Analysis and correction of ill-conditioned model in multivariable model predictive control

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Abstract: Ill-conditioned model usually appears in modelling high purity and complicated production processes such as the high-purity distillation column. Model predictive control (MPC) via closed-loop feedback is a class of methods for which a plant model is used to forecast the outputs or states of controlled systems and to then solve a linear or nonlinear programming with multivariable and constraints by so-called receding horizon optimisation. Ill-conditions can cause many negative effects on the implementation of MPC including system instability and controller failure. In this paper, these phenomena such as output static error and stability decreasing of ill-conditioned model caused in MPC are found by simulating simple examples, and the close correlation between the movement direction of the controlled system output and the characteristics of ill-conditioned model is also observed. The geometry tools and singular value decomposition (SVD) in linear algebra are used to analyse the essential cause of ill-conditioned model generation. A new offline quantitative strategy is proposed to improve the ill-conditioned model. Based on modified model of reengineering and implementation of model predictive control, the closed-loop control performance and stability can be significantly enhanced.

Keywords: model predictive control; MPC; model ill-conditioned problem; singular value decomposition; SVD; model identification; model mismatch.

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1 Introduction

Model predictive control (MPC) is a class of multivariable and constrained control methods based on the model. MPC can be explicitly handled in a united linear or nonlinear programming formula and is first applied to control in oil refining processes (Qin and Badgwell, 1997). At present, as MPC has the strong robustness and can significantly reduce the variance of the control process output (Qin and Badgwell, 2003), it has been widely applied to many industries, including petroleum, chemical industry, paper making, food processing, and aviation. The classic algorithm of MPC mainly includes the dynamic matrix control (DMC) (Cutler and Ramaker, 1980), the model algorithm control (MAC) (Richalet et al., 1978), and the generalised predictive control (GPC) (Clarke et al., 1987). Currently, the majority of industrial MPC software is based on DMC.

The model in MPC has a great influence on the control performances. The plant model and errors were usually obtained by system identification, and sometimes the resulting model may be ill-conditioned. This paper focuses on the stability and performances of MPC based on the ill-conditioned model. In conventional sense, ill-conditioned model is singular and uncontrollable. Here, ill-condition is specified to nearly ill-conditioned, i.e., the ill-conditioned model in this paper is approximate ill-condition, which is close to critical ill-condition. Large condition number of model and strong correlation between input and output are the intuitive feature of ill-conditioned model, which leads to difficult control of the process. The ill-condition of plant model can be divided into the following two cases. First, there is strong correlation in the process itself (Skogestad and Postlethwaite, 1996), which is an ill-conditioned process physically, such as the certain distillation towers in the chemical process. Second, when the model of the multivariate process is obtained by system identification, the recognition accuracy of the identified results is highly dependent on inputs and outputs data, and the fitted process models may contain ill-conditioned sub-models by identification of certain specific inputs and outputs data. The model ill-conditioned problem is common in chemical engineering process (Skogestad et al., 1988), which has great influence on the stability of process in many serious cases (Matonoha and Papáček, 2015).

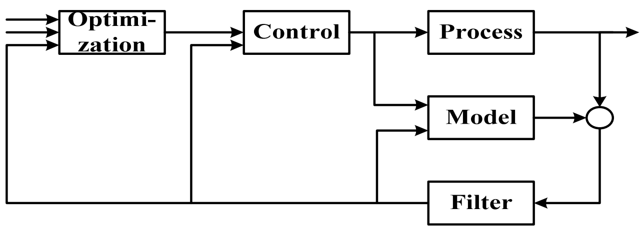
There are several methods to eliminate the influence of ill-conditioned model. For the square system (the number of inputs equal to outputs), Grosdidier et al. (1988) removed the outputs of linear correlation, and made them not participate in the calculation of control function in rolling optimisation process, which leaded to the output offset. Another solution was to use output range control strategy by relaxing the control requirements of some outputs, set points were extend to a certain range, which adapted to the process of low output demand. Therefore, this method was widely applied in industry. In fact, this solution was same as Grosdidier's method. Based on the standard regularisation theory, Marroquin et al. (1987) proposed a new random method to handle model ill-conditioned problem by numerical calculation. In addition, Honeywell's advanced control software adopted singular value thresholding (SVT) method to eliminate the influence of the ill-conditioned model on the control effect. Based on the threshold set by controller configuration, this method will neglect the input of smaller singular value in the corresponding direction, and thus avoid input that may possibly lead to the instability (Cai et al., 2010). AspenTech's advanced control software DMCplus adopted input move suppression (IMS) control strategy to solve the model ill-conditioned problem (Mohamed et al., 2015; Huo et al., 2013). This method decreased the inputs action by directly increasing the values of diagonal elements in LS problem and decreasing the condition number. Because of the two-layer predictive controller in DMCplus (Qin and Badgwell, 2003; Kassmann et al., 2000; Nikandrov and Swartz, 2009; Hu et al., 2010), this strategy was successful to solve ill-conditioned problem.

The methods mentioned above were all online solution strategies. These methods did not solve the essential problem or analyse the fundamental reasons for ill-conditioned model. Therefore, this paper adds to the literature with an offline strategy as a complementary method. This proposed strategy can well handle the model ill-conditioned problem in certain extent by integrating the above online strategies. This paper uses geometry and linear algebra tools to analyse why ill-conditioned model influenced the control performances. Then, this paper proposes a method to improve the process model. Finally, this paper uses numerical simulation to verify the effectiveness of proposed method.

