\dot{w}}, \quad Z_{\dot{q}} = \frac{\rho}{2} L^4 Z'_{\dot{q}}, \quad M_{\dot{w}} = \frac{\rho}{2} L^4 M'_{\dot{w}}, \quad M_{\dot{q}} = \frac{\rho}{2} L^5 M'_{\dot{q}},$$

$$Z_w = \frac{\rho}{2} L^3 Z'_{w|q}|q_0| + \frac{\rho}{2} L^2 (Z'_w u_0 + Z'_{w|w}|w_0| + Z'_{ww} w_0), \quad Z_q = \frac{\rho}{2} L^4 Z'_{q|q}|q_0| + mz_G q_0 + \frac{\rho}{2} L^3 Z'_q u_0,$$

$$M_w = \frac{\rho}{2} L^3 M'_{ww} u_0 + \frac{\rho}{2} L^3 M'_{w|w}|w_0|, \quad M_q = \frac{\rho}{2} L^5 M'_{q|q}|q_0| + \frac{\rho}{2} L^4 M'_q u_0 + \frac{\rho}{2} L^4 M'_{|w|q}|w_0| - mz_G w_0,$$

$$d_q = \frac{\rho}{2} L^3 (M'_s u_0^2 + M'_{|w|} u_0 |w_0| + M'_{ww} w_0^2 + M'_{\delta_s} u_0^2 \delta_s) - x_G W, \quad d_w = \frac{\rho}{2} L^2 (Z'_s u_0^2 + Z'_{|w|} u_0 |w_0| + Z'_{\delta_s} u_0^2 \delta_s).$$

where  $m$ ,  $L$  and  $I_y$  are the mass, overall length and moment of inertia, respectively.  $\rho$  is the density of sea water.  $\theta$ ,  $u$ ,  $w$  and  $q$  are the pitch angle, horizontal speed, vertical speed, pitch angular velocity, respectively.  $\delta_s$  is the horizontal rudder angle.  $\dot{w}$  and  $\dot{q}$  are the vertical acceleration and pitch angular acceleration, respectively.  $\tau_Z$  and  $\tau_M$  are the heave force and pitch moment, respectively.  $d_w$  and  $d_q$  are the total disturbance of vertical velocity section and pitch control section, respectively. The other parameters are hydrodynamic coefficients.

Since the matrix  $H$  is reversible, the equation (7) can be rewritten as

$$\dot{\mathbf{x}} = P' \mathbf{x} + B' [\tau_Z \quad \tau_M]^T + R', \quad (8)$$

where,  $\mathbf{x} = [w \quad q \quad \theta]^T$ ,  $P' = H^{-1}P$ ,  $B' = H^{-1}B$ ,  $R' = H^{-1}R$ .

### 3.2 Controller design

In order to eliminate and compensate internal and external uncertainty, TC controller of single variable can be designed as

$$\begin{cases} u = -\sum_{i=0}^{r-1} h_i z_{i+1} - \hat{d}, \quad \hat{d} = \xi + \sum_{i=0}^{r-1} k_i z_{i+1}, \\ \dot{\xi} = -k_{r-1} \xi - k_{r-1} \sum_{i=0}^{r-1} k_i z_{i+1} - \sum_{i=0}^{r-2} k_i z_{i+2} - k_{r-1} u, \end{cases} \quad (9)$$

where  $r$  is the relative degree;  $k_i$  ( $i = 0, \dots, r-2$ ) are arbitrary constants;  $k_{r-1} = \text{sign}(f_i) \mu$ , with  $\mu$  being a suitable positive constant and determining the system stability;  $h_i$ ,  $i = 0, \dots, r-1$  are adjustable dynamic parameters and determines the response time. The relative degree of vertical velocity system is  $r = 1$  and the state variable  $z_1$  is  $w - w^*$ . The relative degree of pitch angle system is  $r = 2$  and the state variables  $z_1$  and  $z_2$  are  $\theta - \theta^*$ ,  $q$ , respectively.

