Analysis and Correction of Ill-Conditioned Model in Multivariable Model Predictive Control

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Abstract. The ill-conditioned model is a common problem in model predictive control. The model ill-conditioned can lead to control performance declining obviously from steady-state model of process in this paper. The direction of output movement is relevant to whether the model is ill-conditioned by simulation and analysis. Model mismatch also leads to model ill-conditioned becoming more serious. The geometry tools and SVD in linear algebra are used to analyze the essential reason of ill-conditioned model, and an offline strategy is proposed which can solve the ill-conditioned model problem together with existing online strategies. Finally, the simulations are used to prove the conclusions which presented in this paper are correct.

Keywords: Model predictive control · Model ill-conditioned problem · Singular value decomposition (SVD) · Model identification · Model mismatch

1 Introduction

Model predictive control (MPC)is a kind of constrained、multivariable and model-based control method and was first applied to control the oil refining process [1]. At present, MPC has been widely applied to many industries, for example, petroleum, chemical industry, paper making, food processing, aviation and so on, which can significantly reduce the variance of the control process output and have the strong robustness [2]. The classic algorithm of MPC mainly includes the Dynamic Matrix Control (DMC) [3], the Model Algorithm Control (MAC) [4], and the Generalized Predictive Control (GPC) [5]. Currently, the most industrial MPC software is based on DMC, and to extend widely.

MPC is a model-based control, so the model has a great influence on the control effect. The controller model was usually obtained through system identification in practice, but the model of identification is ill-conditioned or has error. This article focuses on the control effect of the ill-conditioned MPC controller model. The intuitive performance of ill-conditioned model is the large condition number of the process model, has strong correlation between input and output to lead to difficult control of the process. The ill-conditioned of controller model can be divided into the following two cases: 1, There is strong correlation in the process itself [6], which is a pathologi-

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cal process physically, such as the distillation tower in the chemical process; 2, As the model of the multivariate process is obtained through system identification, the subunits of the model gained by fitting the input and output data may be sick.

The model ill-conditioned problem is more common in process control, and has great influence on the stability of the process [7]. The output set point of a certain direction movement probably brings about dramatic action of the controller, therefore it is necessary to eliminate or reduce the influence of the model ill-conditioned on the process.

There are several methods to eliminate model ill-conditioned and are introduced in following. For the square system(the number of inputs equal to outputs'), Grosdidier P [8] removed the outputs of linear correlation, and made them not participate in the calculation of control function in rolling optimization processes, which leaded to the output offset. Another solution was to use output range control strategy by relaxing the control requirements of some outputs, that is, set points were extend to a set range, which adapted to the process of low output demand and was widely applied in industry. In fact, this solution was same as Grosdidier's method. In light of the standard regularization theory, J.Marroquin [9] came up with a new random method to handle model ill-conditioned problem in numerical calculation. In addition, Honeywell's advanced control software adopted SVT (Singular Value Thresholding) method to eliminate the influence of the model ill-conditioned on the control effect, which avoided the inputs possibly leading to the instability by neglecting the input which the singular value in the corresponding direction smaller than the threshold which was set by controller conFiguration [10]. AspenTech's advanced control software DMCplus adopted IMS(Input Move Suppression) control strategy to solve the model ill-conditioned problem, which decreased the inputs action by directly increasing the values of diagonal elements in LS problem and decreasing the condition number. Thanks to the two-layer predictive controller in DMCplus [2, 11-12, 19], this strategy was useful. However, the methods mentioned above were all online solution strategies. Therefore, this paper put up with an offline strategy as a complementary method, it can quite well handle the model ill-conditioned problem by cooperating with the online strategies above.

Starting with the steady state gain matrix of the multivariable process, this paper will use geometry and linear algebra tools to analysis the reason why ill-conditioned model influenced the control effect, then proposed the method to improve the model, and used the simulation to verify the effectiveness of the given method at last.