2 Brief introduction of TMPV

The main feature of the 4th generation predictive control technology is the two-layer model predictive control (TMPC). TMPC adds a steady state optimisation (SSO) layer above dynamic control layer of traditional MPC. Previous literature (Matonoha and Papáček, 2015) introduced a new generation state space controller of AspenTech. The structure of TMPC is shown in Figure 1.

Figure 1 Structure of TMPV



Compared with the traditional MPC, TMPC enhances steady-state optimisation function modules by including external targets, optimising input parameters and output feedback items. Steady-state optimisation has an optimisation function to achieve economic performance. This function serves as an automatic optimisation process to locate adjacent the steady-state operating point as the optimal process set value. This process is called the steady-state target calculation, which requires the use of steady-state model of process.

Considered an invariant open-loop stable linear system, its transfer function model of dynamic characteristics is

$$\boldsymbol{y}(s) = \boldsymbol{G}(s)\boldsymbol{u}(s). \tag{1}$$

The steady-state model is

$$\Delta \boldsymbol{y}_s = \boldsymbol{K} \Delta \boldsymbol{u}_s, \tag{2}$$

where $\boldsymbol{K}$ represent steady-state gain matrix. The constraints of inputs and outputs are

$$\begin{cases} \boldsymbol{u_{LL}} \le \boldsymbol{u}_s(k) \le \boldsymbol{u}_{HL} \\ \boldsymbol{y_{LL}} \le \boldsymbol{y}_s(k) \le \boldsymbol{y}_{HL} \end{cases} \quad k \ge 0, \tag{3}$$

where $\boldsymbol{u}_{LL}$ are low limit of inputs, $\boldsymbol{u}_{HL}$ are high limit of inputs, $\boldsymbol{y}_{LL}$ are low limit of outputs and $\boldsymbol{y}_{HL}$ are high limit of outputs. The constraints involve hard constraints and soft constraints. Soft constrains can be flexible within engineering permitted range when there is no solution. And the hard constraints should be ensured.

Given the inputs and outputs constraint conditions of the controlled process, the steady-state optimisation of TMPC is linear programming (LP) or quadratic programming (QP) problem. This paper solves LP problem as example, the problem descriptions are

$$\begin{cases} \min\limits_{\Delta \boldsymbol{u}_s(k)} \boldsymbol{J} = \boldsymbol{c}^T \Delta \boldsymbol{u}_s \\ \text{s.t. } \Delta \boldsymbol{y}_s(s) = \boldsymbol{K} \Delta \boldsymbol{u}_s(k) + \boldsymbol{e}_k \\ \boldsymbol{u_{LL}} \le \boldsymbol{u}_s(k-1) + \Delta \boldsymbol{u}_s(k) \le \boldsymbol{u_{HL}} \\ \boldsymbol{y_{LL}} \le \boldsymbol{y}_s(k-1) + \Delta \boldsymbol{y}_s(k) \le \boldsymbol{y_{HL}} \end{cases}, \tag{4}$$

where $\boldsymbol{c}$ represent the price factors, $\boldsymbol{e}_k$ can be computed by dynamic predictive errors of process,

$$\boldsymbol{e}_k = \boldsymbol{y}_k - \tilde{\boldsymbol{y}}(k|k-1), \tag{5}$$

$\boldsymbol{y}_k$ represent output measured values of k time, $\tilde{\boldsymbol{y}}(k|k-1)$ represent output predictive values of k time in $k - 1$ time.

Feasibility analysis is need to solve the steady-state optimisation problem. If there are no viable solutions, feasibility determination and soft constraints adjustment will be done and constraint boundary of output variables will be relaxed appropriately according to the priority order.

Under assumption that steady-state optimisation condition is feasible, the final optimal control input can be calculated by the steady-state target calculation, and the optimal process output values can be further obtained. Optimal input and output values are transferred to lower layer of MPC as the set values. MPC will track the changing of each set point. If set point is given manually, the steady-state optimisation represents the shortest distance in the least squares between the initial steady-state working point and mathematical expectation.

The lower layer of steady-state optimisation is the dynamic control layer which uses DMC algorithm. Under double two-layer structure prediction control, penalty term of steady-state target values of the control inputs will be added to the objective function of DMC in the framework of TMPC, and the formation of the following forms of control objective function

$$\begin{aligned} \boldsymbol{J}(k) = & \left\| \boldsymbol{w}(k) - \tilde{\boldsymbol{y}}_{PM}(k) \right\|_{\boldsymbol{Q}}^2 + \left\| \boldsymbol{u}(k) - \boldsymbol{u}_{ss}(k) \right\|_{\boldsymbol{V}}^2 \\ & + \left\| \Delta \boldsymbol{u}_M(k) \right\|_{\boldsymbol{R}}^2, \end{aligned} \tag{6}$$

where $\boldsymbol{w}(k)$ are the set values, $\tilde{\boldsymbol{y}}_{PM}(k)$ are prediction values of k time, $\Delta \boldsymbol{u}_M(k)$ are the optimal control inputs increment of k time, $\boldsymbol{Q}$ is the weight matrix of outputs error, $\boldsymbol{R}$ is the weight matrix of control increment, $\boldsymbol{V}$ is the weight matrix of control inputs error.

There are not obvious differences in model prediction and feedback collection between TMPC and traditional MPC.

