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# The consensus of nonlinear multi-agent system with switching topologies and communication failure

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**Abstract**. In this paper, for nonlinear multi-agent systems under switching topologies with communication failure, the leader-following consensus problem is studied. Firstly, several switching topologies including a null graph are considered. Secondly, the novel consensus protocols for followers are given considering the communication failure (null graph). Then the convergence analysis is presented and some sufficient conditions are derived for the consensus tracking in a leader-followers multi-agent network. The analysis tools developed in this paper are based on matrix theory and control theory. Finally, the simulations show that by the proposed theorem, the consensus can be achieved and the control gain can influence the speed of the consensus.

Keywords: Multi-agent systems, communication failure, consensus, switching topologies

## 1. Introduction

In recent years, the control of multi-agent systems has received compelling attentions and emerged as a challenging research field [1–16]. There are many open problems in consensus of multi-agent system, such as the nonlinear system, the data missing, the time delay and even the fault diagnosis [17–19]. Many control method can be used in this field such as adaptive control, sliding control, data driven control and so on. Especially when the priori physical and mathematical knowledge of multi-agent systems cannot be gotten sufficiently, the data driven schemes could be applied [20, 21].

Nonlinear systems are very common which attracts more researchers' attention. In [6], a second-order consensus problem for multi-agent systems is considered. Each agent adjusts their own position and speed by local information of their neighbors. Some sufficient conditions are derived for reaching second-order consensus in multi-agent system. In [7], the distributed containment control problem for multiple nonlinear systems with multiple dynamic leaders is studied. The author makes the states of the followers convergent to the convex set which is composed by leaders. In [8], the consensus tracking problem with a switching directed topology is presented. It is shown that consensus tracking can be achieved if there always exists a directed path from the leader to each follower. In [9], a pinning control algorithm is proposed to achieve leader-following consensus for the network with nonlinear second-order dynamics. In particularly, the network can neither be strongly connected nor have a directed spanning tree. The authors analyze the typology of the network and the leader should be chosen as the node.

Switching topology means that the multi-agent communication topology transfers from one mode to others.

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In the past studies, the problem could be simplified by assuming that the topology is fixed. But in fact, the communication topology is time-varying. For example, when the disturbance happens and the communication distance changes, some of the links would disconnect. Therefore, the study of the agent with the switching topology is more practical. In [10], flocking algorithms for a network of a second-order agent with bounded control inputs and switching topology is proposed and analyzed. In [11], for the second-order multi-agent systems with dynamically random switching directed typologies, the local consensus problem is considered. The authors present two cases with and without time-delay respectively. It proves that the agent system can achieve consensus if there exists one directed spanning tree in the network with suitable parameters. In fact, the controller of the agent system may fail due to the communication failure. There are several studies consider this problem. In [12], the nonlinear agent consensus problem is studied considering the disturbance and control failure.

Motivated by the above discussions, the main purpose of this paper is to study the leader-following consensus problem for nonlinear multi-agent systems. Meanwhile, the switching topologies are also considered. And then the consensus protocols for followers are given. By analyzing the system, some sufficient conditions are obtained. The contributions of this paper are threefold. Firstly, we propose a leader-following flocking algorithm and the communication failure is described as the null graph. Secondly, we assume a switching topology rather than fixed network topology. Finally, the third contribution concerns with the nonlinear intrinsic dynamics. As far as we know, no results consider all the three factors for the consensus problem of multi-agent systems.

The rest of this paper is organized as follows: Section 2 addresses the dynamic model of multi-agent system with a leader and some necessary definitions and lemmas are given. In Section 3, the conditions for the multi-agent systems with switching topologies to achieve consensus are given. Then in Section 4, we present simulation results which support our theorem. Finally, the conclusions are drawn in Section 5.