2 Brief Introduction of Two-Layer Model Predictive Control

Currently， predictive control technology has developed to the fourth generation, the main feature is the two-layer model predictive control(TMPC). TMPC adds a steady state optimization (SSO) layer above dynamic control layer of traditional MPC. Literature [22] introduces a new generation state space controller of AspenTech, the structure of TMPC is shown in Fig.1.

Compared with the traditional MPC, TMPC increases steady-state optimization function modules including external targets, optimizing input parameters and output feedback items. Steady-state optimization has optimization function for economic performance which can be automatic optimization nearby the steady-state operating point and find the optimal process set value, this process is called the steady-state target calculation. Steady-state target calculation needs to use steady-state model of the process.

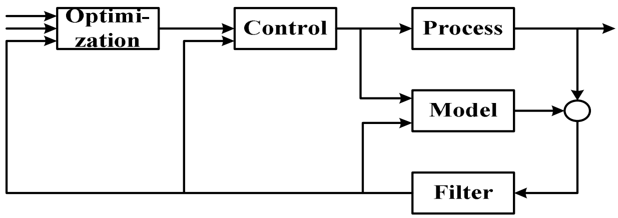

Fig. 1. Structure of two-layer model predictive control

Considered invariant open-loop stable linear system, its transfer function model of dynamic characteristics is

$$\boldsymbol{y}(s) = \boldsymbol{G}(s)\boldsymbol{u}(s) \tag{1}$$

and steady-state model is

$$\Delta \boldsymbol{y}_s = \boldsymbol{K} \Delta \boldsymbol{u}_s \tag{2}$$

where, $\boldsymbol{K}$ represent steady-state gain matrix. The constraint of inputs and outputs are

$$\begin{cases} \boldsymbol{u}_{LL} \le \boldsymbol{u}_s(k) \le \boldsymbol{u}_{HL} \\ \boldsymbol{y}_{LL} \le \boldsymbol{y}_s(k) \le \boldsymbol{y}_{HL} \end{cases} \quad k \ge 0 \tag{3}$$

The constraint conditions involve hard constraints and soft constraints, the soft constrains can relax in engineering permitted range when could not find a feasible solution, but the hard constraints can not relax.

Considered the inputs and outputs constraint conditions of controlled process, the steady-state optimization of TMPC is linear programming (LP) or quadratic programming (QP) problem. This paper solves LP problem as example, the problem descriptions are

$$\begin{cases} \min\limits_{\Delta \boldsymbol{u}_s(k)} \boldsymbol{J} = \boldsymbol{c}^T \Delta \boldsymbol{u}_s, \\ \text{s.t. } \Delta \boldsymbol{y}_s(s) = \boldsymbol{K} \Delta \boldsymbol{u}_s(k) + \boldsymbol{e}_k, \\ \boldsymbol{u}_{LL} \le \boldsymbol{u}_s(k-1) + \Delta \boldsymbol{u}_s(k) \le \boldsymbol{u}_{HL} \\ \boldsymbol{y}_{LL} \le \boldsymbol{y}_s(k-1) + \Delta \boldsymbol{y}_s(k) \le \boldsymbol{y}_{HL} \end{cases} \tag{4}$$

where, $\boldsymbol{e}_k$ can be concluded by dynamic predictive errors of process,

$$\boldsymbol{e}_k = \boldsymbol{y}_k - \tilde{\boldsymbol{y}}(k|k-1) \tag{5}$$

where, $\boldsymbol{y}_k$ represent output measured values of k time, $\tilde{\boldsymbol{y}}(k|k-1)$ represent output predictive values of k time in $k-1$ time.

In order to solve steady-state optimization problem, analysis of feasibility is needed. If the viable solutions are nonexistent, feasibility determination and soft constraints adjustment will be done and constraint boundary of output variables will be relaxed appropriately according to the priority order.

