Proceedings of the 2015
IEEE Conference on Robotics and Biomimetics
Zhuhai, China, December 6-9, 2015

A New Survey Adjustment Method for Laser Tracker Relocation

An Wan, Jing Xu*, Zonghua Zhang, Ken Chen

Abstract— **The relocation function is frequently used in the application of Laser Tracker. However, in large component assembly, since the laser tracker would be blocked by the large-size component, the number and distribution of fiducial points can hardly meet the requirement of laser tracker relocation, resulting in a low relocation accuracy. To solve this problem, we present a new laser tracker relocation method based on survey adjustment, which reduces the fiducial point location error by survey adjustment, thereby improving the laser tracker relocation accuracy. Simulations show that the proposed method improves the laser tracker relocation accuracy clearly especially in large component assembly.**

I. Introduction

In aircraft and automobile manufacturing, large component assembly is an important step that has a great influence on the total manufacture precision. Assembly robots are widely used in large component assembly due to the advantages of high speed, high accuracy and low cost[1]-[3]. In automated large component assembly, there are normally one fixed part and one moving part, and the moving part is carried by assembly robot to dock with the fixed part. To achieve a high assembly accuracy, the measurement of large part requires high accuracy, so the laser tracker is usually used[4]-[6].

Before assembly, the fixed part is measured by laser tracker to get its pose and the target pose of the moving part; then the moving part is measured to get its actual position and orientation; and finally the moving path can be calculated with the actual and target pose of the moving part. Because the absolute moving accuracy of the assembly robot is not high enough, the moving part cannot reach the target pose in one step, so the laser tracker needs to track the moving part's pose and correct the robot trajectory.

However in practice, due to huge size of the fixed and moving parts, the laser tracker would be blocked, which leads to a problem that the laser tracker cannot complete the measurement of both the fixed and the moving parts in one location. So the laser tracker relocation is required, that is the laser tracker measures the fixed part in one station, and then moves to another station to complete the measurement of the moving part. Finally the measured values in two stations can be unified into one coordinate system by laser tracker relocation[7][8].

The point-based rigid registration is usually used for laser tracker relocation[9][10]. First several common fiducial points are set in a certain range around the laser tracker, then the laser tracker measures the fiducial points respectively in two stations, obtaining two groups of measurement values, and finally, the two groups of measurement values are matched with the point-based rigid registration method to build the relationship between the above two stations, completing the laser tracker relocation (see Fig 1).

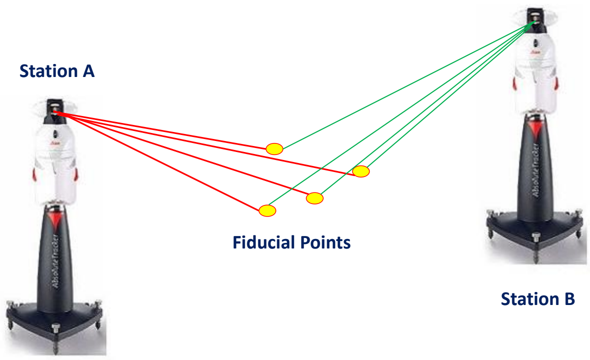

Fig. 1. The usually used laser tracker relocation method

Fitzpatrick studied the error distribution of the point-based rigid registration, and claimed that the registration error at a target point in space (i.e. the relocation error) relates to the fiducial location error (FLE), the fiducial point number and distribution[11][12]. The smaller FLE, the more fiducial points, and more uniform the fiducial points spread in space, so the target registration error (TRE) is smaller. The FLE is mainly the measurement error of fiducial points, which is difficult to reduce, so the current method for reducing the TRE is to increase the fiducial point number and look for the optimal fiducial points distribution.

However, in large component assembly, since the laser tracker would be blocked by large-size parts, few fiducial points can be set and the fiducial points distribution is difficult to optimize, which seriously reduces the accuracy of point-based rigid registration and laser tracker relocation. To solve this problem, we propose a new laser tracker relocation method based on survey adjustment, which improves the registration accuracy of target point by reducing the FLE.

