Modeling hysteresis for piezoelectric actuators

Lianwei Ma1, Yu Shen2, Jinrong Li1, Hui Zheng1 and Tao Zou3

Abstract
In this article, the concept of constraint factor is proposed, and a new hysteretic operator consisting of a constraint factor and a polynomial is presented. Based on the constructed hysteretic operator, the one-to-multiple mapping of hysteresis is transformed into one-to-one mapping by expanding the input space, so that the mapping between the expanded input space and the output space comprises one-to-one and multiple-to-one mappings. Finally, a neural network is employed to approximate hysteresis for piezoelectric actuators. The validation performance of experimental tests suggests that the proposed approach is effective.

Keywords
hysteretic operator, hysteresis, expanded-space method, piezoelectric actuator

Introduction
Piezoelectric materials were extensively used in many fields in the past decades (Lam and Chan, 2005; Wu et al., 2014; Zhou and Zhang, 2014). Among piezoelectric materials, piezoelectric ceramic materials are commonly employed to form sensors, actuators, and transducers due to the high electromechanical coupling factor and piezoelectric coefficients (Hu et al., 2014; Yin et al., 2011; Zhang and Eitel, 2013). For the numerical simulation of the systems involved with piezoelectric devices, it is essential to use appropriate models (Liu et al., 2014; Sutor et al., 2013; Zhang et al., 2012). However, hysteretic behavior exists in piezoelectric devices, which brings a challenge when modeling them. Hence, considerable effort has been put into construction of accurate hysteresis models.

Among hysteresis models, the Preisach model (Ahn and Kha, 2007; Han et al., 2007; Li et al., 2014; Mayergoyz, 1986; Xiao and Li, 2013), Krasnosel’ski–Pokrovskii (KP) model (Krasnosel’ski and Pokrovskii, 1989; Zhou et al., 2014), Prandtl–Ishlinskii (PI) model (Chen et al., 2013; Macki et al., 1993; Su et al., 2005), Duhem model (Duhem, 1879; Oh and Bernstein, 2005), and Bouc–Wein model (Ortiz et al., 2013; Wang et al., 2011; Wen, 1976) are most frequently used in nonlinear control systems. The former three models are operator-based models and the latter two models are differential equation models. However, with the development of the science and technology, control systems with hysteresis need more accurate models. Neural networks (NNs) have always been of great importance in modeling hysteresis in the last decade because of their merits such as self-learning, associative memory, and high-speed optimization solution. A hysteretic operator (HO)–based neural hysteresis model using the expanded-space method was presented (Ma et al., 2008), and it was proved that hysteresis could be identified using NN if the mapping between the expanded input space and the output space only consists of one-to-one and multiple-to-one mappings (Ma and Shen, 2014). Moreover, Zhao and Tan (2008) and Zhang et al. (2010) presented, respectively, new HOs and constructed NN-based hysteresis models.

The HO is a key factor in the neural hysteresis models mentioned above. As a matter of fact, the similarity between HO and branch of hysteric loop is a reliable indicator of the model precision. In this article, first of all, the concept of constraint factor is proposed, and a new HO consisting of a constraint factor and an m-order polynomial is presented. And then, it is proved that the one-to-multiple mapping of hysteresis can be transformed into one-to-one mapping by expanding...
the input space based on the HO so that the mapping between the expanded input space and the output space only comprises one-to-one and multiple-to-one mappings. Finally, an NN is employed to approximate a set of real data measured from a piezoelectric ceramic actuator. The experimental results approve the proposed approach.

**Construction of HO**

**Definition of HO**

In this article, two main points are considered about construction of HO in the expanded-space method, and the detailed information can be found in Ma and Shen (2014). On one hand, the shape of branch of hysteresis loop is similar to that of parabola. In Ma and Shen (2014), a cubic function was used in order to reduce complexity of the parameters’ calculation, which degrades the accuracy of the hysteresis model. Thus, an $m$-order polynomial function is employed in this article. On the other hand, the curve of the HO should pass through the origin in every minor coordinate system. The constant term of the cubic function was set to be 0 in order to meet this point in Ma and Shen (2014), which reduces the adaptability of hysteresis model. Therefore, in this article, the HO comprises a constraint factor $(1 - e^{-x})$ and an $m$-order polynomial with constant term. The constraint factor, whose output range is $[0, 1)$, reflects the variation tendency of the branch of hysteretic loops. The constraint factor can not only adjust the amplitude and shape of the HO but also ensure that the curve of the HO passes through the origin. Therefore, the HO in the $i$th minor coordinate system is

$$f(x_i) = (1 - e^{-x}) \sum_{j=0}^{m} a_j x_i^j$$  

(1)

where $x_i$ and $f$ are any input and the corresponding output of HO in the $i$th minor coordinate system, respectively.