Under feasible of steady-state optimization condition, the final optimal control input can be calculated by the steady-state target calculation, and the optimal process output values can be obtained further more. Optimal input and output values as the set values are transferred to lower layer of MPC, and MPC will track changing of set point. If set point is given manually, the steady-state optimization represents the shortest distance in the least squares between the initial steady-state working point and mathematical expectation.

The lower layer of steady-state optimal layer is dynamic control layer which uses DMC algorithm. the double structure prediction control needs to be The penalty term of steady-state target values of the control inputs need to add to the objective function of DMC in the framework of TMPC, and the formation of the following forms of control objective function

$$\boldsymbol{J}(k)=\left\|\boldsymbol{w}(k)-\tilde{\boldsymbol{y}}_{PM}(k)\right\|_{\boldsymbol{Q}}^{2}+\left\|\boldsymbol{u}(k)-\boldsymbol{u}_{ss}(k)\right\|_{\boldsymbol{V}}^{2}\left\|\Delta\boldsymbol{u}_{M}(k)\right\|_{\boldsymbol{R}}^{2} \tag{6}$$

There are not obvious differences in model prediction and feedback collection between TMPC and traditional MPC.

3 Affect of Ill-Conditioned of Steady-State Gain Model for Control Performance

The function of prediction model is based on the established mathematical model to predict outputs, including steady-state prediction and dynamic prediction. Steady-state optimization is a process of calculation of the optimal operating point by steady-state target calculation, the steady-state prediction needs steady-state model. By model identification, a step response coefficient of inputs and outputs model will be built. At present, most of the large-scale production process with continuous, stable production characteristics, identified model can characterize the input-output relationship of processes more accurately, so that the steady-state calculation has good application effect. Meanwhile, the steady-state analysis is relative simple, and can generally reflect the dynamic performance and controllability of the system. Therefore, steady-state gain matrix which including steady-state gain coefficient information of processes is to be an example,

$$\boldsymbol{K}=\begin{bmatrix} k_{11} & \cdots & k_{1m} \\ \vdots & \ddots & \vdots \\ k_{p1} & \cdots & k_{pm} \end{bmatrix}$$

where, $\boldsymbol{K}$ represent steady-state gain matrix, m, p represent the number of inputs and outputs respectively. This paper proposes a strategy that tests the singularity of steady-state models and its subunits (2×2 matrix) before identified models are used as controller model [13-15]. If the models are ill-conditioned, the models will be modified and updated to avoid the problems of ill-conditioned model.

At first, the influence of ill-conditioned model to control is analyzed. It is assumed that a subunit model of process was as following. The heavy oil separation column is used to be object of study that is common in chemical field, its identification, process model is written as following by identifying.

$$\boldsymbol{G}(s)=\begin{bmatrix} \frac{-12.62}{50s+1}e^{-27s} & \frac{9.84}{60s+1}e^{-28s} & \frac{5.88}{50s+1}e^{-27s} \\ \frac{9.84}{(50s+1)}e^{-18s} & \frac{-6.88}{(60s+1)}e^{-14s} & \frac{6.9}{(40s+1)}e^{-15s} \\ \frac{4.38}{33s+1}e^{-20s} & \frac{4.42}{44s+1}e^{-22s} & \frac{7.20}{19s+1}e^{-24s} \end{bmatrix}$$

Then, sub models of process model are used to be object of study,

$$\boldsymbol{G}_{2\times2}=\begin{bmatrix} \frac{-12.62}{50s+1}e^{-27} & \frac{9.84}{60s+1}e^{-28} \\ \frac{9.84}{50s+1}e^{-18} & \frac{-6.88}{60s+1}e^{-14} \end{bmatrix}$$

and the steady-state gain matrixes are

$$\boldsymbol{K}=\begin{bmatrix} -12.62 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}$$

The condition numbers can be calculated by steady-state gain matrixes, and the equation is written as following.

$$cond(\boldsymbol{K})=\frac{\sigma_{\max}}{\sigma_{\min}} \tag{7}$$

Where, σ is singular value of matrix, $\sigma_{\max}$ is maximum singular value of matrix, and $\sigma_{\min}$ is minimum singular value of matrix. The condition number of this matrix is 40. If the condition number is big, this model will be considered as ill-conditioned by knowledge of linear algebra. In order to analyze the affect of control performance when the controller models are ill-conditioned, two different sets of data are chosen to simulate.