3 Influence of ill-conditioned steady-state gain model upon control performances

Function of the prediction model is to forecast system outputs based on the established mathematical model. The prediction model includes steady states and dynamic prediction. SSO is an approach to be working at the optimal working point of systems based on steady state predication, thus steady-state model is required for such operations. The step response coefficient of the input and output model will be usually gotten through model identification works (Courreges and Chassagnolle, 2014; Limebeer et al., 1993; Lin, 1987; Gui et al., 201). At present, most of the large-scale production process has a continuous and stable production characteristic, the input-output relationship of processes can be characterised more accurately by the resulting model through identification, so that the steady state calculations have better effect on control and optimisation applications. Meanwhile, the steady-state analysis is relatively simple, and can generally reflect the dynamic performance and controllability of the system. Thus, we begin with an example of the steady state gain coefficient matrix containing gain information, as shown in equation (7).

$$\boldsymbol{K} = \begin{bmatrix} k_{11} & \cdots & k_{1m} \\ \vdots & \ddots & \vdots \\ k_{p1} & \cdots & k_{pm} \end{bmatrix}, \tag{7}$$

where $\boldsymbol{K}$ denotes steady-state gain matrix; m, p are the number of inputs and outputs respectively. For the existing ill-conditioned model, this paper proposes a solution strategy is before the operation of the identified model as the controller, the singularity test (Gui et al., 2011; Wu and Liu, 2004; Sanliturk and Cakar, 2005) will be carried out upon the steady-state model and its subunits (2 × 2 matrix). If the sub model is ill-conditioned, we will correct and update model, which can avoid the problem of ill-conditioned model.

Firstly, the influences of ill-conditioned model upon control are analysed. The heavy oil separation column is selected which commonly used in chemical engineering for the study object, and the process model is as following by identifying,

$$\boldsymbol{G}(s) = \begin{bmatrix} \dfrac{-12.62}{50s+1}e^{-27s} & \dfrac{9.84}{60s+1}e^{-28s} & \dfrac{5.88}{50s+1}e^{-27s} \\ \dfrac{9.84}{(50s+1)}e^{-18s} & \dfrac{-6.88}{(60s+1)}e^{-14s} & \dfrac{6.9}{(40s+1)}e^{-15s} \\ \dfrac{4.38}{33s+1}e^{-20s} & \dfrac{4.42}{44s+1}e^{-22s} & \dfrac{7.20}{19s+1}e^{-24s} \end{bmatrix}. \tag{8}$$

Then, sub model of process model is used to be study object,

$$\boldsymbol{G}_{2\times 2} = \begin{bmatrix} \dfrac{-12.62}{50s+1}e^{-27} & \dfrac{9.84}{60s+1}e^{-28} \\ \dfrac{9.84}{50s+1}e^{-18} & \dfrac{-6.88}{60s+1}e^{-14} \end{bmatrix}. \tag{9}$$

The steady-state gain matrix is

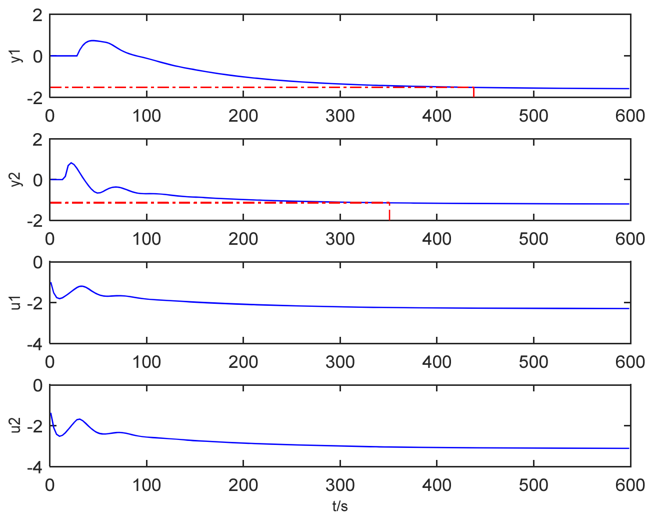
$$\boldsymbol{K} = \begin{bmatrix} -12.62 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}. \tag{10}$$

The condition number can be calculated by steady-state gain matrix, and the equation is as following,

$$cond(\boldsymbol{K}) = \frac{\sigma_{\max}}{\sigma_{\min}}, \tag{11}$$

where σ is singular value of matrix; $\sigma_{\max}$ is maximum singular value of matrix, and $\sigma_{\min}$ is minimum singular value. The condition number of this matrix is 40. If the condition number is big, this model will be considered as ill-conditioned by knowledge of linear algebra. In this paper, 60 is the critical value of condition number. If condition number is greater than 60, the model will be complete ill-condition and uncontrollable. We only discuss the case when the condition number is less than 60, the model is approximate ill-condition. In order to analyse the affect of control performance when the controller models are ill-conditioned, two different sets of data are chosen for demonstration of performance and stability.

