Robust stabilization of a class of time-delay nonlinear systems with Markovian jump

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Abstract: The problem of robust stabilization of time-delay nonlinear systems with Markovian jump with a triangular structure is studied. Based on backstepping method, we prove that the closed system is almost surely asymptotically stable and the controller and the associated Lyapunov function is independent of the generator of the Markov process.

Key Words: Backstepping method, time-delay nonlinear systems, Markovian jump

1 Introduction

There have been several examples showing the importance of dynamic systems subject to abrupt changes in their structures or parameers. This is in part due to the fact that very often dynamic systems are inherently vulnerable to component failures or repairs, sudden environmental disturbances, changing subsystem interconnections, abrupt variations of the operating point of a nonlinear plant, etc.. In many situations, Markovian jump systems comprise an important class of stochastic dynamic hybrird systems which can model the above problems. The theory of stability and control theorey has recently received a lot of attention, for example, in [9], [10], [11], [12], [1], [8] and the references therein.

It is well-known that time-delay cannot be avoid in practice, such as electrical networks, turbojet engines, microwave oscillators, nuclear reactors, hydraulic systems, and so on, and it often results in instability and poor performance. So far, the stability analysis and robust control for these dynamic time-delay systems have attracted a number of researchers over the past years, see,for example, [13], [14], [15], [16]. Similar to Markovian jump systems, more attentions are paid to linear time-delay systems and the results are often obtained in the form of linear matrix inequalities [17], [18]. Backstepping method can be used to construct both feedback control laws and associated Lyapunov functions systematically. S. K. Nguang [2] and Y. Fu [3] studied the state feedbak stabilization of a class of time-delay nonlinear systems using backstepping method. C. Hua [5] and Y. Fu [4] studied the output feedback stabilization of a class of time-delay nonlinear systems.

In this paper, we investigate the robust stabilization of a class of time-delay nonlinear systems with Markovian jump. The system considered is in the strict-feedback form which is more general than [14] and [3]. We show that the controller and associated Lyapunov function obtained are independent of the generator of the Markov process and the closed-loop system is almost surely asymptotically stable.

$Notations$: The notation used in this paper is quite standard. We use $\mathbb{R}^n$ to denote the n dimensional Euclidean space, $|A|$ to denote the 2-norm of a vector A, and A^T to denote the transpose of a matrix A. If A is square, we use $Tr(A)$ to denote its trace. And let $C^{2,1}(\mathbb{R}^n \times [-\tau,\infty)\times S;\mathbb{R}_+)$ denote the family of all nonnegative functions $V(x,t,i)$ on $\mathbb{R}^n \times [-\tau,\infty) \times S$ which are continuously twice differentiable in x and once differentiable in t, where $S = \{1,2,\cdots,N\}$ and N is a positive integer. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},\mathbb{P})$ be a complete probability space with a filtration $\{\mathscr{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is increasing and right continuous while $\mathscr{F}_0$ contains all $\mathbb{P}$-null sets), where $\mathbb{P}$ is the probability measure defined on the algebra of events $\mathscr{F}$, and we use $\mathbb{E}$ to denote the expectation operator. Let $\tau > 0$ and $C([-\tau,0];\mathbb{R}^n)$ denote the family of all continuous $\mathbb{R}^n$-valued functions ξ on $[-\tau,0]$ with the norm $\|\xi\| = \sup\{|\xi(s)|,-\tau \leq s \leq 0\}$. Let $C^b_{\mathscr{F}_0}([-\tau,0];\mathbb{R}^n)$ denote the family of all $\mathscr{F}_0$-measurable bounded $C([-\tau,0];\mathbb{R}^n)$-valued random variables $\xi = \{\xi(s),-\tau \leq s \leq 0\}$. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

2 Preliminaries

Consider the Markovian jump time-delay nonlinear systems

$$\begin{aligned} dx = f(x(t),x(t-d(t)),t,r(t))dt \\ +g(x(t),x(t-d(t)),t,r(t))dw, \quad t\geq 0 \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$ is the state with initial data $x_0 = \xi \in C^b_{\mathscr{F}_0}([-\tau,0];\mathbb{R}^n)$ for $-\tau \leq t \leq 0$, $d(t) \in \mathbb{R}_+$ is the time-varying time-delay which satisfy $\dot{d}(t) \leq \eta < 1$ and $d(0) = \tau > 0$. $r(t), t \geq 0$ is a right-continuous Markov chain on the probability space $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq 0},\mathbb{P})$ taking values in finite state space S. $w(t)$ is m-dimensional Wiener process defined on the same probability space which is independent of the Markov chain $r(t)$. f and g are locally Lipschitz continuous functions with proper dimensions with $f(0,0,t,i) = 0$ and $g(0,0,t,i) = 0$ for all $t \geq 0$ and $i = 1,2,\cdots,N$, so the equation admits a trivial solution $x(t;0) = 0$.