This paper used standard DMC algorithm to simulate. The set point 1 is $[-1.6,-1.2]^{\mathrm{T}}$, set point 2 is $[-1.6,1.2]^{\mathrm{T}}$, and they will use in all of simulation. The weight coefficient matrix Q, R are unit matrixes, the input and output curves are shown in Fig. 2 as below.

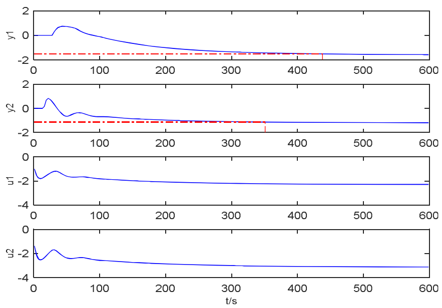

Fig. 2. Input and output curves of set point 1

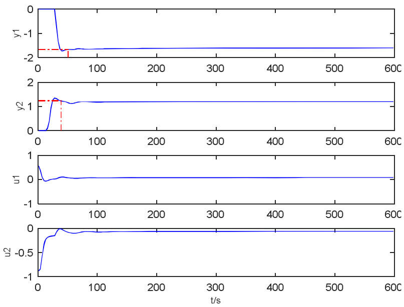

Fig. 3. Input and output curves of set point 2

It is seen from Fig.2 that the outputs are finally stabile in set point 1 $[-1.6,-1.2]^{\mathrm{T}}$, where point of intersection of dotted line represents the two output variables stabilize the range of $\pm 5\%$ error. In other words, setting time t_s are 438 seconds and 351 second respective, which indicates the controller is very slow response for process.

The weight coefficient matrix Q, R are still unit matrixes, and use set point 2 to simulate, the input and output curves are shown in Fig.3.

By the simulation shows that the outputs are finally stabile in $[-1.6,1.2]^{\mathrm{T}}$, and the process reaches the set point 2 quickly, meanwhile setting time t_s are only 51 seconds and 39 seconds. There are two very different controller's action and control performance for the same ill-conditioned model by two different set points. In particular, the response time have a lot of difference. The reason of this case is only on account of changing of moving direction of the outputs, and the detailed explanation will be introduced next.

The model mismatch is very common in practice. There are many reasons for the model mismatch such as the accuracy of the identification results, characteristic's changing in control processes [16, 20, 21], and lead to controller performance decline. Therefore, it is necessary to analyze model ill-conditioned problem under model mismatch condition.

The actual process model assumes the existence model mismatch of a $\pm 20\%$ range for k_{11}, the remaining parameters does not exist mismatch, $\pm 20\%$ mismatch are defined as mismatch mode 1 and mode 2 mismatch, the model gain matrixes are written as following.

$$\boldsymbol{K} = \begin{bmatrix} -10.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}, \boldsymbol{K}_1 = \begin{bmatrix} -15.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}$$

where, mismatch mode 1 represents forward mismatch, which means the condition number of matrix decrease after model mismatched. The final condition number of matrix $\boldsymbol{K}$ is 14.9, which smaller than 40 that has been condition number before model mismatched. The mode 2 is opposite to mode 1, the final condition number of matrix $\boldsymbol{K}_1$ is 14.9