Figure 2 Input and output curves of set point one (see online version for colours)

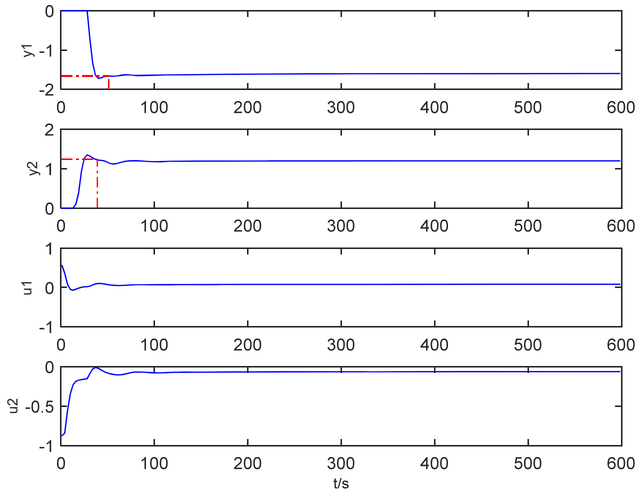


This paper uses the standard DMC algorithm to do the simulation experiments, the relevant parameters are as following. The set point one is $[-1.6, -1.2]^T$, set point two is $[-1.6, 1.2]^T$, and will use in all of simulation. The weight coefficient matrix Q, R are chosen to be a unit diagonal matrix (take this value for all), the input and output curves are shown in Figure 2.

It is seen from Figure 2 that the outputs finally stay on set point one $[-1.6, -1.2]^T$, where point of intersection of dotted line represents that the two output variables can be stabilised between the range of ±5% error. In other words, setting times t_s are 438 seconds and 351 second respectively, which indicates the controllers behave is a very slow response for this process.

Use set point two to simulate, the input and output curves are shown in Figure 3.

Figure 3 Input and output curves of set point two (see online version for colours)



By the simulation shows that the outputs are finally stay on set point two $[-1.6, 1.2]^T$, and the process reaches the set point 2 quickly, meanwhile setting times t_s are only 51 seconds and 39 seconds. Distinctions of controller action and a significant different control result, especially the response times, are observed by comparison of two different groups of set points based on the same ill-conditioned model. The changing of system outputs moving direction is only the cause of this phenomenon occurrence, the fundamental reason will be analysed in the following.

The mismatch between plant and its model is very common in practice. There are many reasons for the mismatch such as the accuracy of the identification results, characteristic's changing over time in controlled processes (Zou et al., 2010; Chen et al., 2011), and all these lead to decline in controller performances. Therefore, it is very necessary to analyse model ill-conditioned problems under the mismatch circumstances.

It is assumed that the mismatch is limited to a ±20% range for normal value of k_{11}, the remaining parameters are matched. Mismatch mode 1 is defined for +20% gain error and mode 2 is defined for –20% gain error, the resulting model gain matrixes are

$$\boldsymbol{K} = \begin{bmatrix} -10.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix} \text{ and}$$

$$\boldsymbol{K}_1 = \begin{bmatrix} -15.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}.$$

Mismatch mode 1 represents forward mismatch, which means the condition number of matrix decreases after model mismatch. The condition number of matrix $\boldsymbol{K}$ of this case is 14.9, which smaller than 40, which is the condition number of normal conditions. However the mode 2 is opposite to mode 1, the condition number of matrix $\boldsymbol{K}_1$ is 66.4.

In order to analyse model ill-conditioned problem under the mismatch condition, set point 1 and set point 2 are used to simulate, and the input and output curves are shown in Figures 4 and 5.

Figure 4 Input and output curves of set point one under mismatch mode 1 (see online version for colours)

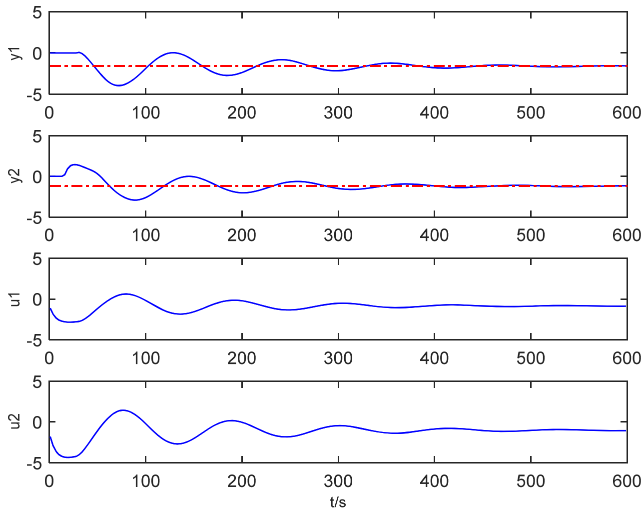
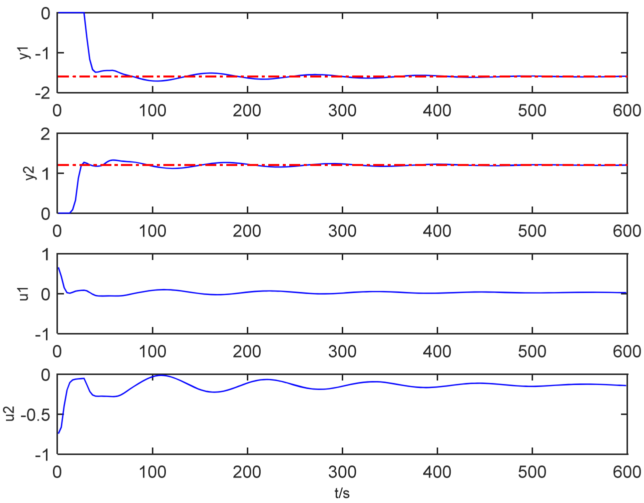


Figure 5 Input and output curves of set point two under mismatch mode 1 (see online version for colours)