In order to copy with the contradiction between the response time and overshoot, the desired value is dealt with tracking differentiator (TD)<sup>[5]</sup> before being transmitted to TC.

### 4. Numerical simulation

To validate the performances of this controller, we carried out two simulations that are leveling control simulation and tracking control simulation. The diving plane model parameters are shown in table 1. The TC parameters for vertical velocity system are  $k_0 = 252.9636$  and  $h_0 = 0.1953$ ; The TC parameters for pitch angle system are  $k_0 = 221.2037$ ,  $h_0 = 10$ ,  $k_1 = 500.9934$  and  $h_1 = 0.2201$ . The TC parameters for leveling control simulation and tracking control simulation are same.

Table 1 The hydrodynamic coefficients of the AUV

$X$	$Z$	$M$
$L = 6.3 \text{ m}$	$m = 1300 \text{ kg}, \rho = 1020.00 \text{ kgm}^3$	$I_y = 3740 \text{ kgm}^2, g = 9.81 \text{ ms}^{-2}$
$x_G = -0.002 \text{ m}$	$Z'_w = -7.256 \times 10^{-3}, Z'_q = 0.030 \times 10^{-3}$	$M'_w = 0.030 \times 10^{-3}$
$W = 13000 \text{ N}$	$Z'_{w q} = -11.20 \times 10^{-3}, Z'_w = -13.21 \times 10^{-3}$	$M'_q = -0.340 \times 10^{-3}, M'_w = 3.96 \times 10^{-3}$
	$Z'_{w w} = -74.70 \times 10^{-3}, Z'_{ww} = 0.75 \times 10^{-3}$	$M'_{w w} = -21.36 \times 10^{-3}, M'_{q q} = -1.67 \times 10^{-3}$
	$Z'_{q q} = -3.78 \times 10^{-3}, Z'_q = -7.00 \times 10^{-3}$	$M'_q = -3.80 \times 10^{-3}, M'_{ w q} = -13.68 \times 10^{-3}$
	$Z'_* = 0.04 \times 10^{-3}, Z'_{ w } = 0.93 \times 10^{-3}$	$M'_* = 0.009 \times 10^{-3}, M'_{ w } = 0.17 \times 10^{-3}$
	$Z'_{\delta_s} = -5.30 \times 10^{-3}, z_G = 0.005 \text{ m}$	$M'_{ww} = -1.00 \times 10^{-3}, M'_{\delta_s} = -2.5 \times 10^{-3}$

4.1 Leveling control simulation

In the leveling control simulation, both the desired vertical velocity and pitch angle are set as zero, with the initial conditions of vertical velocity and pitch angle being 0.15 m/s and 0.3 rad, respectively.

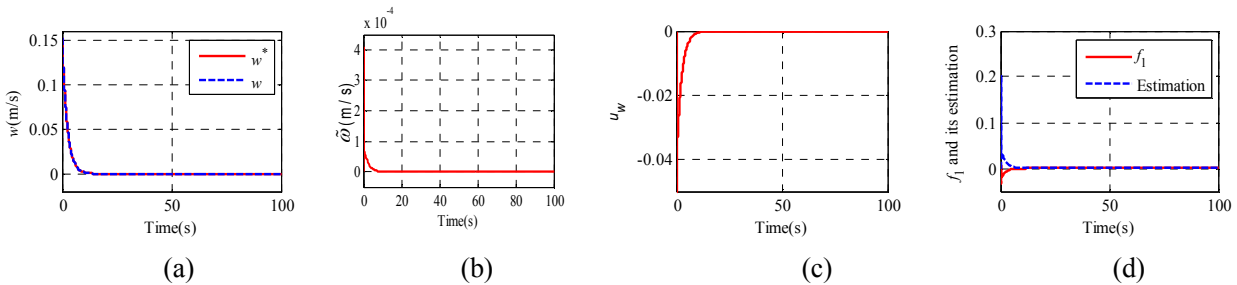


Fig.2. Vertical velocity, (a)references and outputs, (b) tracking errors, (c) control inputs, (d) disturbance and estimation.