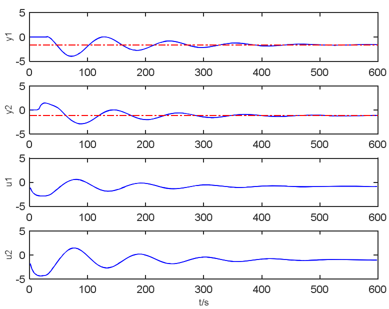

Fig. 4. Input and output curves of set point 1 under mismatch mode 1

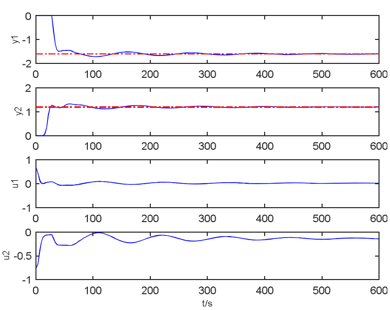

Fig. 5. Input and output curves of set point 2 under mismatch mode 1

In order to analyze model ill-conditioned problem under model mismatch condition, use set point 1 and set point 2 to simulate, Q, R are still unit matrixes, and, the input and output curves are shown in Fig.4 and Fig.5 as above.

From Fig. 4 and Fig.5, it is seen that the process has a turbulence which is still related to the direction of outputs movement. Setting times before model mismatched are larger than after model mismatched for two sets of set point. Meanwhile, the potential static state input point is $u_s = [-2.2816, -3.0888]^{\mathrm{T}}$ which corresponding to the set point $[-1.6, -1.2]^{\mathrm{T}}$, the output value of actual process steady-state model is

$$\boldsymbol{y}_{process} = \boldsymbol{K}_{process\_gain} \times \boldsymbol{u}_s = [-7.3496, -1.2]^{\mathrm{T}}$$

It is seen that 20% model mismatch in k_{11} is amplified 459.35% in output, which is caused by a large movement of control input. That is to say that the range of the controller action amplifies the mismatch of the model, which causes the controller become very sensitive for model mismatch. And then, changing the direction of model mismatch and use mismatch model 2, set point 1 and set point 2 to simulate, Q, R are still unit matrixes, and the input and output curves are shown in Fig.6 and Fig.7 as below.

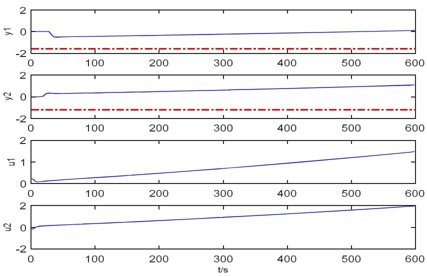

Fig. 6. Input and output curves of set point 1 2 under mismatch mode 2.

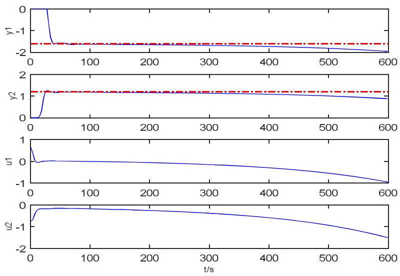

Fig. 7. Input and output curves of set point under mismatch mode 2.

It is seen from Fig.6 and Fig.7 that two groups of output do not reach set point under model mismatch, and model mismatch leads to controller action more drastic and causes the curves of input and output detergency. When movement direction of set point is changing, the condition number of matrix will change, and model ill-conditioned level will change, and then the diverging direction of curve will change too. All these phenomena show that the changing of output movement direction will affect control performance under model ill-conditioned.

So far, two conclusions can be obtained. (1) The ill-conditioned model can lead to extreme action of the controller, which can bring about poor control performance. (2) The changing of output movement direction will affect control performance under model ill-conditioned.

4 Analysis of Poor Control Performance Under Model Ill-Conditioned

For ill-conditioned model, there is a relationship between whether the controller produces a large action and movement direction of output. For the 2-input 2-output square system (sub process model is a steady-state process), when set point is given, the potential steady-state input point has been determined [22].To solve input set point actual is a process for solving linear equations group, and in geometrically speaking, the solution is the intersection point of the two lines.