From Figures 4 and 5, the process has an oscillation, and the magnitude of the oscillation is also related to the direction of the output movement. It is found that the response time of the two set-points for follow-up tracing under mismatch condition is greater than the response time of normal model. At the same time, it is also found that the control input of the potential corresponding to the set point is $u_s = \begin{bmatrix} -2.2816 \\ -3.0888 \end{bmatrix}$ which corresponding to the set point $[-1.6, -1.2]^T$. For this group of input steady state, the output value of steady-state model is

$$\boldsymbol{y}_{process} = \boldsymbol{K}_{process\_gain} \times \boldsymbol{u}_s = \begin{bmatrix} -7.3496 \\ -1.2 \end{bmatrix}.$$

It is seen that the mismatch 20% in k_{11} is amplified 459.35% in the output, which is caused by a large movement of control input. That is to say that the mismatch is amplified because of the range of the controller action, which causes the controller to become very sensitive to the mismatch. And then, changing the direction of the mismatch and using mismatch model 2, set point 1 and set point 2 for next simulations, and the input and output curves are shown in Figures 6 and 7.

Figure 6 Input and output curves of set point one under mismatch mode 2 (see online version for colours)

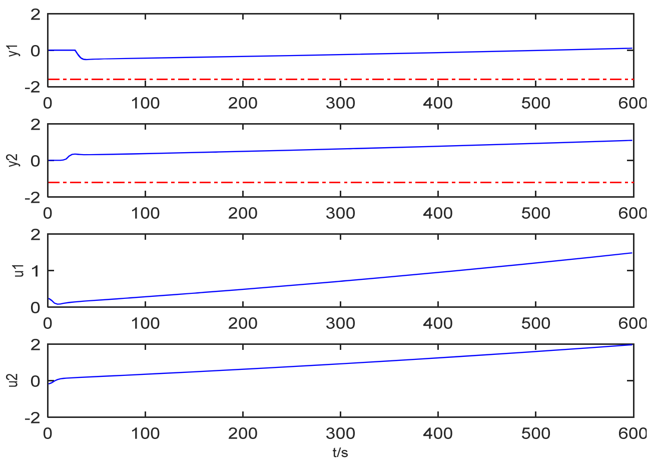
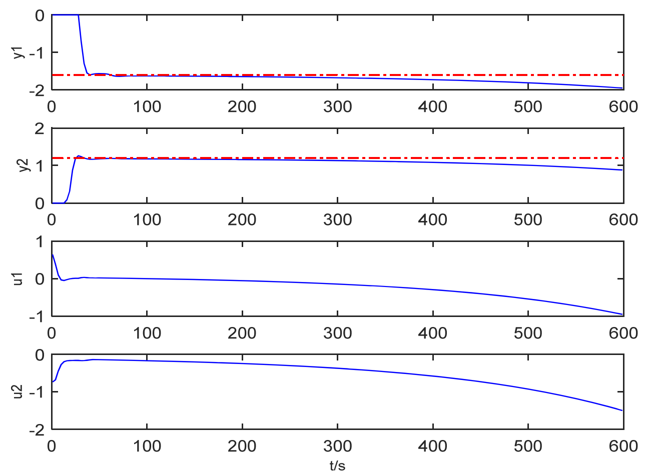


Figure 7 Input and output curves of set point two under mismatch mode 2 (see online version for colours)



From Figures 6 and 7, it can be seen that the output of the two sets of the mismatch is not to reach the set point, the mismatch causes the controller action is too large, so that the divergence of input and output curve is observed finally. When the moving direction of the two set points is changed, the condition number of the matrix is also changed, which leads to the change of the degree of the model morbidity.

So far, we can draw two hypotheses:

1 the ill conditioned nature of the model can lead to the too violent controller's action, then making the control effect becomes worse

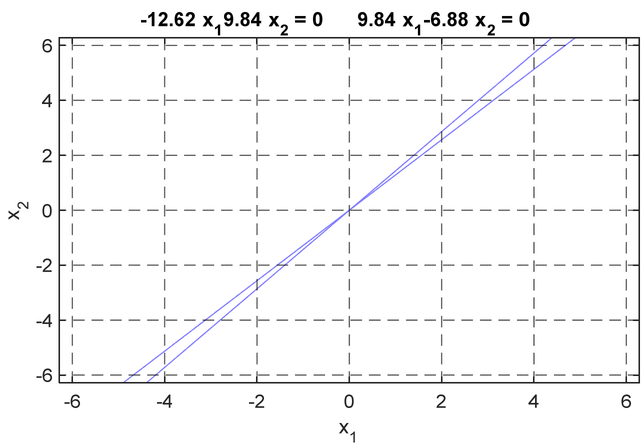
2 the direction of the output movement may affect the effect of the ill conditioned model-based predictive control.

4 Analysis of poor control performance under ill-conditioned model

For ill-conditioned model, there is a certain relationship between the large controller action and the movement direction of controlled systems output. For the 2-input 2-output square system (here it is assumed sub process model is stable), when set point is given, the potential steady-state input point is already determined (Harrison and Qin, 2009; Froisy, 2006). And the solution of the steady state input point is a process of solving the equation set of two variables, and geometrically speaking, the solution is the intersection point of the two lines.

For two groups of set points given in Section 2, the geometrical solution is shown as following. For output initial state $[0, 0]^T$, its geometrical explanation is shown in Figure 8.