The generator of the Markov process r_t is $\Pi = (\pi_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+s)=j|r(s)=i\} = \begin{cases} \pi_{ij}t + o(t), & if \quad i\neq j \\ 1+\pi_{ii} + o(t), & if \quad i=j \end{cases}$$

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for any $s, t \geq 0$. Here $\pi_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while

$$\pi_{ii} = -\sum_{j=1, j\neq i}^{N} \pi_{ij}.$$

If $V \in C^{2,1}(\mathbb{R}^n \times [-\tau, \infty) \times S; \mathbb{R}_+)$, introduce the infinitesimal generator for the system (1) by

$$\begin{aligned}\mathcal{L}V(x,t,p) &= V_t(x,t,p) + V_x(x,t,p) f(x(t), x(t-d(t)), t, p) \\ &+ \sum_{q=1}^{N} \pi_{pq} V(x,t,q) + \frac{1}{2} Tr[g^T(x(t), x(t-d(t)), t, p) \\ &* V_{xx}(x,t,p) g(x(t), x(t-d(t)), t, p)]\end{aligned}$$

where

$$\begin{aligned} V_t(x,t,p) &= \frac{\partial V(x,t,p)}{\partial t}, \\ V_x(x,t,p) &= (\frac{\partial V(x,t,p)}{\partial x_1}, \cdots, \frac{\partial V(x,t,p)}{\partial x_n}), \\ V_{xx}(x,t,p) &= \left(\frac{\partial^2 V(x,t,p)}{\partial x_i x_j}_{n\times n}\right). \end{aligned}$$

1 *[8] The trivial solution of* (1) *is said to be almost surely asymptotically stable if for all* $\xi \in C^b_{\mathscr{F}_0}([-\tau, 0]; \mathbb{R}^n)$ *and* $i_0 \in S$

$$\mathbb{P}\{\lim_{t\to\infty} x(t;\xi) = 0\} = 1.$$

3 State Feedback Control

In this section, we consider the following Markovian jump time-delay nonlinear systems

$$\begin{aligned} dx_i &= (x_{i+1} + g_{i1}(\tilde{x}_i, r(t)) + g_{i2}(\tilde{x}_i, \tilde{x}_i(t-d(t)), r(t))dt \\ &+ (h_{i1}(\tilde{x}_i, r(t)) + h_{i2}(\tilde{x}_i, \tilde{x}_i(t-d(t)), r(t)))^T dw, \end{aligned}$$

$$\begin{aligned} dx_n &= (u + g_{n1}(\tilde{x}_n, r(t)) + g_{n2}(\tilde{x}_n, \tilde{x}_n(t-d(t)), t, r(t))dt \\ &+ (h_{n1}(\tilde{x}_n, r(t)) + h_{i2}(\tilde{x}_n, \tilde{x}_n(t-d(t)), r(t))^T dw \end{aligned} \quad (2)$$

where $i = 1, 2, ..., n-1$, $x = (x_1, x_2, \cdots, x_n)^T \in \mathbb{R}^n$ is the state vector, and $\tilde{x}_i = (x_1, \cdots, x_i)^T, i = 1, \cdots, n$. $u \in \mathbb{R}$ is the input of the system. Nonlinear uncertain functions $g_{ij}, h_{ij}, i = 1, \cdots, n, j = 1, 2$ are assumed to satisfy the following conditions [2].

Assumption 1: $|g_{i1}(y,p)| \leq \bar{g}_i(y), |h_{i1}(y,p)| \leq \bar{h}_i(y), i = 1 \cdots, n, p = 1 \cdots, N$ where $\bar{g}_i, \bar{h}_i$ are known smooth nonlinear functions with $\bar{g}_i(0) = 0, \bar{h}_i(0) = 0$.

Assumption 2: $|g_{i2}(y_1, y_2, p)| \leq G_{i1}(y_1) + G_{i2}(y_2), |h_{i2}(y_1, y_2, p)| \leq H_{i1}(y_1) + H_{i2}(y_2)$, $i = 1 \cdots, n, p = 1 \cdots, N$ where $G_{i1}, G_{i2}, H_{i1}, H_{i2}$ are known smooth nonlinear functions with $G_{ij}(0) = 0, H_{ij}(0) = 0, j = 1, 2$.