The HO in the main coordinate system is defined as

$$h(u) = \begin{cases} h(u_{ei}) + f(u - u_{ei}) & u > u_{ei} \\ h(u_{ei}) - f(u_{ei} - u) & u < u_{ei} \end{cases}$$

(2)

where $[u_{ei}, h(u_{ei})]$ are the coordinates of the origin of the $i$th minor coordinate system in the main coordinate system, $u$ is any input, and $h$ is the corresponding output of HO.

**Calculation of the parameters**

It is well-known that the least square method is the best way to determine the parameters of the polynomial functions, so that the samples used to train the NN can be employed to calculate the parameters. It is supposed that the sample’s number is $n$ and the input and output values of any sample are, respectively, $x_i$ and $y_i$ in the $i$th minor coordinate system, and then the residual $\delta_i$ is

$$\delta_i = y_i - f(x_i)$$  

(3)

Consequently, the sum of square residuals is

$$S = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} [y_i - f(x_i)]^2 = \sum_{i=1}^{n} \left[ y_i - (1 - e^{-x_i}) \sum_{j=0}^{m} a_j x_i^j \right]^2$$

(4)

According to the least square method, the sum of square residuals should be minimized. The minimum value can be achieved by setting the partial derivatives of $S$ with regard to $a_0, a_1, \ldots, a_m$ to 0. Since the HO has $(m + 1)$ parameters, there are $(m + 1)$ partial derivative equations

$$\frac{\partial S}{\partial a_k} = -2 \sum_{i=1}^{n} \left[ (1 - e^{-x_i}) \cdot x_i^k \cdot y_i - (1 - e^{-x_i})^2 \sum_{j=0}^{m} (a_j x_i^j + k) \right]$$

$$= 0, \quad k = 0, 1, \ldots, m$$  

(5)

Rearranging equation (5) gives

$$\sum_{j=0}^{m} \left( a_j \sum_{i=1}^{n} [(1 - e^{-x_i})^2 x_i^j + k] \right)$$

$$= \sum_{i=1}^{n} \left[ (1 - e^{-x_i}) \cdot x_i^k \cdot y_i \right] \quad k = 0, 1, \ldots, m$$

(6)

Equation (6) is equivalent to the following equation

$$\begin{cases}
  a_0 \sum_{j=1}^{n} (1 - e^{-x_i})^2 + a_1 \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i + \cdots + a_m \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^m = \sum_{i=1}^{n} [(1 - e^{-x_i})y_i] \\
  a_0 \sum_{j=1}^{n} x_i + a_1 \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^2 + \cdots + a_m \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^{m+1} = \sum_{i=1}^{n} [(1 - e^{-x_i})x_i y_i] \\
  \cdots \\
  a_0 \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^m + a_1 \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^{m+1} + \cdots + a_m \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^n = \sum_{i=1}^{n} [(1 - e^{-x_i})x_i^m y_i] 
\end{cases}$$

(7)
Equation (7) can be written as

$$
\begin{bmatrix}
\sum_{i=1}^{n} (1 - e^{-x_i})^2 & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i & \cdots & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^m \\
\sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^2 & \cdots & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^{m+1} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^m & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^{m+1} & \cdots & \sum_{i=1}^{n} (1 - e^{-x_i})^2 x_i^{2m} \\
\end{bmatrix}
\begin{bmatrix}
a_0 \\
a_1 \\
\vdots \\
a_m \\
\end{bmatrix}
= \begin{bmatrix}
\sum_{i=1}^{n} (1 - e^{-x_i}) y_i \\
\sum_{i=1}^{n} (1 - e^{-x_i}) y_i x_i \\
\vdots \\
\sum_{i=1}^{n} (1 - e^{-x_i}) y_i x_i^m \\
\end{bmatrix}
$$

(8)

that is

$$
X A = Y
$$

(9)

Solving equation (9), the model parameters are given as follows

$$
A = X^{-1} Y
$$

(10)

The existent condition for the unique solution of equation (9) is that $X$ is full rank matrix. Hence, in the following, it will be proved that the matrix $X$ has full rank, that is, $\text{rank}(X) = (m + 1)$, supposing that $(m + 1)$ is smaller or equal to $n$.