The structure of this paper is as follows: In part II, we introduce the usually used point-based rigid registration method for laser tracker relocation. In part III, we pro-

An Wan, Jing Xu, are with State key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang, 110016, China and Department of Mechanical Engineering, Tsinghua University, Beijing, 100084, China
Zonghua Zhang, is with School of Mechanical Engineering, Hebei University of Technology, Tianjin, 300130, China.
Ken Chen, is with Department of Mechanical Engineering, Tsinghua University, Beijing, 100084, China.
Correspondence Email: jingxu@tsinghua.edu.cn

978-1-4673-9675-2/15/$31.00 © 2015 IEEE

pose a survey adjustment relocation method, and perform derivations to show the effect of the proposed method. In part IV, we perform simulations to compare the effect of the usually used direct relocation method and the proposed survey adjustment method. Part V shows and discusses about the simulation result. Finally, part VI is the conclusion.

II. Point-based rigid registration

The point-based rigid registration is usually used for laser tracker relocation. First, set a group of fixed fiducial points $\mathrm{F} = \{f_i\}, i = 1 \ldots N$, and the laser tracker measures all fiducial points in station A to get the measured values $X = \{x_i\}, i = 1 \ldots N$; then, move the laser tracker to station B, and the laser tracker measures all fiducial points again to get the measured values $Y = \{y_i\}, i = 1 \ldots N$. The measured values X and Y have the following theoretical relationship[13]-[15]

$$y_i = Rx_i + T \tag{1}$$

in which R is the rotation matrix, and T is the translation vector. However, taking into account the FLE, there is neither perfect R nor T that can fit the equation for all fiducial points. The point-based rigid registration is to get the optimal R and T that minimize the following equation using the least square method:

$$FRE^2 = \frac{1}{N}\sum_{i=1}^{N}|Rx_i + t - y_i|^2 \tag{2}$$

in which FRE is fiducial registration error, i.e. the distance between the position of a fiducial point in space A and space B after registration[16].

The evaluation criteria of point-based rigid registration is target registration error, which is the distance between the position of a random point not used in calculating the registration in space A and space B after registration[17][18]. Fitzpatrick studied the error distribution of the point-based rigid registration, and obtained the following TRE estimation[11][12]:

$$TRE^2 \approx \frac{FLE^2}{N}\left(1 + \frac{1}{3}\sum_{k=1}^{3}\frac{d_k^2}{f_k^2}\right) \tag{3}$$

All the fiducial points form three inertial principal axes, and f_k is the distance from the various fiducial points to the k-th inertial principal axis, d_k is the distance between the target point and the k-th inertial principal axis, N is the fiducial point number. f_k and d_k relate to the fiducial points distribution, so for a particular target point, the TRE relates to the FLE of each fiducial point, the fiducial point number and the spatial distribution of fiducial points. In large-size component assembly, since the laser tracker is blocked by the huge fixed and moving parts, the number of common fiducial points would be very small, usually less than 6, furthermore it is difficult to uniformly spread the fiducial points, which seriously impacts the effect of point-based rigid registration, resulting in a low relocation accuracy. (In point-based rigid registration, in order to achieve a good registration effect, it is generally required that the fiducial points are more than 6, and spread uniformly in space.)

III. Survey Adjustment

A. The condition adjustment method

To solve this problem, we propose a laser tracker relocation method based on survey adjustment, which reduces the FLE with survey adjustment, thereby reducing the TRE, and improving the laser tracker relocation accuracy. In this paper, the scaleplate showed in Fig 2 is used for laser tracker relocation.