## 2. Problem description

Consider a group with $N$+1 agents, where an agent indexed by 0 is assigned as the leader and the agents indexed by $1, 2, \ldots, N$, are referred to as followers. The dynamics of the leader are given by

$$\dot{x}_0(t) = f(x_0(t), t) \tag{1}$$

where $x_0(t) \in R^n$ indicates the states of the leader, $f(\cdot) \in R^n$ is the non-linear function of the leader. Furthermore, the dynamics of the $i$-th follower are described as

$$\dot{x}_i(t) = f(x_i(t), t) + u_i(t) \; i = 1, 2, \ldots, N \tag{2}$$

where $x_i(t) \in R^n$ indicates the states of the $i$-th follower, $f(\cdot) \in R^n$ describes the intrinsic non-linear dynamics of the $i$-th follower. $u_i(t) \in R^n$ is the control input to be designed.

The control goal here is to design a distributed leader following algorithm based only on the local relative information such that the states of the followers asymptotically reach the leader's states in the sense $\lim_{t\to\infty} \|x_i(t) - x_0(t)\| = 0, \; i = 1, \; 2, \; 3, \ldots, \; N$. It is further assumed that $g^s$ is a null graph, which can be taken as the communications failure between all gents. We assumed that the null graph will be activated many times as the multi-agent system evolves. Then a switching signal $\sigma(t) : [0, \; +\infty) \to \{1, \; 2, \ldots, \; s\}$ is introduced to describe the evolution of the underlying topologies. Let $g^{\sigma(t)}$ be the topology of system (2) at time $t \geq 0$. Suppose that there exits an infinite sequence of uniformly bounded non-overlapping time intervals $[t_k, \; t_{k+1}), \; k \in N$ and $t_1 = 0, \; w_1 > t_{k+1} - t_k \geq w_0, \; w_1 > w_0 > 0$. For each $k \in N$, there exits an finite sequence of time points $t_k = t_k^1 < t_k^2 < \cdots < t_k^{h_k-1} < t_k^{h_k} = t_{k+1}$ such that the underlying topology is time invariant for all $t \in [t_k^i, \; t_k^{i+1}), \; i = 1, 2, \ldots, \; h_k - 1$. Furthermore, it is further assumed that $\sigma(t) \in \{1, \; 2, \ldots, \; s-1\}$ for $t \in [t_k, \; t_k^{h_k-1})$, and $\sigma(t) = s$ for $t \in [t_k^{h_k-1}, \; t_{k+1})$. Based on the above analysis, one knows that the multiple agents may only exchange information with the neighbors during $t \in [t_k, \; t_k^{h_k-1}), \; k \in N$ due to existence of the null graph, which is shown in Fig. 1.

To achieve leader-following consensus, the following distributed tracking protocol is proposed for each follower.

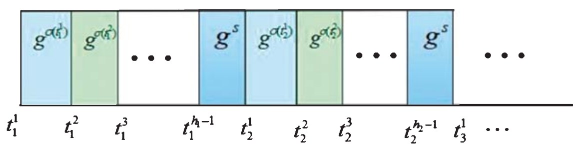


Fig. 1. The diagram of the switching topologies.

$$u_i(t) = \begin{cases} -\alpha \left[ \sum_{j=1}^{N} a_{ij}^{\sigma(t)}(x_i(t) - x_j(t)) + b_i^{\sigma(t)}(t)(x_i(t) - x_0(t)) \right], \ t \in \left[ t_k, t_k^{h_k - 1} \right) \\ 0, \qquad t \in \left[ t_k^{h_k-1}, t_{k+1} \right), k \in N \end{cases} \tag{3}$$

where $\alpha$ is the control gain, $a_{ij}$ is the element of the adjacency matrix.

We define that $e_i(t) = x_i(t) - x_0(t), i = 1, \ldots, N$. Then we can get $e(t) = \mathbf{0}$, if $x_1(t) = x_2(t) = \cdots = x_N(t) = x_0(t)$. The error systems can be described as

**Definition 2.** [6] A non singular real matrix is called $M$-matrix if and only if its non diagonal elements are not positive and each eigenvalue has positive real part.

$$\dot{e}_i(t) = \begin{cases} f(x_i(t), t) - f(x_0(t), t) - \alpha \left[ \sum_{j=1}^{N} l_{ij}^{\sigma(t)}(t) e_j(t) + b_i^{\sigma(t)} e_i(t) \right], \ t \in \left[ t_k, t_k^{h_k-1} \right) \\ f(x_i(t), t) - f(x_0(t), t), \quad t \in \left[ t_k^{h_k-1}, t_{k+1} \right] \end{cases} \tag{4}$$

