COMPLEX NETWORK TRAFFIC SIGNAL OPTIMIZED MODEL AND SIMULATION

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Abstract
In urban road traffic system, an important issue with the expansion of the network, resulting in the increasing complexity of the network, in-depth analysis of complex network operating mechanism is to further optimize and control network traffic and maximize network efficiency is what we are facing, urban traffic network congestion problem is a typical complex network optimization and control problems. In this paper, optimization and modeling of complex transportation network will transport network abstraction for the network nodes and the intersection of the road, the vehicle network communication abstract packets, control timing Intersections, namely switching control network nodes, proposed a distributed message mechanism adaptive optimization method based on network traffic and through mathematical modeling and simulation, to verify its validity.

Keywords: Intelligent Transportation System; Traffic Signal optimization; Distributed network;

1 Introduction
In urban road traffic system, the traffic network is composed of two basic units, road and intersection. The road is connected with different intersections. It is the carrier of vehicles to travel from one intersection to the next intersection nodes, and the intersection realize the function that the vehicles in different directions distribution and transmission. However, urban road traffic capacity is mainly affected by the coordination control ability of the intersection traffic signal, the intersection is also considered to be the most widely distributed traffic bottleneck nodes.

The world contains various kinds of complex network, including Internet, power network, logistics network, traffic network, telecommunication network, including human neural nets, and so on. Although they belong to different fields, they have an irreplaceable role in daily life. Therefore, how to optimize the network to realize the network advantage becomes the current research focus. The network can be abstracted as the basic unit and intersection node. For example, the urban road network can be abstracted as nodes and links to the network. The road is the carrier of vehicles to travel from one intersection to the next intersection nodes, and the intersection realize the function that the vehicles in different directions distribution and transmission. The Internet can be abstracted as the base station and signal transmission channel. The base station sends resources to users, and the channel is carrier for signal transmission. However, with the expansion of the network scale, the complexity of the network has been increasing. It is an important issue to further optimize and control the network. The problem of urban traffic network congestion is a typical complex network optimization and control problem [1].

The traffic network control is abstracted into the problem of complex network resource acquisition and configuration [2]. At different intersections, the vehicles at intersections can get the traffic signal in time. The target of traffic network optimization is that improving the efficiency of the traffic line and avoiding the collision with other vehicles at the intersection. The single node traffic model analysis can be traced back to 1950s [3], the article based on the Poisson distribution model analyzed optimized control signal for on-line vehicles at the single point.

The interaction between more than one intersection introduces new questions. In [4], Macleod C J and Al-Khaili A J present a model for a traffic network with cars that perform a decision about which direction to take based on some probabilistic model at every intersection. By utilizing Cellular Automaton [5,6] and the freeway model presented in [7], models have recently been presented in [8, 9] where the whole network is analyzed at a small scale. This kind of models and approaches have been used widely to generate switching policies.

How to determine when different signal lamps in a network should be switched in order to minimize the delay waiting time in the network is the main question that these papers try to address. A.J. Al-Khaili determines what is the optimal offset between two intersections set in cascade in order to minimize the delay of cars waiting in the intersections in [10,11], but concatenating more than two intersections using this method is infeasible. Many mechanisms have been proposed in order to control traffic light scheduling. [12] presents a review of the different mechanisms that have been devised and implemented. Moreover, distributed and centralized algorithms have been deployed in many cities. Take into account the distributed policies and the information that can be sensed directly from the intersection, an algorithm of this kind is presented in [13].

The late 1970s, China began to research the urban road network coordinated traffic signal. On the one hand, some scholars abroad typical regional control systems (such as SCATS, SCOOT, etc.) for secondary development to adapt the mixed traffic flow of China [14]; on the other hand, some scholars have carried on the theory research to the urban traffic coordination control. The representative researches include: Chang Yuntao et al proposed a system based on the minimum total delay trunk coordinated control model, but the efficiency of the genetic algorithm is not high and the model is not suitable for expan-
sion\textsuperscript{[15]}; Wang Jungang proposed the mathematic model of variable band speed arterial coordination control\textsuperscript{[16]}. The control objective function is green bandwidth and the control decision -making variable is time difference, but the model does not take into account the effect of the left turn on the trunk traffic flow. Li Wei et al proposed the control strategy which combined the traditional control method with the fuzzy logic in order to coordinate the arterial signals\textsuperscript{[17]}. The computational complexity of the algorithm is too large and the algorithm is not suitable for real-time traffic calculation.

Therefore, in this paper, the urban road network can be abstracted as nodes and links to the network. The vehicles are the packets of the network communication and the signal lamp is the switching control time of network nodes. The message is transmitted to the next node, and the delay function is optimized by using the message function in turn to realize the distributed adaptive traffic signal control.