For two groups of set point are given in section II, the geometrically solution is shown as following. For output initial state $[0,0]^T$, the geometrical show is shown in Fig.8(a) as below.

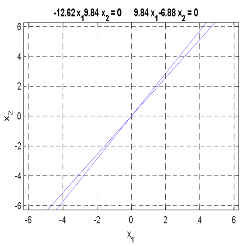

Fig. 8a.

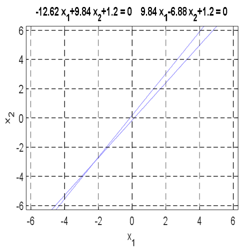

Fig. 8b.

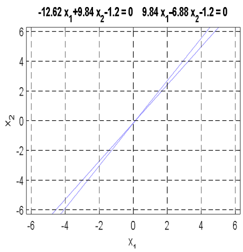

Fig.8c.

Fig. 8. are geometrical show of output initial state, set point 1 and set point 2.

It is seen that the intersection point of two straight lines is the origin point, it means output initial state which corresponding to input initial state is $[0,0]^T$.

For set point 1, the geometrical show is shown in Fig. 8(b).

It is seen that the distance from the intersection point of two lines to origin point is much further than initial state, which means the amplitude of the input initial state will be larger, so controller will generate large motion.

For set point $[-1.6,1.2]^T$, the geometrical show is shown in Fig. 8(c).

It is seen that the intersection point of the two lines is very close to origin point, which means the amplitude of the input initial state is small, so controller will generate little motion.

By the simulation results, a conclusion will be gotten that the movement direction of output will influence the control performance for ill-conditioned model. Because of the changing of output movement direction on geometric changes the distance between input steady state point and the original point, which changes the initial amplitude of input, and lead to the controller action range changed.

It is intuitive to judge whether a dynamic model is ill-conditioned just based on the condition number of model, the condition number is bigger, the model ill-conditioned will become more serious, the control performance will be poorer. This conclusion can be proved through simulation of two groups of different mismatch model in section 3. For mismatch $k_{11} = -10.1$, the condition number of steady-state model will reduce by calculation. Meanwhile, for the mismatch $k_{11} = -15.1$, the number of condition increases, so the former ill-condition is smaller and the control performance is also better. For most of the processes, the steady-state model can represent dynamic nature of the processes, so the ill-conditioned problem of dynamic model can be converted into steady state model to analyze, and the calculation method of condition number has been given above. The linear algebra tools singular value decomposition (SVD) will be used to analyze the model ill-conditioned problem further next.

For the description of steady-state relationship shown in formula (2), the singular value decomposition of steady-state matrix is shown in formula (8) as following.

$$\boldsymbol{K} = \boldsymbol{U}\sum\boldsymbol{V}^T = \sum_{i=1}^{n}\sigma_i u_i \upsilon_i^{\mathrm{T}} \tag{8}$$

If $\boldsymbol{K} \in \boldsymbol{R}^{p*m}$, $\boldsymbol{U} = \begin{bmatrix} u_1 & \cdots & u_p \end{bmatrix}, \boldsymbol{U} \in \boldsymbol{R}^{p*p}$, $\Sigma = diag[\sigma_1 ... \sigma_p], \Sigma \in \boldsymbol{R}^{p*m}$, $\boldsymbol{V} = \begin{bmatrix} v_1 & \cdots & v_m \end{bmatrix}, \boldsymbol{V} \in \boldsymbol{R}^{m*m}$