**Proof.** Let

$$
P = \begin{bmatrix}
1 - e^{-x_1} & 1 - e^{-x_2} & \cdots & 1 - e^{-x_n} \\
(1 - e^{-x_1}) x_1 & (1 - e^{-x_2}) x_2 & \cdots & (1 - e^{-x_n}) x_n \\
\vdots & \vdots & \ddots & \vdots \\
(1 - e^{-x_1}) x_1^m & (1 - e^{-x_2}) x_2^m & \cdots & (1 - e^{-x_n}) x_n^m
\end{bmatrix}
$$

(11)

Since $x_1 \neq x_2 \neq \ldots \neq x_n$ and $(m + 1) \leq n$, the $(m + 1)$ rows of the matrix $P$ are linearly independent, that is, $P$ is a row full rank matrix. Thus, the $(m + 1)$ row rank of a matrix equals the column rank of its transpose, the column rank of $P^T$ is also equal to $(m + 1)$, that is, $P^T$ is a column full rank matrix. Thus, $\text{rank}(P^T) = (m + 1)$.

Since the matrix $X$ can be written as

$$
X = P * P^T
$$

(12)

Therefore, $\text{rank}(X) = \min[\text{rank}(P), (m + 1)]$. And since $\text{rank}(P) = \text{rank}(P^T) = (m + 1)$, it can be obtained that $\text{rank}(X) = (m + 1)$. Therefore, the matrix $X$ has full rank. In conclusion, equation (9) has a unique solution, supposing that the parameter number of the HO is smaller or equal to the sample number, that is, $(m + 1) \leq n$.

**Mapping of the input and output spaces**

It has been proved that the traditional approach of NNs cannot identify one-to-multiple mapping19 (Wei and Sun, 2000), while the mapping of hysteresis comprises multiple-to-one mapping and one-to-multiple mapping. As shown in Figure 1, for two different times $t_1$ and $t_2$, there exist two equal inputs $u(t_1) = u(t_2) = u_1$, while $y(t_1) \neq y(t_2)$. Thus, the two equal inputs $u(t_1)$ and $u(t_2)$ correspond to the two different outputs $y(t_1)$ and $y(t_2)$, that is, so-called one-to-multiple mapping. In this article, the output $h[u(t)]$ of HO and the input $u(t)$ are together fed into NNs so that the input space is expanded from one-dimension to two-dimension. So it is decisive that whether or not the new mapping between the expanded input space and the output space contains one-to-multiple mapping. If the one-to-multiple mapping can be transformed into one-to-one or multiple-to-one mapping using this approach, then the approach is successful and vice versa. In the following, it will be proved that the one-to-multiple mapping can be transformed into one-to-one mapping by expanding input space based on the proposed HO.

**Lemma 1.** Let $u(t) \in S(R)$, where $R = \{t|0 \leq t < \infty \}$ and $S(R)$ are the sets of continuous functions on $R$. For the different times $t_1$ and $t_2$ (shown in Figure 1), while $u(t_1)$ equals $u(t_2)$, $h[u(t_1)]$ is unequal to $h[u(t_2)]$, that is, $u(t_1) = u(t_2)$ leads to $h[u(t_1)] \neq h[u(t_2)]$.