\section*{2 Traffic Model}

The urban traffic system is a complex nonlinear systems, with a strong randomness and uncertainty, it is difficult to propose a precise mathematical model. In this paper, the intersections can be abstracted as nodes. In order to simplify the study, a 10×10 two dimensional network model is established, where, \((i, j)\) represents the location of traffic intersection, \(i, j \in \{0,1,...,9\}\), \(\Delta_x\) represents the road length in the east of the horizontal direction, \(\Delta_y\) represents the road length in the south of the vertical direction. Two dimensional traffic network model is shown in Figure 1.

Based on the above two dimensional traffic network model, the following assumptions are made:

1) The system signal lamp has a uniform cycle and phase, and assume the period \(T=1s\), the phase is \(T/2\);
2) The signal lamp only considers the red light and the green light;
3) The traffic is only taken in two directions, from north to south, from west to east (for the four phase system without loss of generality);
4) The traffic flow length of each node is \(T/2\), and the traffic flow disposable completely get through without delay;
5) The traffic flow of each node in the new cycle is not accumulated, that means the length of the traffic flow is fixed at \(T/2\).

\section*{3 Model Analysis}

Based on the above assumptions, system variables are defined as shown in Table 1.

\begin{table}[h!]
\centering
\begin{tabular}{|l|l|}
\hline
Notation & Meaning \\
\hline
\(W, E, S, N\) & Directins of traffic. Indicates where it comes from \\
\(a_i(i, j)\) & Time of traffic flow reaches node \((i, j)\) from direction \(d, d \in \{W, E, S, N\}\) \\
\(\rho_i(i, j)\) & The phase of traffic signal lamp of node \((i, j)\) \\
\(x_i(i, j)\) & The switching time of signal lamp of node \((i, j)\) \\
\(\Delta_x(i, j)\) & The road length in direction of node \((i, j)\) \\
\(L_d(i, j)\) & The length of traffic flow reaches node \((i, j)\) from direction \(d\) \\
\(\delta_d(i, j)\) & Delay waiting time for a single node \((i, j)\) from direction \(d\) \\
\hline
\end{tabular}
\end{table}

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Two dimensional traffic network model}
\end{figure}

\begin{figure}[h!]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{The system delay model}
\end{figure}

Based on the above model and variable definition, the system delay model can be described as the following situations which are shown in Figure 2 and the delay waiting time for a single node can be described as Formula (1).

\begin{equation}
\delta_d(i, j) = \begin{cases} 
L(x-a+1) & x-a \in (-1-\rho) \\
-(1-\rho)(x-a) & x-a \in [-\rho, 0) \\
L(x-a) & x-a \in [0,1-\rho) \\
-(1-\rho)(x-a-1) & x-a \in [1-\rho,1) 
\end{cases}
\end{equation}
According to the relationship between signal switching time and node delay, the relationship function between \( a_w(i) \) and \( a_N(i) \) is shown in Figure 3. The delay time \( \delta(i,j) = \delta_N(i,j) + \delta_w(i,j) \). Therefore, when the signal light switch time \( x(i,j) \) maintains between \( a_w(i,j) \) and \( a_N(i,j) \), delay waiting time for all nodes in the net-work reaches the minimum.

\[
\delta = f(x(i,j); a_w(i,j), L_{ij}(i,j), \Delta_x(i,j))
\]

(3)

Where, \( \Delta_x(i,j) \) and \( L_{ij}(i,j) \) are static data parameters in the network model. For the same optimization process, the static data parameters will not change. In order to facilitate the representation, the static data parameters are simplified in this paper.

The optimal switching time \( x(i,j) \) which makes the system delay waiting time for all nodes \( \delta(i,j) \) is obtained by optimizing system variables. Based on the assumption of this paper, when the signal light switch time \( x(t) \) maintains \( x(i,j) \in [a_w, a_N] \), delay waiting time for all nodes in the net-work reaches the minimum. With the assumption that the delay is a linear function, the algorithm shifts \( T/2 \) such that \( a_w = a_N \), then the system delay is optimal. The optimization algorithm is as follows:

a) Determine the minimum delay waiting time of the node \( \delta(9,9) \), then determine the switching time of the boundary node \( x^*(9,9) = \arg \min f(x) \); 

b) Based on the switching time of the boundary node \( x^*(9,9) \), the new message function is defined as:

\[
g_{9,9}(a_w) = \min f(x(9,9); a_w, a_N(9,9))
\]

(4)

c) The message is transmitted to the next node, and the delay function \( \delta(8,9) \) is optimized by using the message function

\[
x^*(9,8) = \arg \min \phi(x)
\]