Let $\eta(e_i(t)) = f(x_i(t), t) - f(x_0(t), t)$, $L^{\sigma(t)} = (l_{ij})^{\sigma(t)}$, $B^{\sigma(t)} = diag(b_1^{\sigma(t)}, \ldots, b_n^{\sigma(t)})$, $e(t) = (e_1^{\mathrm{T}}(t), e_2^{\mathrm{T}}(t), \ldots, e_N^{\mathrm{T}}(t))^{\mathrm{T}}$, $\tilde{\eta}(t) = (\eta^{\mathrm{T}}(e_1(t)), \eta^{\mathrm{T}}(e_2(t)) \cdots, \eta^{\mathrm{T}} (e_N(t))$.

Rewriting (4), one has

**Lemma 1.** [11] *For the proper matrix $A$, $B$, $C$ and $D$, their Kronecker products have following properties.*

$$\dot{e}(t) = \begin{cases} \tilde{\eta}(t) - \alpha \left[ L^{\sigma(t)}(t) + B^{\sigma(t)}(t) \right] \otimes I_n e(t) \quad t \in \left[ t_k, t_k^{h_k-1} \right) \\ \tilde{\eta}(t) \qquad t \in \left[ t_k^{h_k-1}, t_{k+1} \right) \end{cases} \tag{5}$$

Let $H_{\sigma(t)}(t) = L_{\sigma(t)}(t) + B_{\sigma(t)}(t)$, then formula (5) can be described as

$$\dot{e}(t) = \begin{cases} \tilde{\eta}(t) - \alpha (H^{\sigma(t)}(t) \otimes I_n) e(t) \quad t \in \left[ t_k, t_k^{h_k-1} \right) \\ \tilde{\eta}(t) \qquad t \in \left[ t_k^{h_k-1}, t_{k+1} \right) \end{cases} \tag{6}$$

Before moving forward, some definitions and lemmas are given as follows.

**Definition 1.** $A = (a_{ij}) \in R^{m \times n}$, $B = (b_{ij}) \in R^{p \times q}$, the following matrix

$$A \otimes B = \begin{bmatrix} a_{11}B & a_{12}B \cdots a_{1n} \ B \\ a_{21}B & a_{22}B \cdots a_{2n} \ B \\ \vdots & \vdots \ \ \cdots \vdots \\ a_{m1}B & a_{m2}B \cdots a_{mn} \ B \end{bmatrix} \in R^{mp \times nq} \tag{7}$$

is Kronecker product, and it can be written briefly as $A \otimes B = (a_{ij}B)_{m \times n}$, so $A \otimes B$ is a block matrix.

(1) $(A + B) \otimes C = A \otimes C + B \otimes C$;
(2) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$;
(3) $(A \otimes B)^{\mathrm{T}} = A^{\mathrm{T}} \otimes B^{\mathrm{T}}$.

**Lemma 2.** [6] *Suppose $A$ is a M-matrix and according to the properties of M-matrix, one gets that $A\Xi + \Xi A^{\mathrm{T}} > 0$,*
where $\Xi = \mathrm{diag}\{\xi_1, \xi_2 \cdots \xi_n\}$, $\xi = (\xi_1, \xi_2 \cdots \xi_n)^{\mathrm{T}} = A^{-1}\mathbf{1}_n$.

Obviously, one has $A^{\mathrm{T}}P + PA > 0$, where $P = \Xi^{-1}$.

**Lemma 3.** [22] *For the leader-follower multi-agent systems, we let $H = L + B$, then $H$ is* a $M$-matrix *if and only if the leader is globally reachable.*

**Lemma 4.** [23] For the proper matrix $X$, $Y$, and a positive definite matrix $Q$, the following inequality is arrived.

$$X^{\mathrm{T}}Y + Y^{\mathrm{T}}X \leq X^{\mathrm{T}}QX + Y^{\mathrm{T}}Q^{-1}Y \tag{8}$$

From Lemma 2 and Lemma 3, we can get Corollary 1.