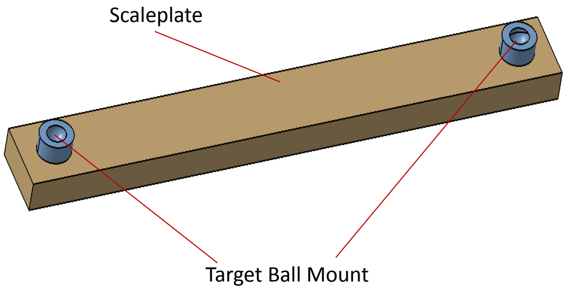

Fig. 2. The Scaleplate

The scaleplate is made of the materials whose shape hardly varies with temperature, humidity and other environmental factors, so its length can be considered fixed. Two laser tracker target ball mounts are respectively installed at each end of the scaleplate, and the distance between the two ball mounts L is measured by high-precision coordinate measuring machine beforehand. The proposed laser tracker relocation method is showed as follows:

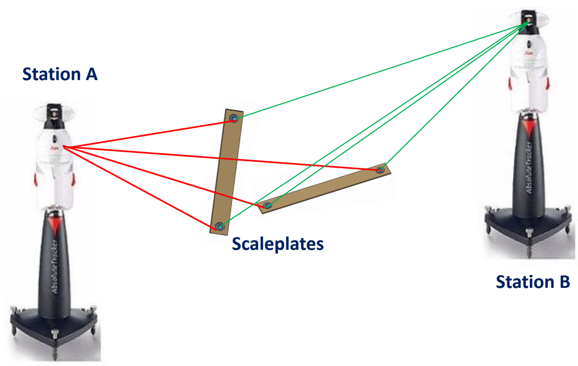

Fig. 3. The proposed laser tracker relocation method

We get the following constraint when the laser tracker measures the two target balls A and B, getting two 3D coordinate values $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$:

$$(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2 = L^2 \tag{4}$$

Then the condition adjustment can be performed on the measured values using the above constraint.

Survey adjustment is a data processing technique in geometry, which uses the least square principle to deal with the redundant data[19][20]. In order to improve the quality of the measurement results, redundant measurements are usually performed, leading to contradictions among the individual measurements. Survey adjustment aims to eliminate these contradictions, and achieve the most reliable result[21][22]. For the measurement of A and B, there are 6 unknowns $(x_1,y_1,z_1,x_2,y_2,z_2)$, and 7 measured values $X = (x_1,y_1,z_1,x_2,y_2,z_2,L)$, in which the distance L is a redundant measurement. Taking into account the measurement error, the above equation is not tenable. The contradiction among the individual measured values, i.e. the closure error is:

$$\begin{aligned} W &= f(x_1,y_1,z_1,x_2,y_2,z_2,L) \\ &= (x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2-L^2 \end{aligned} \tag{5}$$

The purpose of the survey adjustment is to assign the closure error W to the 7 measured values according to a certain rule, thereby eliminating the closure error W, and optimizing the measured values. Set weights for each measured value, and the closure error W will be assigned according to the weighting matrix P. Since the measurements of point A, point B and the distance L are independent, the matrix P has the following form:

$$P = \begin{bmatrix} P_A & & \\ & P_B & \\ & & P_L \end{bmatrix} \tag{6}$$

in which P_A and P_B are 3-by-3 matrixes, presenting the weight of point A and B respectively; and P_L is 1-by-1, presenting the weight of the distance L. In practice, point A and B are far from the laser tracker, and their distance is only about one meter, so the measurement error of them can be considered the same. For convenient calculation, the measurement error is considered to meet the Gaussian distribution, then we have

$$P_A = P_B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{7}$$

and

$$P_L = n = \frac{\sigma_{3D}^2}{\sigma_L^2} \tag{8}$$

in which σ_{3D} is the measurement error of A and B, and σ_L is the measurement error of L. Obviously $\sigma_L \ll \sigma_{3D}$, thus $n \gg 1$. So after simplification, the weighting matrix P is

$$P = diag([1,1,1,1,1,1,n]) \tag{9}$$

For the 7 measured values, the coefficient matrix is

$$\begin{aligned} A &= \frac{\partial f}{\partial X}\Big|_{X_0} \\ &= 2[x_1-x_2,y_1-y_2,z_1-z_2,x_2-x_1,y_2-y_1,z_2-z_1,-L] \end{aligned} \tag{10}$$