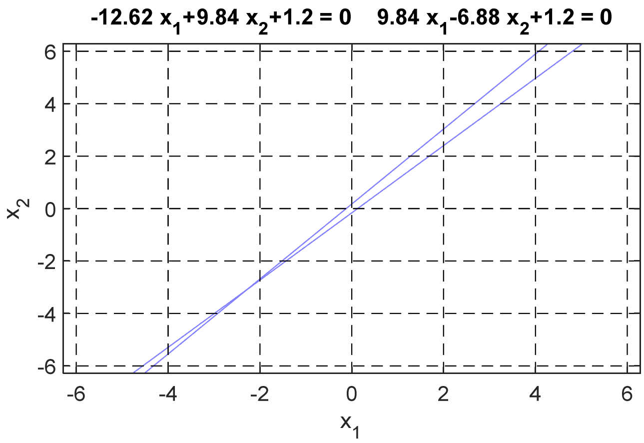
Figure 8 Geometrical explanation of output initial state (see online version for colours)



It is seen that the intersection point of two straight lines is the origin point, it means output initial state which corresponding to input initial state is $[0, 0]^T$.

For set point one, the geometrical explanation is shown in Figure 9.

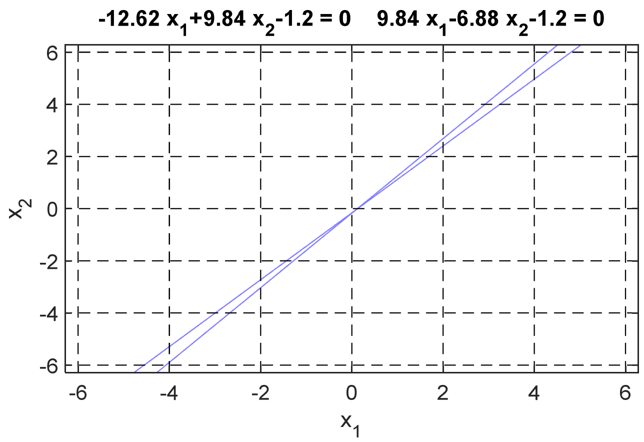
Figure 9 Geometrical explanation of output in set point one (see online version for colours)



It is seen that the distance from the intersection point of two lines to origin point is further away from the initial state, which means the amplitude of the input initial state will be larger, so larger action of controller will be generated.

For set point $[-1.6, 1.2]^T$, the geometrical explanation is shown in Figure 10.

Figure 10 Geometrical explanation of output in set point two (see online version for colours)



It is seen that the intersection point of the two lines is very close to origin point, which means the amplitude of the input initial state is small, so the controller will output smaller actions.

By the simulation results, a conclusion will be gotten that the movement direction of system's output has an influence upon the closed-loop performance for ill-conditioned model-based predictive control. Because changing of the output movement direction geometrically alters the distance between input steady state point and the original point, which changes the initial amplitude of input, it changes the range of controller action.

Whether a dynamic model is ill-conditioned or not by intuitive judgment is based on the condition number of its model, the greater the number of conditions, the more serious the model ill-conditioned, and the worse the control effect. This conclusion can be verified by the simulation of two different models in Section 4. For mismatch $k_{11} = -10.1$, the condition number of steady-state model will reduce by calculation. Meanwhile, for the mismatch $k_{11} = -15.1$, the number of condition increases, so the former is smaller ill-condition and the control performance is also better. For most of the processes, the steady-state model can represent dynamic nature of the processes, so the ill-conditioned problem of dynamic model can be converted into steady state model to analyse, and the calculation method of condition number has been given above. The linear algebra tools singular value decomposition (SVD) will be used to analyse the model ill-conditioned problem further next.

For the description of steady-state relationship shown in equation (2), the SVD of steady-state matrix is shown in equation (12) as following.

$$\boldsymbol{K} = \boldsymbol{U}\Sigma\boldsymbol{V}^T = \sum_{i=1}^{n} \sigma_i u_i \upsilon_i^{\mathrm{T}}. \qquad (12)$$

If $\boldsymbol{K} \in \boldsymbol{R}^{p*m}$; $\boldsymbol{U} = \begin{bmatrix} u_1 & \cdots & u_p \end{bmatrix}, \boldsymbol{U} \in \boldsymbol{R}^{p*p}$;

$$\Sigma = \begin{bmatrix} \sigma_1 & & \\ & \ddots & \\ & & \sigma_p \end{bmatrix}, \Sigma \in \boldsymbol{R}^{p*m};$$

$\boldsymbol{V} = \begin{bmatrix} v_1 & \cdots & v_m \end{bmatrix}$, $\boldsymbol{V} \in \boldsymbol{R}^{m*m}$.

Equation (12) can be gotten by equation (13),

$$\boldsymbol{KV} = \boldsymbol{U}\Sigma. \qquad (13)$$

where $\boldsymbol{U}$ and $\boldsymbol{V}$ are two groups of orthonormal basis vectors. In linear algebra, a matrix represents a linear transformation, and the significance of linear transformation can be explained by two kinds of equivalence.

1 to transform the vector of $x \in \boldsymbol{R}^m$ to the space of $\boldsymbol{R}^p$

2 it provides a representation of the same vector in two different spaces.