The formula (9) can be gotten by formula (8),

$$\boldsymbol{KV} = \boldsymbol{U}\sum \tag{9}$$

where, $\boldsymbol{U}$ and $\boldsymbol{V}$ are two group of orthonormal basis vectors. A matrix represents a linear transformation from the view of linear algebra, and the significance of the linear transformation has two equivalent explanations as following. (1) Transforming a $x \in \boldsymbol{R}^m$ vector to $\boldsymbol{R}^p$ space. (2) The linear transformation provides a representation with a same vector in two different spaces. For steady-state matrix $\boldsymbol{K}$, singular value decomposition just provides a kind of geometric form. For every 2×2 matrix, singular value decomposition can always find a space to another space conversion and corresponding matrix transformation. The steady-state model of the process is $\Delta\boldsymbol{y}_s = \boldsymbol{K}\Delta\boldsymbol{u}_s$, where, $\boldsymbol{V}$ is a set of orthonormal base of $\Delta\boldsymbol{u}_s$, $\boldsymbol{U}$ is a set of orthonormal base of $\Delta\boldsymbol{y}_s$,can be used as a set of orthonormal base of, singular matrix Σ represents the degree of stretch of steady-state gain matrix $\boldsymbol{K}$. Specifically, u_i and v_i are a couple of one-to-one correspondence orthonormal base vector in the formula (8). The singular value describes stretch multiple in different directions, in other word, there are σ_i times stretch from direction of u_i to v_i. From the perspective of the steady state, the nature of control is steady state matrix inversion, the matrix is more close to singular matrix, the determinant value is more small, its inverse will be very big, that means the controller action might be very big, namely

$$\Delta u_s(k) = [\boldsymbol{U}\sum\boldsymbol{V}^T]^{-1} \times \Delta y_s(k) \tag{10}$$

For the model gain matrix of controller in this paper, its singular value decomposition results are written as following.

$$\boldsymbol{U} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}, \Sigma = \begin{bmatrix} 20 & \\ & 0.5 \end{bmatrix}, \boldsymbol{V} = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}$$

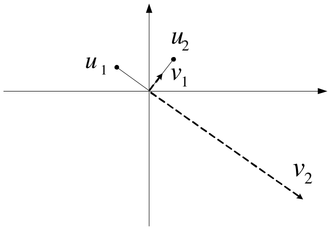

Fig. 9. Coordinate system consisting of base vectors of singular value decomposition

Where, the coordinate system which vertices are original point, are composed of base vector u_1, u_2.The imaginary lines coordinate system that vertices are allows, are composed of base vector v_1, v_2.The stretching amplitude σ_1 , σ_2 in the directions of u_1, u_2 are 0.5 times and 20 times respective, which means if movement output is existent in the direction of v_2, the direction of u_2 need will need twice the input amplitude. Otherwise, the direction of u_2 only will need 0.05 times the input amplitude. This example uses set point 1 and set point 2 which are used to simulate in previous section, the set point 1 $[-1.6, -1.2]^T$ is $(-0.56, -2)$ in the coordinate of which composed of base vector v_1, v_2, and set point 2 $[-1.6, 1.2]^T$ is $(-2, 0)$ .Therefore, if the set point 1 is to be set point, the controller action must be more intense, and this analysis is also proved in the previous simulation.

5 Strategy of Modifying Ill-Conditioned Model

By simulation and analysis in previous section, the fundamental reason of ill-conditioned model is that the last singular value of model gain matrix is small and the solution are shown as following. (1) Make output not affected by direction movement of the model ill-condition influence, it means this strategy ensured magnitude of outputs in the minimum singular value corresponding to the projection of vector direction smaller, online strategy mentioned in introduction is used to this idea in essential. (2) Drastically reduce the model ill-condition by modifying model. The following strategy uses the second method, it is an off-line strategy. After finished identification, the identified models are modified immediately, and qualified modified models will be conFigured as a controller model.

The ill-conditioned model gain matrixes compared to ill-conditioned problem, the errors of model gain coefficient seem unimportant, and it is possible that identified models are different form actual process models. Therefore, a model modified strategy is presented, which means increasing the minimum singular value and reducing model ill-condition by modifying model gain matrixes [17, 18]. The method is shown as following.