Then the error equation of condition adjustment is

$$AV + W = 0 \tag{11}$$

in which V is the correction of condition adjustment. For the above condition adjustment, the adjustment rule is

$$\min(V^T PV) \tag{12}$$

The condition adjustment aims to find the optimal correction V to meet the error equation and the adjustment rule, then the adjustment result is

$$\hat{X} = X + V \tag{13}$$

The mathematical method for this problem is conditional extremum method, namely the indefinite Lagrange multiplier method. In order to minimize $V^T PV$, we build a new extremal function

$$\Phi = V^T PV - 2K^T(AV + W) \tag{14}$$

in which $-2K^T$ is the variable multiplier (Lagrange multiplier), where $K = [k_1,k_2,\cdots k_7]^T$ is called connection number vector. In order to minimize the extremal function Φ, calculate the first derivative of Φ versus V, and set

$$\frac{d\Phi}{dV} = 2V^T P - 2K^T A = 0 \tag{15}$$

Perform transposition for both sides, taking into account that $P^T = P$, then we have

$$PV - A^T K = 0 \tag{16}$$

Since the matrix P is full rank, thus invertible, multiply the above equation by P^{-1}, getting the correction equation

$$V = P^{-1}A^T K \tag{17}$$

Using equation (11) and equation (16), we get the condition adjustment basic equation. The basic equation contains 7+1 unknowns V and K, while the number of equations is also 7 + 1, so the unique V meeting both the error equation and the adjustment rule can be obtained by the basic equation. To solve the basic equation, plug equation (17) into equation (11), then we get

$$AP^{-1}A^T K + W = 0 \tag{18}$$

Set

$$N_{AA} = AP^{-1}A^T \tag{19}$$

then

$$N_{AA}K + W = 0 \tag{20}$$

The above equation is called the connection normal equation of condition adjustment. The coefficient matrix N_{AA} is full rank and symmetric, thus invertible. So we have

$$K = -N_{AA}^{-1}W \tag{21}$$

Plug K into equation (17), then we obtain the correction V, and finally the adjustment result is $\hat{X} = X + V$.

B. The effect of survey adjustment

Plug A and P into the above calculation process, then we get

$$\begin{aligned}N_{AA} =&AP^{-1}A^T\\ =&4[(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2\\ &+(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2+\frac{L^2}{n}]\\ \approx&8L^2\end{aligned} \tag{22}$$

$$K=-N_{AA}^{-1}W\approx\frac{W}{8L^2} \tag{23}$$

$$\begin{aligned}V=&P^{-1}A^TK\\ =&diag([1,1,1,1,1,1,\frac{1}{n}])\cdot\begin{bmatrix}x_1-x_2\\ y_1-y_2\\ z_1-z_2\\ x_2-x_1\\ y_2-y_1\\ z_2-z_1\\ -L\end{bmatrix}\cdot\frac{W}{8L^2}\end{aligned} \tag{24}$$

Set the measurement error of $(x_1,y_1,z_1,x_2,y_2,z_2,L)$ be $\delta_1,\cdots,\delta_7$, then

$$\begin{aligned}W=&(x_1-x_2)^2+(y_1-y_2)^2+(z_1-z_2)^2-L^2\\ =&(\overline{x_1}-\overline{x_2}+\delta_1-\delta_4)^2+(\overline{y_1}-\overline{y_2}+\delta_2-\delta_5)^2\\ &+(\overline{z_1}-\overline{z_2}+\delta_3-\delta_6)^2-(L+\delta_7)^2\\ =&2L[(x_1-x_2)(\delta_1-\delta_4)+(y_1-y_2)(\delta_2-\delta_5)\\ &+(z_1-z_2)(\delta_3-\delta_6)-\delta_7]\end{aligned} \tag{25}$$

in which $(\overline{x_1},\overline{y_1},\overline{z_1},\overline{x_2},\overline{y_2},\overline{z_2})$ are the theoretical coordinates of point A and B. Set α the elevation angle from A to B, and β the azimuth angle, then we get the intermediate parameters

$$\begin{cases}a=\frac{(x_1-x_2)(\delta_1-\delta_4)}{L}=\cos\alpha\cos\beta(\delta_1-\delta_4)\\ b=\frac{(y_1-y_2)(\delta_2-\delta_5)}{L}=\cos\alpha\sin\beta(\delta_2-\delta_5)\\ c=\frac{(z_1-z_2)(\delta_3-\delta_6)}{L}=\sin\alpha(\delta_3-\delta_6)\end{cases} \tag{26}$$

where a, b and c present the measurement error distribution along the three axes of Cartesian coordinate system respectively. Plug the intermediate parameters into V

$$V=-(\frac{a+b+c-\delta_7}{2L})\begin{bmatrix}x_1-x_2\\ y_1-y_2\\ z_1-z_2\\ x_2-x_1\\ y_2-y_1\\ z_2-z_1\\ -L\end{bmatrix} \tag{27}$$