(6)

Where

\[
\phi(x) = f(x(9,8); a_w(9,8), a_N(9,8))
\]

(7)

d) The message function of the horizontal and vertical two directions is derived in turn

\[
g_{(i,j)}(a_w) = \min \left\{ f(x(i,j), a_w, a_N(i,j)) \right\}
\]

(8)

e) Determine the switching time \( x^*(i,j) \) of all the nodes, then determine the optimal delay time \( \delta(i,j) \)

\[
x^*(i,j) = \arg \min_{x} \left\{ f(x(i,j), a_w(i,j), a_N(i,j)) \right\}
\]

(9)

As shown in Figure 4, the initialization delay of the 10×10 two dimensional network is about 25.2s. The total delay waiting time fast converge to 12s after 10 iterations and the total delay waiting time maintains within 6.5s after 50 iterations. The simulation results prove that the algo-
The length of traffic flow divided into \( c \), \( c \). The moment for the \( \tau \). The moment that traffic signal \( t \), \( t \) starts at \( \Delta_{\tau} \). The maximum waiting time at node 2 is \( 1-\rho \) and the traffic flow \( L_{t} \) starts to pass at \( \Delta_{\tau} + (1-\rho) \). The moment for the traffic flow \( L_{t} \) starts from node 1 is \( T \) and the traffic flow \( L_{t} \) reaches the end of the traffic flow \( L_{t} \) at \( T + (\Delta_{\tau} - \rho) \), then the traffic flow will not be merged under the boundary condition.

For the formula (12), the traffic flow is divided into queues \( r, \rho, \rho, \ldots, \rho, \Delta_{\tau} \). The moment that traffic signal lamp of node 1 turns green is the reference zero, then the time for the traffic flow \( L_{t} \) reaches node 2 is \( \Delta_{\tau} \). The maximum waiting time at node 2 is \( 1-\rho \) and the traffic flow \( L_{t} \) starts to pass at \( \Delta_{\tau} + (1-\rho) \). The moment for the traffic flow \( L_{t} \) starts from node 1 is \( T \) and the traffic flow \( L_{t} \) reaches the end of the traffic flow \( L_{t} \) at \( T + (\Delta_{\tau} - \rho) \), then the traffic flow will not be merged under the boundary condition.

The improved model cancels the assumption 4) and 5), then the system model can be transformed into \( A_{t} + 1 \) traffic flow models and the length of traffic flow is \( L' \). Each sub process is optimized independently.

The expression of the message function plays a decisive role in the optimization of the method. To cancel the limit on the length of the traffic flow \( L \) and the phase of traffic signal lamp \( \beta \). Traffic flow size determines the length of time the green lamp \( \rho_{w} \).

For the formula (11), the traffic flow is divided into queues \( \rho, \rho, \rho, \ldots, \rho, A_{t} \). The moment that traffic signal lamp of node 1 turns green is the reference zero, then the time for the traffic flow \( L_{t} \) reaches node 2 is \( \Delta_{\tau} \). The maximum waiting time at node 2 is \( 1-\rho \) and the traffic flow \( L_{t} \) starts to pass at \( \Delta_{\tau} + (1-\rho) \). The moment for the traffic flow \( L_{t} \) starts from node 1 is \( T \) and the traffic flow \( L_{t} \) reaches the end of the traffic flow \( L_{t} \) at \( T + (\Delta_{\tau} - \rho) \), then the traffic flow will not be merged under the boundary condition.

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For example, the initial state of \( N \times N + 1 \) traffic network \( a_{w} = 0.6 \), \( a_{h} = 0.3 \), \( L_{w} = 0.9 \), \( L_{h} = 0.7 \). Ignoring the message function accumulation of other nodes. Figure 6 describes the transfer process of message function. The message function of boundary node \( (N, N) \) have the simplest form. All the changing curves of message function at generating node is shown in Figure 7 and Figure 8. The message function \( g (h) \) is respectively determined by \( a_{w} \) (or \( a_{h} \)) and \( x \). With the message function further transfers, the message function accumulation becomes more complicated.
In order to improve the convergence speed of the algorithm and improve the real-time performance of the system, the non-linear relationship of the message function between the nodes should be reserved as far as possible and the research of message function should be independent of the iterative algorithm, which provides the basis for the further optimization of the system model in the future which are the basis for further optimization of the system model. In this paper, the optimization solution of signal switching time in traffic network $x'(i,j)$ is determined by the matrix form of message function.

The optimization algorithm flow is shown in Figure 9.