1. After models are identified, the authors will examine the subunits of original model ill-condition by minimum scale, and inspection index is condition number.

2. Define the error tolerance factor α, it represents each model gain coefficient is allowed to change the amplitude size. The expression is $|\Delta k_{ij}| \le |k_{ij}| \times \alpha$, it is noted that α can a s allowed factors for special gain, and also be a unified factor for every gain.

3. The initial value of minimum singular value $\sigma_{\min}$ is set $\sigma_{\max}$, and uses dichotomy to doing iterative search by golden section until the maximum allowable error range $\sigma_{\min}$ is found.

4. Update the models.

The error tolerance factor is set 0.1, the modified model was shown as following by application of above method.

$$\boldsymbol{K}' = \begin{bmatrix} -12.3783 & 10.1623 \\ 10.1623 & -6.4503 \end{bmatrix}, \quad \Delta\boldsymbol{K} = \boldsymbol{K}' - \boldsymbol{K} = \begin{bmatrix} 0.2417 & 0.3223 \\ 0.3223 & 0.4297 \end{bmatrix} \le 0.1 \times \boldsymbol{K}$$

$\boldsymbol{K}'$ are configured to controller model, and actual model of process are $\pm 20\%$ mismatched model.

$$\boldsymbol{K} = \begin{bmatrix} -10.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}, \quad \boldsymbol{K}_1 = \begin{bmatrix} -15.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}$$

Using set point 1 and set point 2 to simulate under model mismatch mode 1 and mode 2, Q, R are still unit matrixes, the control effect are shown as Fig.10 as below.

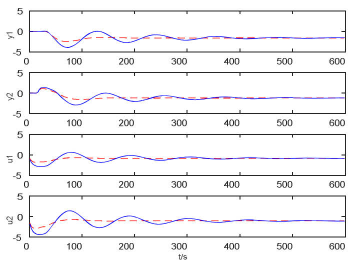

Fig. 10a.

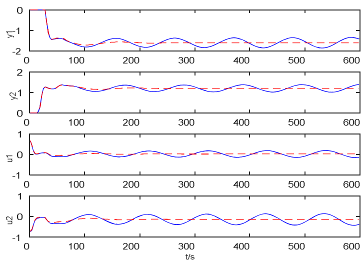

Fig. 10b.

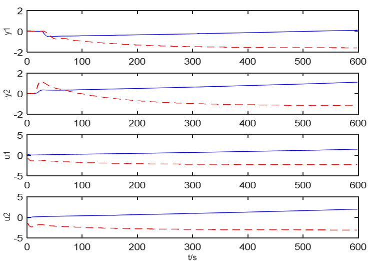

Fig. 10c.

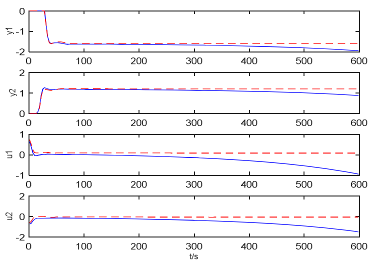

Fig. 10d.

Fig. 10 are Control effect comparison between modified model 1, model 2 and unmodified model under mismatch mode 1, mode 2 and set point 1, set point 2.

In Fig.10, the solid lines represent input and output curves of unmodified models, and the dashed lines represent input and output curves of modified models. It is obviously seen that process shock amplitude and shock time are reduced significantly, while quickly reach the set point, and correct adverse effects for model mismatch Therefore, this strategy is feasible.

6 Conclusions

The ill-conditioned model will lead to control performance declining obviously, and changing of the direction of outputs movement will affect control performance too. Model mismatch problem is a common problem for control, but ill-conditioned model in case of model mismatch will have a greater impact for control performance. In this paper, geometry and linear algebra tools are used to analyze fundamental reason of ill-conditioned, and presents an offline modified strategy to solve model ill-conditioned problem well. Finally, the methods and solutions presented in this paper are proved by simulation.

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