**Proof.** Since the input $u(t_1)$ locates in the minor coordinate system with origin $[u_{t_1}, h(u_{t_1})]$ and $u < u_{t_1}$, according to formula (2), the corresponding output of HO is

![Figure 1. Multi-value mapping of hysteresis. (a) Input signal. (b) Output-input curve.](image-url)
\[ h[u(t_1)] = h(u_{c_1}) - f(u_{c_1} - u_1) \]
\[ = h(u_{c_1}) - (1 - e^{-(u_{c_1} - u_1)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_1)^j \]

(13)

Similarly, \( h(u_{c_2}) \) can be obtained as follows
\[ h(u_{c_2}) = h(u_{c_1}) - f(u_{c_1} - u_2) \]
\[ = h(u_{c_1}) - (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_2)^j \]

(14)

Since the input \( u(t_2) \) locates in the minor coordinate system with origin \([u_{c_2}, h(u_{c_2})]\) and \( u > u_{c_2} \), in accordance with formula (2), the corresponding output of HO is
\[ h[u(t_2)] = h(u_{c_2}) + f(u_{c_2} - u_2) \]
\[ = h(u_{c_2}) + (1 - e^{-(u_{c_2} - u_2)}) \sum_{j=0}^{m} a_j(u_{c_2} - u_2)^j \]

And since \( u_2 = u_1 \)
\[ h[u(t_2)] = h(u_{c_2}) + (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_2)^j \]

(15)

Substituting expression (14) into expression (15) gives
\[ h[u(t_2)] = h(u_{c_1}) - (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_2)^j \]
\[ + (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_1 - u_2)^j \]

(16)

Subtracting expression (16) from expression (13) gives
\[ h[u(t_1)] - h[u(t_2)] = (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_2)^j \]
\[ - (1 - e^{-(u_{c_1} - u_1)}) \sum_{j=0}^{m} a_j(u_{c_1} - u_1)^j \]
\[ - (1 - e^{-(u_{c_1} - u_2)}) \sum_{j=0}^{m} a_j(u_1 - u_2)^j \]

(17)

Let
\[ q = u_1 - u_2 \quad \text{and} \quad r = u_{c_1} - u_1 \]

(18)

then
\[ q + r = u_{c_1} - u_2 \]

(19)

Substituting expressions (18) and (19) into expression (17) gives
\[ h[u(t_1)] - h[u(t_2)] = f(r + q) - f(r) - f(q) \]

(20)

Since the branch of hysteresis loop is similar to a parabola passing through the origin in every minor coordinate system and the curve of the function \( f(x) \) fitted by the least square method is similar to the branch of hysteresis loop, the curve of \( f(x) \) is similar to a parabola. If there are three intersection points at least between the curves of \( f(x) \) and \( g(x) = kx \) besides the origin, \( f(q + r) = f(q) + f(r) \). However, there is only one intersection point at most between \( f(x) \) and \( g(x) \) as shown in Figure 2. Therefore, \( f(q + r) \neq f(q) + f(r) \), that is, \( h[u(t_1)] \neq h[u(t_2)] \). Thus, for two particular times \( t_1 \) and \( t_2 \), even \( u(t_1) = u(t_2) \), it can lead to \( h[u(t_1)] \neq h[u(t_2)] \).

**Lemma 2.** For two different times \( t_1 \) and \( t_2 \), \( u(t_2) - u(t_1) \to 0 \) if \( h[u(t_2)] - h[u(t_1)] \to 0 \) in any minor coordinate system.

**Proof:** In any minor coordinate system, considering
\[ \frac{h[u(t_2)] - h[u(t_1)]}{u(t_2) - u(t_1)} = k, \quad k \in (0, +\infty) \]
then
\[ u(t_2) - u(t_1) = \frac{h[u(t_2)] - h[u(t_1)]}{k} \]

(22)

It is clear that if \( h[u(t_2)] - h[u(t_1)] \to 0 \), then \( u(t_2) - u(t_1) \to 0 \).

**Theorem 1.** For any hysteresis, the multiple-to-one mapping can be transformed into a continuous one-to-one mapping by expanding the input space based on the HO.

**Proof:** For the two different times \( t_1 \) and \( t_2 \), both \( u(t_1) \) and \( u(t_2) \) are equal to \( u_1 \), but \( y(t_1) \) is unequal to \( y(t_2) \), that is, the mapping is one-to-multiple.