So the adjustment result is

$$\begin{aligned}\begin{bmatrix}\hat{x}_1\\ \hat{y}_1\\ \hat{z}_1\\ \hat{x}_2\\ \hat{y}_2\\ \hat{z}_2\\ \hat{L}\end{bmatrix}&=\begin{bmatrix}x_1\\ y_1\\ z_1\\ x_2\\ y_2\\ z_2\\ L\end{bmatrix}+V\\ &=\begin{bmatrix}x_1\\ y_1\\ z_1\\ x_2\\ y_2\\ z_2\\ L\end{bmatrix}-(\frac{a+b+c-\delta_7}{2L})\begin{bmatrix}x_1-x_2\\ y_1-y_2\\ z_1-z_2\\ x_2-x_1\\ y_2-y_1\\ z_2-z_1\\ -L\end{bmatrix}\end{aligned} \tag{28}$$

Next, we will compare the FLE before and after survey adjustment.

The FLE before survey adjustment is

$$\begin{aligned}FLE_2^2=&(\hat{x}_1-\overline{x_1})^2+(\hat{y}_1-\overline{y_1})^2+(\hat{z}_1-\overline{z_1})^2\\ &+(\hat{x}_2-\overline{x_2})^2+(\hat{y}_2-\overline{y_2})^2+(\hat{z}_2-\overline{z_2})^2\\ =&(x_1+v_1-\overline{x_1})^2+(y_1+v_2-\overline{y_1})^2+(z_1+v_3-\overline{z_1})^2\\ &+(x_2+v_4-\overline{x_2})^2+(y_2+v_5-\overline{y_2})^2+(z_2+v_6-\overline{z_2})^2\\ =&(\mathrm{r}\cos\alpha\cos\beta+\delta_1)^2+(\mathrm{r}\cos\alpha\sin\beta+\delta_2)^2\\ &+(\mathrm{r}\sin\alpha+\delta_3)^2+(-\mathrm{r}\cos\alpha\cos\beta+\delta_4)^2\\ &+(-\mathrm{r}\cos\alpha\sin\beta+\delta_5)^2+(-\mathrm{r}\sin\alpha+\delta_6)^2\\ \approx&2\mathrm{r}^2(\cos^2\alpha\cos^2\beta+\cos^2\alpha\sin^2\beta+\sin^2\alpha)\\ &+2r(-2r+\delta_7)+\sum_{i=1}^{6}\delta_i\end{aligned} \tag{29}$$

in which

$$r=-\frac{a+b+c-\delta_7}{2} \tag{30}$$

Then

$$\begin{aligned}\Delta FLE^2=&FLE_2^2-FLE^2\\ \approx&-2\mathrm{r}^2+2r\delta_7\end{aligned} \tag{31}$$

Plug $r=\frac{a+b+c-\delta_7}{2}$ into the above equation, and simplify it

$$\Delta FLE^2=-\frac{1}{2}[(a+b+c)^2-\delta_7^2] \tag{32}$$

Since δ_7 is very small compared to a, b and c, thus can be ignored, so we have

$$\Delta FLE^2=-\frac{1}{2}(a+b+c)^2<0 \tag{33}$$

That is

$$FLE_2^2<FLE^2 \tag{34}$$

showing that the FLE of A and B can be reduced by the survey adjustment.

We can see from the above derivations that the condition adjustment is to adjust the 3-dimensional coordinates of A and B towards the true values, based on the constraint L. For convenient calculation, we set the weighting matrix

$$P = diag([1,1,1,1,1,1,n]) \tag{35}$$

thus the coordinates are adjusted uniformly. Actually when P is set according to the measurement characteristic of laser tracker, the weight of each coordinate will not be the same. Thus when adjusted, the coordinate with higher accuracy moves litter, while the change of the coordinate with lower accuracy is large, so that the spatial position of A and B are adjusted closer to the true values, thus reducing the FLE.