**Corollary 1.** For the multi-agent systems with switching topologies, and in the possible underlying topologies, the leader is globally reachable, then $H^{\sigma(t)}$ is $M$-matrix. Therefore, there exists $\xi^{\sigma(t)} = \left\{\xi_1^{\sigma(t)}, \xi_2^{\sigma(t)}, \ldots, \xi_N^{\sigma(t)}\right\}$ satisfying

$H^{\sigma(t)}\xi^{\sigma(t)} = \mathbf{1}_N$ and $\Xi^{\sigma(t)}H^{\sigma(t)} + \left(H^{\sigma(t)}\right)^{\mathrm{T}}\Xi^{\sigma(t)} > 0$,

where $\Xi^{\sigma(t)} = diag\{\, 1\big/\xi_1^{\sigma(t)}, 1\big/\xi_2^{\sigma(t)}, \cdots 1\big/\xi_N^{\sigma(t)}\}$ .

**Assumption 1.** [8] For any $i \in N$, $f_i(\cdot)$ satisfies Lipschitz conditions:

$$\|f(x(t)) - f(y(t))\| \le \rho \|x(t) - y(t)\| \tag{9}$$

**Assumption 2.** [6] For $i \in \{1, 2, \ldots, s-1\}$, the leader is globally reachable for every underlying topology $g^i$.

## 3. Main results

**Theorem 1.** *For the multi-agent systems with s topologies, every $g^i$ for $i = 1, 2, \ldots, s-1$ satisfying the assumption 2 and $g^s$ is a null graph. Then, the leader-following consensus problem of multi-agent system can be solved by the protocol (3) if for given $\varepsilon$, $\rho$ and proper $\alpha$ the following conditions hold*:

$$\varepsilon^{-1} + \varepsilon\rho^2 - \alpha\lambda_0\xi_0 < 0 \tag{10}$$

$$\beta u_k - \gamma v_k - (h_k - 1)\ln r_0 > 0, k = 1, 2\cdots \tag{11}$$

where $\beta = -\varepsilon^{-1} - \varepsilon\rho^2 + \alpha\lambda_0\xi_0 > 0$, $\gamma = \varepsilon^{-1} + \varepsilon\rho^2 > 0$, $u_k = t_k^{h_k-1} - t_k$, $v_k = t_k^{h_k} - t_k^{h_k-1}$, $r_0 = \xi_1\big/\xi_0$, $\lambda_0 = \min_{i=1,2\cdots s-1}\lambda_{\min}\left(\Xi^i H^i + (H^i)^{\mathrm{T}}\Xi^i\right)$, $\xi_0 = \min_{i,j}\xi_j^i, i \in \{1, \ldots, s-1\}$, $j = 1, 2, \ldots, N$, $\xi_1 = \max_{i,j}\xi_j^i, i \in \{1, \ldots, s-1\}$ $j = 1, 2, \ldots, N$.

**Proof.** Construct the following multiple Lyapunov function candidate for the switched systems (6) as

$$V(t) = \begin{cases} e^{\mathrm{T}}(t)\Xi^{\sigma(t)} \otimes I_n e(t), & t \in \left[t_k, t_k^{h_k-1}\right) \\ e^{\mathrm{T}}(t)e(t), & t \in \left[t_k^{h_k-1}, t_{k+1}\right) \end{cases} \tag{12}$$

For $t \in \left[t_1, t_1^2\right)$, taking the time derivative of $V(t)$, it induces

$$\begin{aligned} \dot{V}(t) &= e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)\dot{e}(t) + \dot{e}^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &= e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)\left(\tilde{\eta}(t) - \alpha(H^{\sigma(t)}(t) \otimes I_n)e(t)\right) \\ &\quad + \left(\tilde{\eta}(t) - \alpha(H^{\sigma(t)}(t) \otimes I_n)e(t)\right)^{\mathrm{T}}(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &= 2e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)\tilde{\eta}(t) - \alpha e^{\mathrm{T}}(t) \\ &\quad \left[\left(\Xi^{\sigma(t)}H^{\sigma(t)}(t) + (H^{\sigma(t)}(t))^{\mathrm{T}}\Xi^{\sigma(t)}\right) \right. \\ &\quad \left. \otimes I_n\right] e(t) \end{aligned} \tag{13}$$