When the input space is expanded from one-dimension to two-dimension based on the HO, in terms of Lemma 1, for two different times \( t_1 \) and \( t_2 \)
And since $y(t_1) \neq y(t_2)$, $(u(t_1), h[u(t_1)]) \neq (u(t_2), h[u(t_2)])$ \hspace{1cm} (23)

Therefore, the one-to-multiple mapping is transformed into a one-to-one mapping. Next, it will be proved that the new mapping is continuous.

In terms of the article (Gorbert, 1997)

$$u(t_2) - u(t_1) \to 0 \Rightarrow h[u(t_2)] - h[u(t_1)] \to 0$$ \hspace{1cm} (24)

Then, considering Lemma 2

$$h[u(t_2)] - h[u(t_1)] \to 0 \Rightarrow u(t_2)$$

$$-u(t_1) \to 0 \Rightarrow y(t_2) - y(t_1) \to 0$$ \hspace{1cm} (25)

So the new mapping is continuous. Therefore, it can be concluded that the one-to-multiple mapping is transformed into a continuous one-to-one mapping.

Remark 1. Theorem 1 indicates that the multi-value mapping of hysteresis can be transformed into a continuous mapping comprising one-to-one mapping and multiple-to-one mapping by expanding the input space based on the proposed HO.

As is well-known, any continuous one-to-one or multiple-to-one mapping can be approximated to arbitrary accuracy on a compact set using a three-layer NN that contains a sufficient number of hidden nodes (Funahashi, 1989).

**Model validation**

In the following, the proposed model was validated by being compared with the PI model in experimental tests. The experimental platform consists of a piezoelectric ceramic actuator (PZT-753.21C; PI Corp.), a voltage amplifier (E505.00), a noncontact capacitive sensor (E-509.Cxa; PI Corp.), and a multifunction board (PC-7483 from Advantech Corp.) with 16-bit analog-to-digital (A/D) and digital-to-analog (D/A) converters. Driven by an input voltage in the range from 0 to 120 V, the actuator has a nominal displacement ranging from 0 to 20 μm. Two sets of data were collected, wherein one set containing 1045 input–output pairs was used for model identification and another set containing 1002 input–output pairs was used for model validation. The input signal for model identification is shown in Figure 3(a) and the input signal for model validation is shown in Figure 3(b). It can be seen that the two input signals are apparently different.

**Proposed model**

In this section, the conjugate gradient algorithm with Powell–Beale restart method will be employed to train the NN, so that the convergent rate and the model performance could be improved. A three-layer feed-forward NN, whose activation functions of the hidden and output layers are, respectively, the sigmoid and linear functions, is employed to approximate the measured data. The mean square errors (MSEs) with the degree of the proposed HO from 1 to 10 are listed in Table 1. The results illustrate that the model would give the best result when the degree equals 5. Thus, $m = 5$.

### Table 2. Performances of NN with different numbers of hidden nodes in the proposed model.

<table>
<thead>
<tr>
<th>No. of hidden nodes</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.0049</td>
</tr>
<tr>
<td>21</td>
<td>0.0061</td>
</tr>
<tr>
<td>8</td>
<td>0.0062</td>
</tr>
</tbody>
</table>

MSE: mean square error.

![Figure 3. Input signals.](image-url)
Because the optimal number of hidden nodes has to be determined through experiments and over-many hidden nodes will weaken the NN for fault tolerance, the number of hidden nodes is tried from 1 to 100 in this experiment. Taking into account the length of the article, only the best three performances are listed in Table 2. As shown in Table 2, the NN gives the best result when the number of hidden nodes becomes five. Therefore, an NN consisting of two input nodes, five hidden nodes, and one output node is employed to approximate the hysteresis nonlinearity for the piezoelectric ceramic actuator. After 191 iterations, the training procedure finishes. The parameters of HO are shown in Table 3. Then, the NN is used to predict the output signal for the input signal $b$. Figures 4 and 5 show the validation result and absolute errors, respectively. The MSE, maximum absolute error, mean relative error, and maximum relative error are shown in Table 7.

### PI model

In this section, the PI model was also used to predict the output signal for the input signal $b$. First of all, the first set of data was used to determine the weights of the PI model. The thresholds were determined according to

$$r_i = \frac{i - 1}{n} \left[ \max(u(k)) - \min(u(k)) \right]$$

where $n$ is the number of backlash operators and $i = 1, 2, \ldots, n$. Table 4 shows the MSEs with the number of backlash operators from 5 to 15. It illustrates that the PI model would give the best result when the number equals 12. Thus, $n = 12$. The weights were determined based on the least square method. The weights and thresholds are shown in Table 5.