For each scaleplate, the FLE of the two fiducial points can be reduced after survey adjustment. According to the equation

$$TRE^2 \approx \frac{FLE^2}{N}(1+\frac{1}{3}\sum_{k=1}^{3}\frac{d_k^2}{f_k^2}) \tag{36}$$

the TRE is finally reduced, and higher laser tracker relocation accuracy is achieved.

IV. Simulation

To test the proposed survey adjustment relocation method, we perform the following simulations. Set several scaleplates, and for each scalplate, the theoretical coordinates of the two target balls $A(x_1,y_1,z_1)$, $B(x_2,y_2,z_2)$ and the distance L are given. Then set the rotation matrixes and translation vectors form the world coordinate system to the two laser tracker stations R_1, R_2, T_1 and T_2 . Then for the fiducial points $F=\{f_i\}, i=1\ldots 2N$, the theoretical values in station A are $x_i = R_1 f_i + T_1 i = 1\cdots 2N$, and theoretical values in station B are $y_i = R_2 f_i + T_2 i = 1\cdots 2N$. x_i and y_i have the following relationship:

$$y_i = R_2 R_1^{-1} x_i - R_2 R_1^{-1} T_1 + T_2 \tag{37}$$

Add measurement error to x_i and y_i to get the actual measured values mx_i and my_i in two stations. Then perform condition adjustment for the actual measured values to obtain the adjusted values $\overline{mx}_i$ and $\overline{my}_i$. Then point-based rigid registration is performed for mx_i, my_i and $\overline{mx}_i$, $\overline{my}_i$ respectively. The obtained $\overline{R_x}$, $\overline{T_x}$ and R_x, T_x are the laser tracker relocation results with and without survey adjustment.

Randomly chose three target points within a cube with an edge of 10 meters, then we can get the theoretical values V in world coordinate system, V_1 in station A coordinate system, and V_2 in station B coordinate system. Then V_1 is transferred to station B coordinate system using R_x, T_x and $\overline{R_x}$, $\overline{T_x}$ respectively, getting the measured values V_x and $\overline{V_x}$. So the laser tracker relocation error with and without survey adjustment is:

$$TRE_{adj}^2 = \|\overline{V_x} - V_2\|^2 \tag{38}$$

$$TRE^2 = \|V_x - V_2\|^2 \tag{39}$$

The scaleplates are about 1 meter length, and spread within a cube with an edge of 3 meters. Then the rotation matrixes R_1 and R_2, transformation vectors T_1 and T_2, target points V are randomly given. Finally 10,000 times of simulation are performed to obtain the mean and variance of TRE.

V. Results and Discussion

Table I and Table II compare the relocation error with and without survey adjustment for different scaleplate number. Fig 4 and Fig 5 show the mean and variation of relocation error of the two methods varies with the scalplate number N respectively. We can see that the relocation error using the two methods both decrease with the greater number of scalplate number N, but the proposed survey adjustment relocation method always achieves clearly smaller TRE, showing a higher accuracy.

TABLE I
MEAN OF TRE FOR TWO METHODS

Scaleplate Number	Mean of TRE /mm	Mean of TRE_{adj} /mm
2	0.1301	0.1099
3	0.0564	0.0538
4	0.0384	0.0374
5	0.0321	0.0319

TABLE II
VARIATION OF TRE FOR TWO METHODS

Scaleplate Number	Var of TRE /mm	Var of TRE_{adj} /mm
2	0.0122	0.0084
3	0.0024	0.0022
4	0.0009	0.0008
5	0.0005	0.0005