According to lemma 4, we can get

$$\begin{aligned} 2e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)\,\tilde{\eta}(t) &\le \varepsilon^{-1}e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &\quad + \varepsilon\tilde{\eta}^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)\tilde{\eta}(t) \end{aligned} \tag{14}$$

Take formula (14) into (13), and from $\lambda_0 = \min_{i=1,2\cdots s-1}\lambda_{\min}\left(\Xi^i H^i + (H^i)^{\mathrm{T}}\Xi^i\right)$, $\xi_0 = \min_{i,j}\xi_j^i$, $i \in \{1, 2\cdots, s-1\}$, $j \in \{1, \ldots, N\}$, then we can get

$$\begin{aligned} \dot{V}(t) &\le \varepsilon^{-1}e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &\quad + \varepsilon\rho^2 e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) - \alpha\lambda_0 e^{\mathrm{T}}(t) \otimes I_n e(t) \\ &\le \varepsilon^{-1}e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) + \varepsilon\rho^2 e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &\quad - \alpha\lambda_0\xi_0 e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &= \left(\varepsilon^{-1} + \varepsilon\rho^2 - \alpha\lambda_0\xi_0\right)e^{\mathrm{T}}(t)(\Xi^{\sigma(t)} \otimes I_n)e(t) \\ &= \left(\varepsilon^{-1} + \varepsilon\rho^2 - \alpha\lambda_0\xi_0\right)V(t) \end{aligned} \tag{15}$$

Because $-\beta = \varepsilon^{-1} + \varepsilon\rho^2 - \alpha\lambda_0\xi_0$, then (15) can be written as

$$\dot{V}(t) \le -\beta V(t) \tag{16}$$

Note that the closed-loop multi-agent system (2) with tracking protocol (3) switches when $t = t_1^2$. It thus follows from the above analysis that

$$V(t_1^{2-}) < V(t_1)e^{-\beta(t_1^2 - t_1)} \tag{17}$$

According to (12), one gets that

$$V(t_1^2) \le r_0 V(t_1^{2-}) \tag{18}$$

Thus, it induces that

$$\begin{aligned} V(t_1^2) &< r_0 e^{-\beta(t_1^2 - t_1)}V(0) \\ &= e^{\left[-\beta(t_1^2 - t_1) + \ln r_0\right]}V(0) \end{aligned} \tag{19}$$

By recursion and (6), we get

$$V(t_1^{h_1-1}) < \xi_1 e^{\left[-\beta(t_1^{h_1-1}-t_1)+(h_1-2)\ln r_0\right]} V(0) \quad (20)$$

Because $\varepsilon^{-1}+\varepsilon\rho^2 > 0$ and $\gamma=\varepsilon^{-1}+\varepsilon\rho^2$, we can get

$$V(t_2^-) \le e^{\gamma(t_2-t_1^{h_1-1})} V(t_1^{h_1-1}) \quad (21)$$

Since the closed-loop multi-agent system (2) with protocol (3) switches when $t=t_2$, and $u_k = t_k^{h_k-1} - t_k$, $v_i = t_k^{h_k} - t_k^{h_k-1}$, one has the following conclusion: when $k=1$, we have

$$\begin{aligned} V(t_2) &\le (1/\xi_0) V(t_2^-) \\ &= (\xi_1/\xi_0) e^{[-\beta u_1+\gamma v_1+(h_1-2)\ln r_0]} V(0) \\ &= e^{[-\beta u_1+\gamma v_1+(h_1-1)\ln r_0]} V(0) \\ &= e^{-K_1} V(0) \end{aligned} \quad (22)$$

where $K_1 = \beta u_1 - \gamma v_1 - (h_1-1)\ln r_0$, $K_1 > 0$. Similar to the above analysis, one can conclude that, for any given $k \in N$, we have

$$V(t_{k+1}) \le e^{-\sum_{j=1}^{k} K_j} V(0) \quad (23)$$

where $K_j = \beta u_j - \gamma v_j - (h_j-1)\ln r_0 > 0$, $j = 1, 2 \cdots k$.