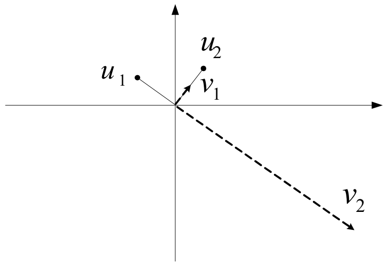
For steady-state matrix $\boldsymbol{K}$, SVD provides a manifestations of geometry. For any 2 × 2 matrix, it can always find a space to another space conversion and matrix transformation corresponds. The steady-state model of the process is $\Delta\boldsymbol{y}_s = \boldsymbol{K}\Delta\boldsymbol{u}_s$, where $\boldsymbol{V}$ is a set of orthonormal base of $\Delta\boldsymbol{u}_s$, $\boldsymbol{U}$ is a set of orthonormal base of $\Delta\boldsymbol{y}_s$, can be used as a set of orthonormal base of, singular matrix Σ represents the degree of stretch of steady-state gain matrix $\boldsymbol{K}$. Specifically, u_i and v_i are a couple of one-to-one correspondence orthonormal base vector in equation (12). From the perspective of steady-state, the essence of control is the inversion calculation of steady-state matrix. The matrix close to singular, the value of determinant will be very small, its inverse will be great, it means the motion of controller is large, that is, $\Delta u_s(k) = [\boldsymbol{U}\Sigma\boldsymbol{V}^T]^{-1} \times \Delta y_s(k)$.

For the model gain matrix of controller in this paper, the results of its SVD are

$$\boldsymbol{U} = \begin{bmatrix} -0.8 & 0.6 \\ 0.6 & 0.8 \end{bmatrix}, \Sigma = \begin{bmatrix} 20 & \\ & 0.5 \end{bmatrix}, \boldsymbol{V} = \begin{bmatrix} 0.8 & 0.6 \\ -0.6 & 0.8 \end{bmatrix}.$$

The coordinate system consisting of base vectors of SVD is shown in Figure 11.

Figure 11 Coordinate system consisting of base vectors of SVD



Here, the coordinate system, which vertex is circle-dot are composed of the base vectors u_1, u_2. The coordinate system, which vertex is allows of dotted lines are composed of the base vectors v_1, v_2. The stretching amplitudes σ_1, σ_2 in the directions of u_1, u_2 are 0.5 times and 20 times respectively, which means if the output moves in the direction of v_2, the direction of u_2 will need twice the amplitude of the input. Otherwise, the direction of u_2 only will need 0.05 times the input amplitude. Taking set point 1 and set point 2 which are simulated in previous section for example, the set point 1 $[-1.6, -1.2]^T$ is (–0.56, –2) in the coordinate system of which composed of the base vector v_1, v_2, and set point 2 $[-1.6, 1.2]^T$ is located at (–0.56, –2) in point (–2, 0). Therefore, if the set point 1 is selected, the controller action must be intensified, and this analysis is also proved in the previous simulation.

5 Strategy of modifying ill-conditioned model

By simulation and analysis in previous section, the main cause of ill-conditioned model appearance is that the minimum singular value of model gain matrix is too small, so the clear solution is shown as following.

1. make output not affected by movement direction of the ill-conditioned model, it means this strategy ensured magnitude of outputs in the minimum singular value corresponding to the projection of vector direction smaller, online strategy mentioned in introduction is used in terms of this idea
2. drastically reduce the ill-conditioned level of model by modifying and updating, which is used as the following strategy that is an offline approach.

After finished identification, the identified models are modified immediately, and qualified modified models will be configured as a controller model.

For the ill-conditioned model gain matrices compared with the ill-conditioned model, modelling errors of model gain coefficient seem unimportant, and it is possible that identified models have different models form for the same plant. Therefore, a model modified strategy is presented through increasing the minimum singular value and reducing model ill-condition by modifying model gain matrices (Hu et al., 2010; Courreges and Chassagnolle, 2014). The method is shown as following.

1. After models are identified, which subunits of original model is ill-condition will be examined by taking 2 × 2 sub-model as a basic unit, and condition number is taken as inspection index.
2. Define the error tolerance factor α, it represents which each model gain coefficient is allowed to change the amplitude size. The expression is $|\Delta k_{ij}| \le |k_{ij}| \times \alpha$, it is noted that α can be as allowed factors for special gain, and also be a unified factor for every gain.
3. At the beginning σ_{max} is selected as initial value of minimum singular value σ_{min}, and the iterative search is carried out by using the dichotomy in accordance with the golden section until the maximum allowable error range σ_{min} is found.
4. Update the models.

Here the error tolerance factor is set to 0.1, the modified model was

$$\boldsymbol{K}' = \begin{bmatrix} -12.3783 & 10.1623 \\ 10.1623 & -6.4503 \end{bmatrix}$$

by application of above method, where

$$\Delta\boldsymbol{K} = \boldsymbol{K}' - \boldsymbol{K} = \begin{bmatrix} 0.2417 & 0.3223 \\ 0.3223 & 0.4297 \end{bmatrix} \le 0.1 \times \boldsymbol{K}.$$

$\boldsymbol{K}'$ are configured to controller model, and the ±20% mismatch is assumed, which are $\boldsymbol{K} = \begin{bmatrix} -10.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}$ and $\boldsymbol{K}_1 = \begin{bmatrix} -15.1 & 9.84 \\ 9.84 & -6.88 \end{bmatrix}$ respectively.