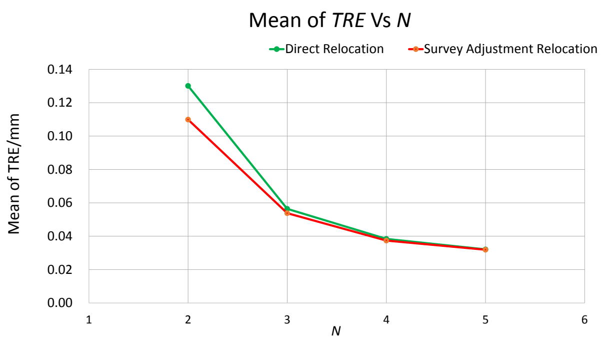

Fig. 4. Mean of TRE versus N for the two methods

Table III and Fig 6 show the reduction of TRE for the two methods. In which

$$RED_m = \frac{mean(TRE) - mean(TRE_{adj})}{mean(TRE)} \times 100\% \tag{40}$$

and

$$RED_v = \frac{var(TRE) - var(TRE_{adj})}{var(TRE)} \times 100\% \tag{41}$$

are the reduction of the mean and the variation of TRE using the proposed survey adjustment method instead of the direct relocation method respectively. We can see that the effect of the proposed method reduces with the greater number of scaleplate, because the direct relocation method can achieve a good effect when the fiducial points are more than 6. But

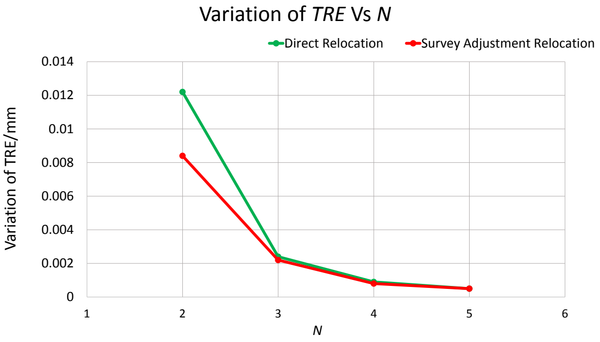

Fig. 5. Variation of TRE versus N for the two methods

when there are few scaleplates, the improvement in relocation accuracy is extremely clear, proving the effectiveness of the proposed method using in large component assembly.

TABLE III

THE REDUCTION OF TRE FOR TWO METHODS

Scaleplate Number	RED_m /%	RED_v/%
2	15.53	31.15
3	4.61	8.33
4	2.60	11.11
5	0.62	0.00

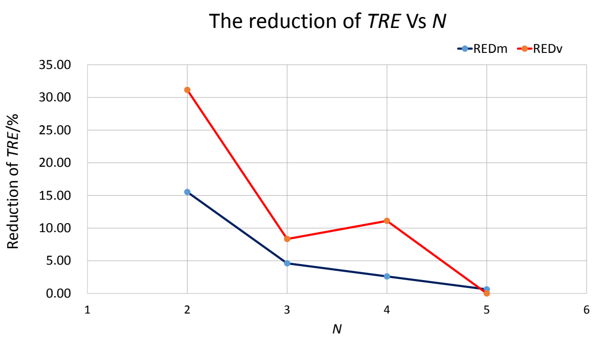

Fig. 6. The reduction of $mean(TRE)$ versus N for the two methods

VI. CONCLUSION

We presented a new survey adjustment algorithm for solving the laser tracker relocation problem when used in large component assembly in this paper. Simulations have shown that the new algorithm always performs better than the direct point-based rigid registration method, and performs a substantial improvement when used for laser tracker relocation in large component assembly, clearly showing the effectiveness of the proposed survey adjustment relocation method.

ACKNOWLEDGEMENT

The work is partially supported by the State Key Laboratory of Robotics and National Science Foundation of China with No. 51105218 .