For the case that $t > t_2$, and based on the above analysis, one gets

$$\begin{aligned} V(t) &< e^{-\beta(t-t_k)+(h_k-2)\ln r_0} V(t_k) \\ &< e^{(h_k-2)\ln r_0} V(t_k) \\ &\le \varsigma_0 e^{-\frac{(k-1)K_0 t}{k\omega_1}} V(0) \end{aligned} \quad (24)$$

where $\varsigma_0 = e^{(h_k^{\max}-2)\ln r_0}$, $h_k^{\max} = \sup_{k\in N} h_k$, $K_0 = \inf_{j\in N} K_j$ and $\omega_1$ is a positive scalar such that $(t_{k+1} - t_k) \le \omega_1$, $k \in N$. Since $k > 2$, it thus follows from (24) that

$$V(t) < \varsigma_0 e^{-\frac{K_0 t}{2\omega_1}} V(0),\ t \in (t_k, t_k^{h_k-1}) \quad (25)$$

when $t = t_k^{h_k-1}$, we can get

$$V(t) < r_0 \varsigma_0 e^{-\frac{K_0 t}{2\omega_1}} V(0) \quad (26)$$

when $t \in \left(t_k^{h_k-1}, t_{k+1}\right]$, we get

$$V(t) \le e^{-\frac{K_0 t}{\omega_1}} V(0) \quad (27)$$

In conclusion, we can get that $V(t) \to 0$ when $t \to \infty$, so $e(t) \to 0$. Thus, the leader-following consensus problem is solved by protocol (3). This completes the proof.

## 4. Numerical simulation

In this section, a numerical example is provided to verify the theoretical analysis. In the simulations, we take the multi-agent systems with 5 agents for example, the non-linear function $f(\cdot)$ is selected as:

$$f(x_i(t)) = \begin{bmatrix} -x_{i1}(t) + x_{i2}(t) - l(x_{i1}(t)) \\ 0.1(x_{i1}(t) - x_{i2}(t) + x_{i3}(t)) \\ -1.8x_{i2}(t) \end{bmatrix}$$

where $l(x_{i1}(t)) = bx_{i1}(t) + 0.5(a-b)(|x_{i1}(t)+1| - |x_{i1}(t)-1|)$, $a = -4/3$, $b = -3/4$. We take the topologies case $a$, case $b$ and case $c$ as $g^1$, $g^2$, $g^3$ and it is obvious that $c$ is a null graph which describes the communication failure and the leader is global reachable in case $a$ and case $b$, shown in Fig. 2.

From Fig. 2, we get $\xi^1 = [2\ 3\ 1\ 3]$, $\xi^2 = [2\ 3\ 2\ 3]$, $\lambda_0 = 0.1826$, $\xi_0 = 1$ and