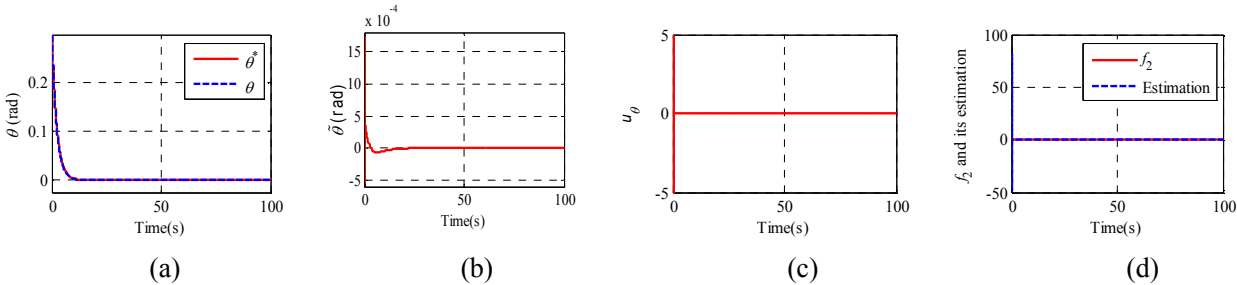


Fig.3. Pitch angle, (a) references and outputs, (b) tracking errors, (c) control inputs, (d) disturbance and estimation.

Simulation results are depicted in Figure 2 and 3. Figure 2(a) and 3(a) illustrate the time behavior of  $w$  and  $\theta$ . The practical values can almost track the desired values without overshoot. The tracking errors converge to zero rapidly as shown in Figure 2(b) and 3(b). Two TC control inputs have no chattering in Figure 2(c) and 3(c). TC can estimate the system uncertainties with short time and high accuracy shown in Figure 2(d) and 3(d).

4.2 Tracking control simulation

In the leveling control simulation, the desired vertical velocity is set as 0.15 m/s in [0, 30s], 0 m/s [30s, 60s], -0.15 m/s in [60s, 100s] and 0.5 m/s in [100s, 150s]. And the desired pitch angle is set as 0.3 rad in [0, 30s], 0 rad [30s, 60s], -0.3 rad in [60s, 100s] and 0.1 rad in [100s, 150s]. The initial conditions of vertical velocity and pitch angle are 0.15 m/s and 0.3 rad, respectively.

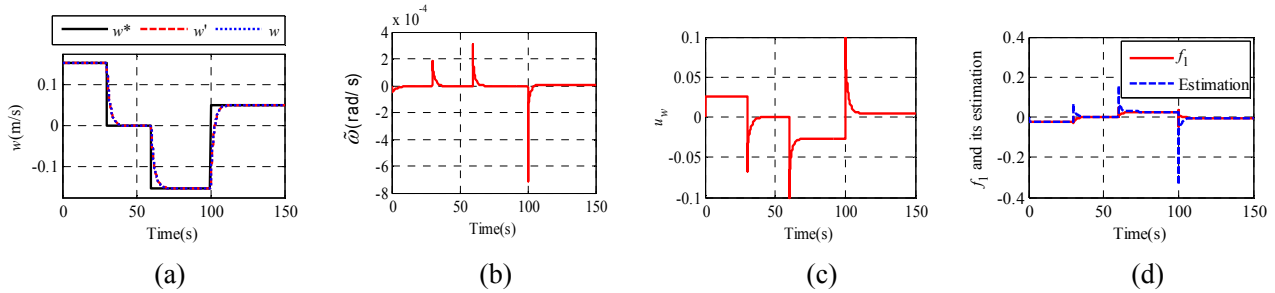


Fig.4. Vertical velocity, (a) references and outputs, (b) tracking errors, (c) control inputs, (d) disturbance and estimation.