The thresholds shown in Table 6 were computed based on formula (26) when the PI model was used to predict the output for the input signal $b$. Figures 6 and 7 show, respectively, the validation result and absolute errors. The MSE, maximum absolute error, mean relative error, and maximum relative error are shown in Table 7.

### Comparison

As shown in Table 7, in comparison with the PI model with respect to the MSE, maximal absolute error, mean relative error, and the maximal relative error, the proposed model can better approximate the measured data.

### Conclusion

In this article, a new HO comprising a constraint factor and an $m$-order polynomial is proposed. Based on the constructed HO, the one-to-multiple mapping of hysteresis is transformed into a continuous one-to-one mapping by expanding the input space from one-dimension to two-dimension. It is proved that the mapping between the expanded input space and the output space only contains one-to-one mapping and multiple-to-one mapping. Finally, an NN is employed to
approximate the real measured data. The experimental performance shows that the proposed approach is effective.

Declaration of Conflicting Interests
The authors declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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Table 4. Variation of MSE along with the increase in the number of backlash operators.

<table>
<thead>
<tr>
<th>n</th>
<th>MSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>0.1244</td>
</tr>
<tr>
<td>6</td>
<td>0.0989</td>
</tr>
<tr>
<td>7</td>
<td>0.0969</td>
</tr>
<tr>
<td>8</td>
<td>0.0247</td>
</tr>
<tr>
<td>9</td>
<td>0.0300</td>
</tr>
<tr>
<td>10</td>
<td>0.0222</td>
</tr>
<tr>
<td>11</td>
<td>0.0259</td>
</tr>
<tr>
<td>12</td>
<td>0.0221</td>
</tr>
<tr>
<td>13</td>
<td>0.0252</td>
</tr>
<tr>
<td>14</td>
<td>0.0344</td>
</tr>
<tr>
<td>15</td>
<td>0.4560</td>
</tr>
</tbody>
</table>

MSE: mean square error.

Table 5. Weights and thresholds of PI model (n = 12) for signal a.

<table>
<thead>
<tr>
<th>i</th>
<th>r_i</th>
<th>w_i</th>
<th>i</th>
<th>r_i</th>
<th>w_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.0704</td>
<td>7</td>
<td>16.6345</td>
<td>-0.0759</td>
</tr>
<tr>
<td>2</td>
<td>2.7724</td>
<td>0.0145</td>
<td>8</td>
<td>19.4069</td>
<td>0.0774</td>
</tr>
<tr>
<td>3</td>
<td>5.5448</td>
<td>0.0057</td>
<td>9</td>
<td>22.1793</td>
<td>-0.0132</td>
</tr>
<tr>
<td>4</td>
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<td>0.0085</td>
<td>10</td>
<td>24.9518</td>
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</tr>
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<td>0.0053</td>
<td>11</td>
<td>27.7242</td>
<td>0.0294</td>
</tr>
<tr>
<td>6</td>
<td>13.8621</td>
<td>0.0064</td>
<td>12</td>
<td>30.4966</td>
<td>0.1811</td>
</tr>
</tbody>
</table>

Table 6. Thresholds of PI model (n = 12) for signal b.

<table>
<thead>
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<td>27.4470</td>
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<td>5</td>
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<td>33.5463</td>
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</table>

PI: Prandtl–Ishlinskii.

Table 7. Comparison of the proposed model with the PI model.

<table>
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<tr>
<th>Models</th>
<th>Mean square error</th>
<th>Maximal absolute error</th>
<th>Mean relative error</th>
<th>Maximal relative error</th>
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<td>0.27%</td>
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<tr>
<td>PI model</td>
<td>0.0221</td>
<td>0.0649</td>
<td>0.36%</td>
<td>1.22%</td>
</tr>
</tbody>
</table>

PI: Prandtl–Ishlinskii.

Figure 6. Comparison of the PI model prediction and the real data.

Figure 7. Absolute errors of the PI model.
References