$$L_1 = [1, -1, 0, 0; -1, 1, 0, 0; 0, 0, 0, 0; 0, -1, -1, 2],$$

$$L_2 = [1, -1, 0, 0; -1, 1, 0, 0; 0, 0, 1, -1; 0, 0, -1, 1],$$

$$B_1 = B_2 = [1, 0, 0, 0; 0, 0, 0, 0; 0, 0, 1, 0; 0, 0, 0, 0].$$

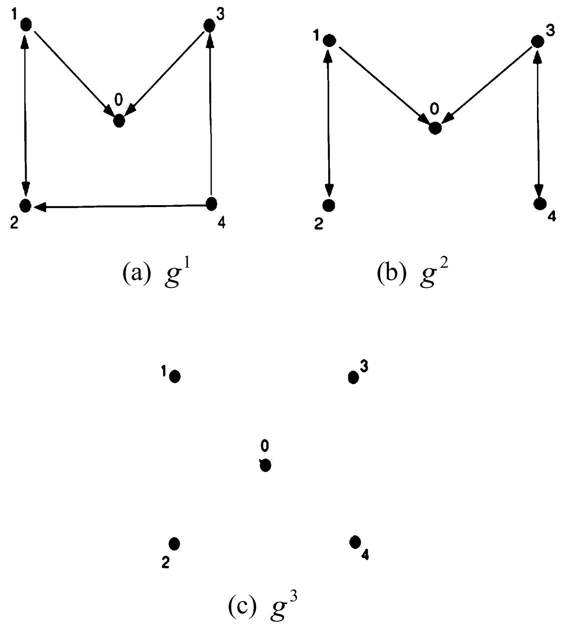
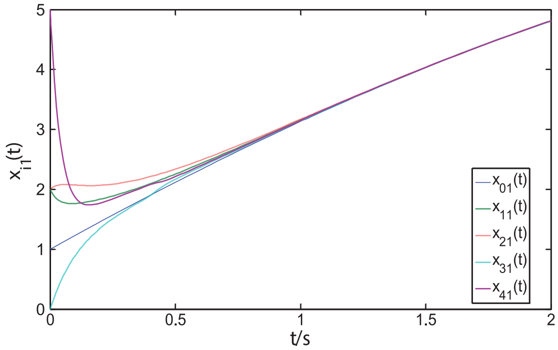
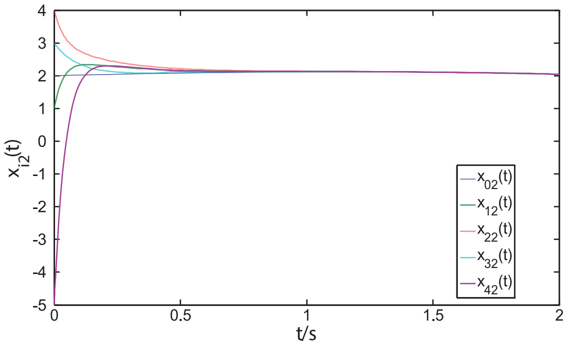


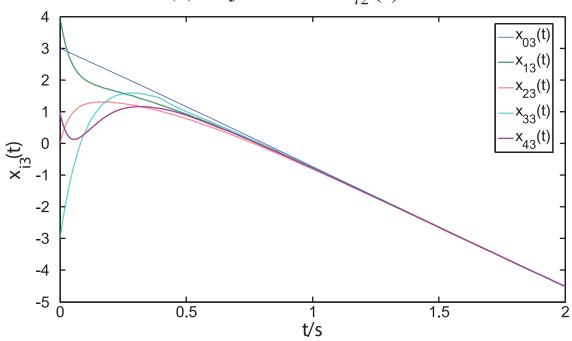

Fig. 2. Communication topologies $g^1$, $g^2$ and null graph of multi-agent.




(a) Trajectories of $x_{i1}(t)$

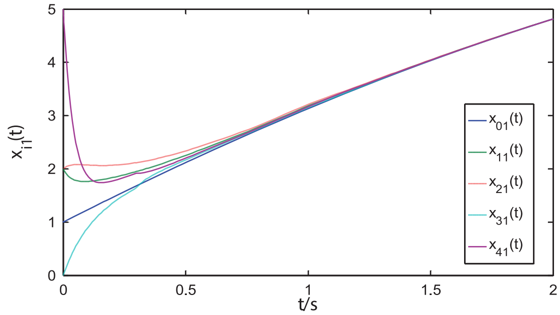



(b) Trajectories of $x_{i2}(t)$

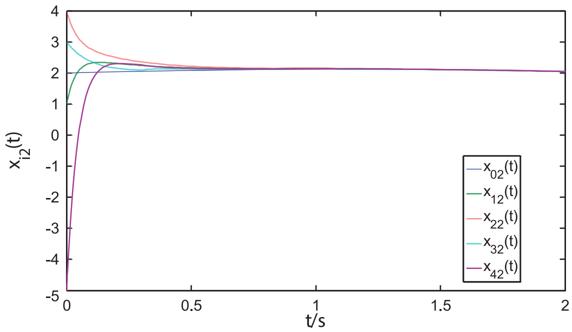



(c) Trajectories of $x_{i3}(t)$

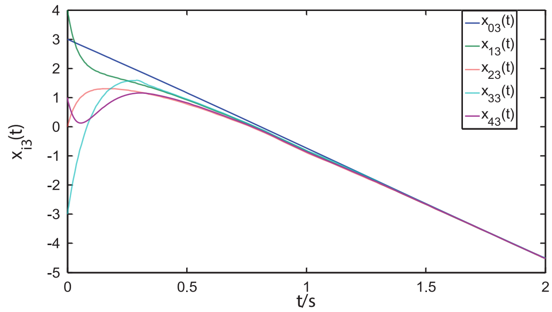
Fig. 3. The trajectories of each agent with no null graph.