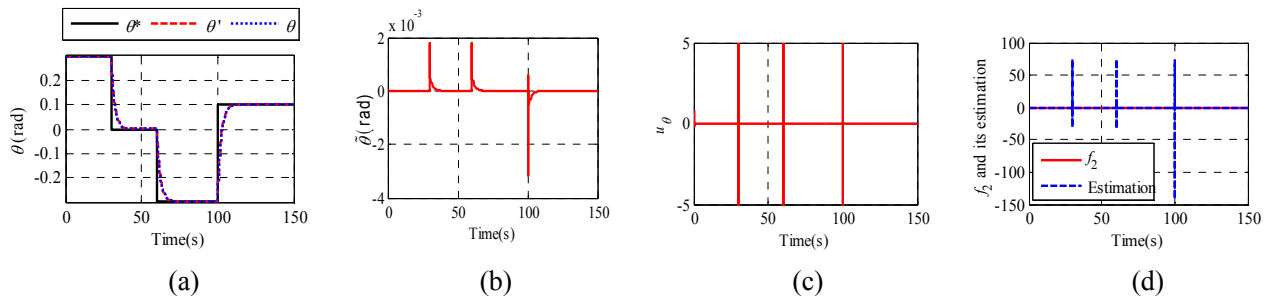


Fig.5. Pitch angle, (a) references and outputs, (b) tracking errors, (c) control inputs, (d) disturbance and estimation.

Figure 4 and 5 illustrate the tracking performance of vertical velocity and pitch angle. Figure 4(a) and 5(a) show the desired, transient and practical vertical velocity and pitch angle, respectively. Figure 4(c) and 5(c) display the activity of control inputs. TC can handle large changing desired inputs and estimate system uncertainties and make most compensation.

## 5. Conclusion

The paper described the design of TCs for vertical velocity and pitch angle of AUV in the diving plane. The methodology achieved multivariable decoupling control. The performance of the controller developed was evaluated by leveling control simulation and tracking control simulation. Simulations have shown that the control system has high accuracy and stability, and performs well with robustness.

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## References

- [1] Silvestre C, Pascoal A. Depth control of the INFANTE AUV using gain-scheduled reduced order output feedback [J]. *Control Engineering Practice*, 2007, 15(7): 883-895.
- [2] Lapierre L. Robust diving control of an AUV [J]. *Ocean Engineering*, 2009, 36(1): 92-104.
- [3] Bessa W M, Dutra M S, Kreuzer E. Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller[J]. *Robotics and Autonomous Systems*, 2008, 56(8): 670-677.
- [4] Tornambe A, Valigi P. A decentralized controller for the robust stabilization of a class of MIMO dynamical systems [J]. *Journal of Dynamic Systems, Measurement, and Control*, 1994, 116(2): 293-304.
- [5] A. Levant. Robust exact differentiation via sliding mode technique [J]. *Automatica*, 1998, 34: 379-384.

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## **Multivariable Decoupling Control Based on TC Control in the Diving and Floating Process of AUV**

10.4028/www.scientific.net/AMM.741.720

### **DOI References**

[1] Silvestre C, Pascoal A. Depth control of the INFANTE AUV using gain-scheduled reduced order output feedback [J]. *Control Engineering Practice*, 2007, 15(7): 883-895.

<http://dx.doi.org/10.1016/j.conengprac.2006.05.005>

[2] Lapierre L. Robust diving control of an AUV [J]. *Ocean Engineering*, 2009, 36(1): 92-104.

<http://dx.doi.org/10.1016/j.oceaneng.2008.10.006>

[3] Bessa W M, Dutra M S, Kreuzer E. Depth control of remotely operated underwater vehicles using an adaptive fuzzy sliding mode controller[J]. *Robotics and Autonomous Systems*, 2008, 56(8): 670-677.

<http://dx.doi.org/10.1016/j.robot.2007.11.004>

[4] Tornambe A, Valigi P. A decentralized controller for the robust stabilization of a class of MIMO dynamical systems [J]. *Journal of Dynamic Systems, Measurement, and Control*, 1994, 116(2): 293-304.

<http://dx.doi.org/10.1115/1.2899223>