Figure 12 Control effect comparison between modified model and unmodified model under mismatch mode 1 and set point one (see online version for colours)

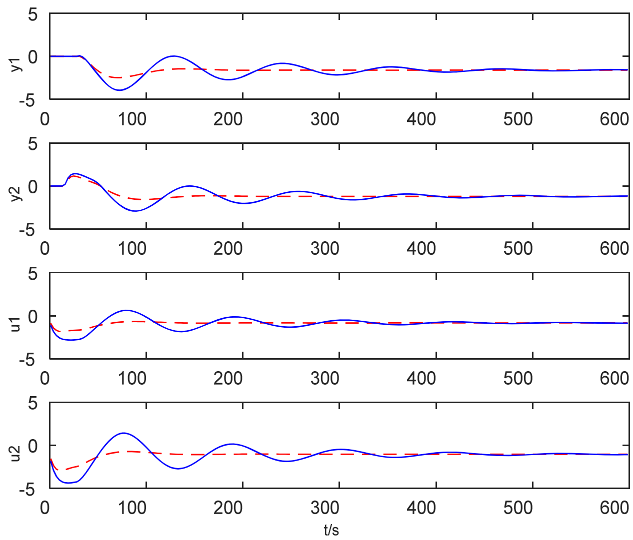


Figure 13 Control effect comparison between modified model and unmodified model under mismatch mode 1 and set point two (see online version for colours)

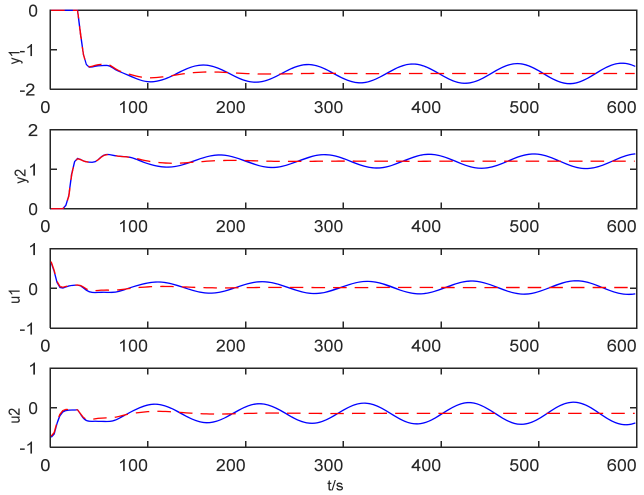


Figure 14 Control effect comparison between modified model and unmodified model under mismatch mode 2 and set point one (see online version for colours)

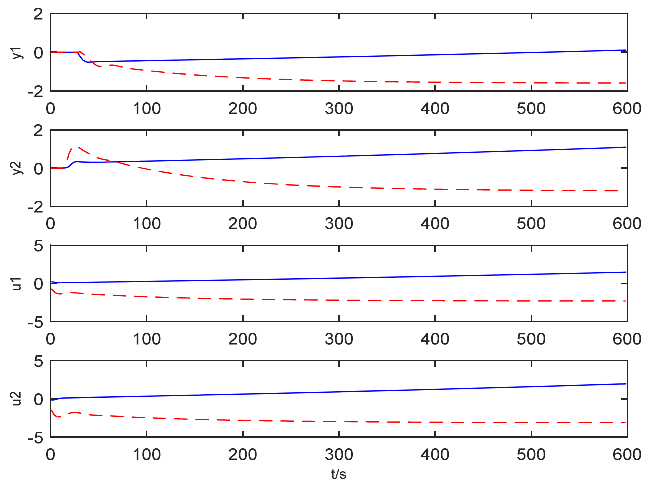
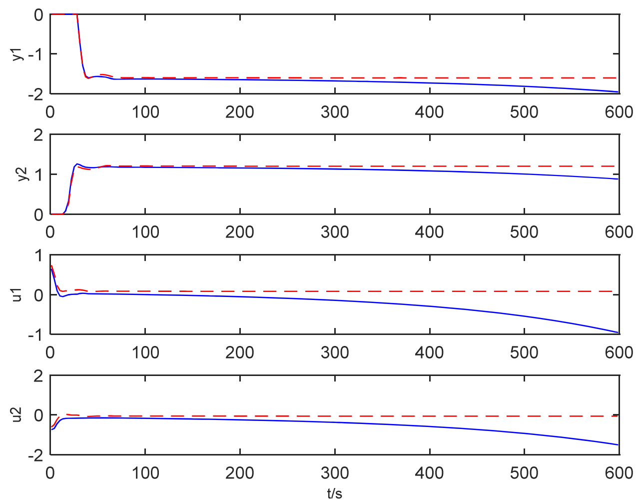


Figure 15 Control effect comparison between modified model and unmodified model under mismatch mode 2 and set point two (see online version for colours)



Using set point 1 and set point 2 to simulate under the mismatch mode 1 and mode 2, the control effects are shown as Figures 12 to 15.

In Figures 12 to 15, the solid lines represent input and output curves of unmodified models, and the dashed lines represent input and output curves of modified models. It is obviously seen that the oscillation amplitude and time are reduced significantly, while the set point following-up is realised quickly, and adverse effects because of the mismatch is corrected Therefore, our suggested strategy is simple and feasible.

6 Conclusions

The inherent strong correlation of processes and the presence of its modelling error through identification from certain input and output data are the root cause of ill-conditioned model problems. The ill-conditioned model will adversely affect the stability of multivariable predictive control, especially when some output set-point to move in certain directions, will further exacerbate instability of the controller, thus it is necessary to eliminate or reduce the bad impact upon the stability and performance of a control system which model is ill-conditioned during the course of implementing MPC projects. By geometric tools and Singular Value Analysis, the theory basis of ill-conditioned model which deteriorate the control result is given, and the new offline strategies and methods that can improve the dynamic performance of the closed-loop system to a certain extent are suggested. Finally, the solutions proposed in the present work are verified by simulation studies.

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