(a) Trajectories of $x_{i1}(t)$




(b) Trajectories of $x_{i2}(t)$




(c) Trajectories of $x_{i3}(t)$

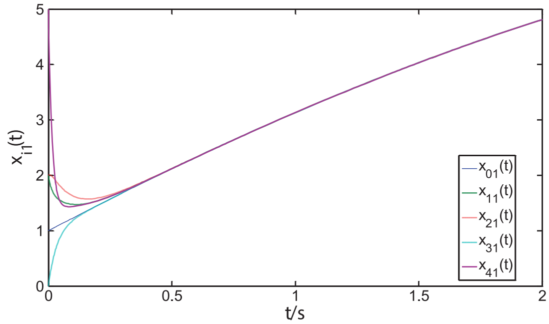
Fig. 4. The trajectories of each agent when the controller gain is 2.

Firstly, we suppose there's no null graph. We take the control gain $\alpha = 2$, furthermore, suppose that $g^{\sigma(t)} = g^1$ when $t \in [k, k + 0.4)$, $g^{\sigma(t)} = g^2$ when $t \in [k + 0.4, k + 1)$. The state trajectories of the multiple agents are shown in Fig. 3.

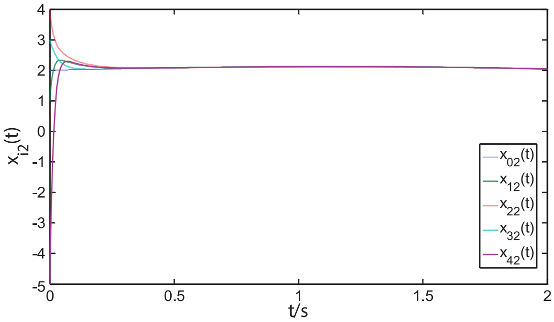
From Fig. 3, we can see that the leader-follower nonlinear multi-agent system can achieve consensus. Then we consider the situation that there's null graph in the switching topologies. We suppose $g^{\sigma(t)} = g^1$ when $t \in [k, k + 0.3)$, $g^{\sigma(t)} = g^2$ when $t \in [k + 0.3, k + 0.8)\,s$, $g^{\sigma(t)} = g^3$ when $t \in [k + 0.8, k + 1)\,s$, the conditions in Theorem 1 are satisfied. The state trajectories are shown in Fig. 4.

From Fig. 4, we can see that the leader-follower nonlinear multi-agent system can achieve consensus even with the null graph and the results verify the Theorem 1. In order to show the influence of the control gain, then we set $\alpha = 10$. The state trajectories are shown in Fig. 5.

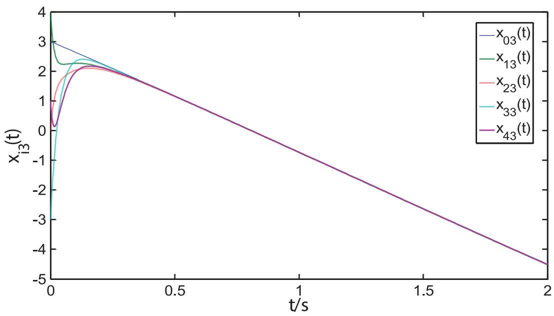
From Figs. 4 and 5, we can see that the speed of the consensus is faster if the control gain is bigger. Then we can conclude that the control can influence the consensus speed.




(a) Trajectories of $x_{i1}(t)$




(b) Trajectories of $x_{i2}(t)$




(c) Trajectories of $x_{i3}(t)$

Fig. 5. The trajectories of each agent when the controller gain is 10.

## 5. Conclusion

In this paper, the leader-following consensus problem for nonlinear multi-agent systems under switching topologies with communication failure is studied. The nonlinear models for leader and followers are given firstly. And then the control protocol is proposed to achieve leader-following consensus with switched topologies. Finally several numerical simulation results show that the obtained results are quite useful and we can control the consensus speed by setting the control gain.

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