**Figure 9 The optimization algorithm**

First, establish the network model based on the actual traffic data. The matrix form of the static data parameters can be expressed as

$$
\Delta_E = \begin{bmatrix}
\Delta_E(0,0) & \cdots & \cdots & \cdots \\
\Delta_E(1,0) & \cdots & \cdots & \cdots \\
\cdots & \cdots & \Delta_E(N-2,M-3) & \Delta_E(N-2,M-2) \\
\cdots & \cdots & \Delta_E(N-1,M-3) & \Delta_E(N-1,M-2)
\end{bmatrix}
$$

(14)

$$
\Delta_S = \begin{bmatrix}
\Delta_S(0,0) & \cdots & \cdots & \cdots \\
\Delta_S(1,0) & \cdots & \cdots & \cdots \\
\cdots & \cdots & \Delta_S(N-3,M-2) & \Delta_S(N-3,M-1) \\
\cdots & \cdots & \Delta_S(N-2,M-2) & \Delta_S(N-2,M-1)
\end{bmatrix}
$$

(15)

Where, $\Delta_E$ is the road length in the direction $W \leftrightarrow E$, $\Delta_S$ is the road length in the direction $N \leftrightarrow S$.

$$
L_d = \begin{bmatrix}
L_d(0,0) & \cdots & \cdots & \cdots \\
L_d(1,0) & \cdots & \cdots & \cdots \\
\cdots & \cdots & L_d(N-2,M-2) & L_d(N-2,M-1) \\
\cdots & \cdots & L_d(N-1,M-2) & L_d(N-1,M-1)
\end{bmatrix}
$$

(16)
The assumption of this paper, the algorithm constituted by \( a_N a_M a_N a_M \) is based on the fact that the bigger the iteration step size, the faster the network node is to converge. The matrix of the message function \( \delta(i,j) \) is calculated according to the message function structure is too complex, the algorithm avoids the disadvantages of the error caused by the approximate calculation. The iterative process is performed until the system is stable, and the switching time of signal lamp in the network node is optimization solution \( x'(i,j) \).

### 5 Simulations

Based on the assumption of this paper, the algorithm is verified by simulations. The same as above, initialize a 10×10 two-dimensional traffic network model. The system signal lamp has a uniform cycle and phase, and assume the period \( T=1s \), the phase is \( T/2 \). The signal lamp only considers the red light and the green light. The traffic is only taken in two directions, from north to south, from west to east.

The simulation results prove that the algorithm based on the parallel equidistant distribution has strong convergence and high efficiency. The message transfer function matrix of each node is calculated according to the road length between the nodes. As shown in Figure 10, the total delay waiting time fast converge to 6.34s after 4 iterations. Simulation shows that the algorithm has a fast convergence speed and good stability.

\[
\begin{bmatrix}
  a_d(0,0) & \cdots & \cdots & \cdots \\
  a_d(1,0) & \cdots & \cdots & \cdots \\
  \vdots & \ddots & \ddots & \ddots \\
  a_d(N-1, M-2) & a_d(N-2,M-1) & \cdots & a_d(N-1,M-1)
\end{bmatrix}
\]

(17)

According to the message transfer function, select the iterative step, establish the basic matrix of the message function which is constituted by \( a_w, a_x \) and \( x \), where, \( a_w, a_x, x \in [0, T] \). Then calculate the message function matrix of each node at once. Take the message function \( g_{(i,j)}(a_w) \) as an example, the steps are as follows:

a) According to the basic matrix of the message function and the the current iteration value \( a_g \), get the optimal value of the current iteration \( x'(i,j) \), calculate delay waiting time for this node;

b) According to the message function matrix of the downstream nodes in the east and the south, calculate the total delay waiting time;

c) The sum of delay waiting time for this node and the total delay waiting time is the the message function matrix of this node.

The accuracy of the optimal value \( x'(i,j) \) is independent of the delay function \( \delta(i,j) \)'s form, which is related to the choice of the step size. Although the function structure of formula (3) function structure is too complex, the algorithm avoids the disadvantages of the error caused by the approximate calculation. The iterative processing is performed until the system is stable, and the switching time of signal lamp in the network node is optimization solution \( x'(i,j) \).

### 6 Conclusion

The paper proposed a distributed message mechanism adaptive optimization method based on the complex network information model. The urban road network can be abstracted as nodes and links to the network. The vehicles are the packets of the network communication and the signal lamp is the switching control time of network nodes. The message is transmitted to the next node, and the delay function is optimized by using the message function in turn to realize the distributed adaptive traffic signal control.

In this paper, the modeling of the method is realized and the simulation results prove that the algorithm based on different distributions has strong convergence and high efficiency. The more in-depth work is that the more phase information, the starting delay of the vehicle, the accumu-
lation of vehicle queue and other information can be added to the system model and the algorithm needs to be further improved.

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