REFERENCES

[1] B. G. Ho , M.Y. Geun, Y.S. Min, L. M.Cheol "Development of Process of A Force Sensorless Interference fit Assembly Robot System using Sliding Perturbation Observer", Journal of the Korean Society for Precision Engineering, 31(3), pp243-251, 2014

[2] W.j. Zhang,T. Freiheitb, H.S. Yang, "Dynamic scheduling in flexible assembly system based on timed Petri nets model", Robotics and Computer-Integrated Manufacturing, 21(6), pp550C558, 2005

[3] J. Krgera, R. Bernhardtb, D. Surdilovicb, G. Spurc, "Intelligent Assist Systems for Flexible Assembly", CIRP Annals - Manufacturing Technology, 55(1), pp29-32, 2006

[4] A. Nubiola, I.A. Bonev, "Absolute calibration of an ABB IRB 1600 robot using a laser tracker", Robotics and Computer-Integrated Manufacturing, 29(1), pp236-245, 2013

[5] J.H. Burge, P. Su, C.Y. Zhao, T. Zobrist, "Use of a commercial laser tracker for optical alignment", Optical System Alignment and Tolerancing, 2007

[6] B. Hughes, A. Forbes, A. Lewis, Wenjuan Sun, D. Veal, and K. Nasr, "Laser tracker error determination using a network measurement", Measurement science and Technology, 2011

[7] T.L. Zobrist, J.H. Burge, and H.M. Martin, "Accuracy of laser tracker measurements of the GMT 8.4m off-axis mirror segments", Modern Technologies in Space- and Ground-based Telescopes and Instrumentation, 2010

[8] K.Lau, R.J. Hocken, and W.C. Haight, "Automatic laser tracking interferometer system for robot metrology", Precision Engineering, 8(1), pp3-8, 1986

[9] C. Studholme, D.L.G. Hill, D.J. Hawkes, "Automated 3-D registration of MR and CT images of the head", Medical Image Analysis, 1(2), pp163-175, 1996

[10] J.B. West, J.M. Fitzpatrick, S.A. Toms, C.R. Maurer, R.J. Maciunas, "Fiducial Point Placement and the Accuracy of Point-based, Rigid Body Registration", 48(4), pp810-817, 2001

[11] J.M. Fitzpatrick, J.B. West, and C.R. Maurer, "Predicting error in rigid-body point-based registration", IEEE Transacions on Medical Imaging, 17(5), pp694-702, 1998

[12] J.M. Fitzpatrick, J.B. West, "The distribution of target registration error in rigid-body point-based registration", IEEE Transacions on Medical Imaging, 20(9), pp917-927, 2001

[13] M.H. Moghari, P. Abolmaesumi, "Distribution of target registration error for anisotropic and inhomogeneous fiducial localization error", IEEE Transactions on Medical Imaging, 28(6), pp799-813, 2009

[14] M.H. Moghari, P. Abolmaesumi, "Point-based rigid-body registration using an unscented Kalman filter", IEEE Transactions on Medical Imaging, 26(12), pp1708-1728, 2007

[15] R. Balachandran, J.M. Fitzpatrick, and R.F. Labadie, "Fiducial registration for tracking systems that employ coordinate reference frames", SPIE Medical Imaging 2005, pp134C145, 2005

[16] A. Lorusso, D.W. Eggert, and R.B. Fisher, "A comparision of four algorithms for estimating 3-D rigid transfermations", British Mach Vision Conference, 1995

[17] A.D. Wiles, A. Likholyot, D.D. Frantz, and T.M. Peters "A statistical model for point-based target registration error with anisotropic fiducial localizer error", IEEE Transacions on Medical Imaging, 27(3), pp378-390, 2007

[18] R. Balachandran1, J.M. Fitzpatrick, "Iterative solution for rigid-body point-based registration with anisotropic weighting", Medical Imaging 2009: Visualization, Image-Guided Procedures, and Modeling, 2009

[19] B. Schaffrin, A. Wieser, "On weighted total least-squares adjustment for linear regression", Journal of Geodesy, 85(7), pp415-421, 2001

[20] B. Schaffrin, Y.A. Felus, "Multivariate Total Least C Squares Adjustment for Empirical Affine Transformations", VI Hotine-Marussi Symposium on Theoretical and Computational Geodesy, 132, pp238-242, 2007

[21] O. Akyilmaz, "Total Least Squares Solution of Coordinate Transformation", Survey Review, 39(303), pp68-80, 2007

[22] N. Crocettoa, A. Vettoreb, "Best unbiased estimation of variance-covariance components: from condition adjustment to a generalized methodology", Journal of Information and Optimization Sciences, 22(1), pp113-122, 2001