Under Assumptions 1 and 2, we shall deal with a state feedback stabilizing problem for the Markovian jump time-delay nonlinear systems (2). In the backstepping design, the error variables z_i are give by

$$z_i = x_i - \alpha_{i-1}(\tilde{x}_{i-1}), \quad i = 1, \cdots, n$$

with $\alpha_0 \equiv 0$ and α_i are continuous functions to be determined later. Because $g_{ij}(0) = 0, h_{ij}(0) = 0$, it is required that α_i vanish at $\tilde{x}_i = 0$ as well as $\tilde{z}_i = 0$, where $\tilde{z}_i = [z_1, \cdots, z_i]$. Thus, we can have the following decompositions

$$\begin{aligned} \alpha_i(\tilde{x}_i) &= \sum_{j=1}^{i} z_j \alpha_{ij}(\tilde{x}_j), \\ \bar{g}_i(\tilde{x}_i) &= \sum_{j=1}^{i} z_j \bar{g}_{ij}(\tilde{x}_j), \\ \bar{h}_i(\tilde{x}_i) &= \sum_{j=1}^{i} z_j \bar{h}_{ij}(\tilde{x}_j) \\ G_{ik}(\tilde{x}_i) &= \sum_{j=1}^{i} z_j G_{ijk}(\tilde{x}_j), \\ H_{ik}(\tilde{x}_i) &= \sum_{j=1}^{i} z_j H_{ijk}(\tilde{x}_j), \\ & k = 1, 2 \end{aligned}$$

According to $It\hat{o}$'s differential rule, we have

$$\begin{aligned} dz_i &= d(x_i - \alpha_{i-1}(\tilde{x}_{i-1})) \\ &= (x_{i+1} + g_{i1} + g_{i2} \\ &- \frac{1}{2}\sum_{j,k=1}^{i-1} \frac{\partial^2 \alpha_{i-1}}{\partial x_j \partial x_k}(h_{j1} + h_{j2})^T (h_{k1} + h_{k2}). \\ &- \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(x_j + g_{j1} + g_{j2}))dt \\ &+ \left((h_{i1} + h_{i2})^T - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(h_{j1} + h_{j2})^T\right) dw \end{aligned}$$

where $x_{n+1} = u$. We employ a Lyapunov function of the form

$$V = \frac{1}{4}\sum_{i=1}^{n} z_i^4 + \int_{t-d(t)}^{t} q(z(s))ds$$

where $q(z)$ is a positive function to be determined. Along the trajectories of (2), we have

$$
\begin{aligned}
\mathcal{L}V &= \sum_{i=1}^{n-1} z_i^3(x_{i+1}+g_{i1}+g_{i2}\\
&- \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}(h_{j1}+h_{j2})^T(h_{k1}+h_{k2})\\
&- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_j+g_{j1}+g_{j2}))dt\\
&+ z_n^3\left(-\frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}(h_{j1}+h_{j2})^T(h_{k1}+h_{k2})\right.\\
&- \left.\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_j+g_{j1}+g_{j2})+u+g_{n1}+g_{n2}\right)\\
&+ \frac{3}{2}\sum_{i=1}^{n} z_i^2\left((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T\right)^T\\
&\times ((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T)\\
&+ q(z(t))-(1-\dot{d}(t))q(z(t-d(t)))\\
&= \sum_{i=1}^{n-1} z_i^3(\alpha_i+g_{i1}+g_{i2}\\
&- \frac{1}{2}\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}(h_{j1}+h_{j2})^T(h_{k1}+h_{k2})\\
&- \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(x_j+g_{j1}+g_{j2}))dt\\
&+ z_n^3(-\frac{1}{2}\sum_{j,k=1}^{n-1}\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}(h_{j1}+h_{j2})^T(h_{k1}+h_{k2})\\
&- \left.\sum_{j=1}^{n-1}\frac{\partial\alpha_{n-1}}{\partial x_j}(x_j+g_{j1}+g_{j2})+u+g_{n1}+g_{n2}\right)\\
&+ \sum_{i=1}^{n-1} z_i^3 z_{i+1}\\
&+ \frac{3}{2}\sum_{i=1}^{n} z_i^2\left((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T\right)^T\\
&\times \left((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T\right)\\
&+ q(z(t))-(1-\dot{d}(t))q(z(t-d(t))).
\end{aligned}
$$

By the assumption and Young's inequalities, we have

$$
\begin{aligned}
z_i^3 g_{i1} &\le |z_i^3||\bar{g}_i| = |z_i^3|\left|\sum_{j=1}^{i} z_j\bar{g}_{ij}\right| \le \frac{3i}{4}z_i^4\left(\bar{g}_{ij}^{4/3}\right)+\sum_{j=1}^{i}\frac{1}{4}z_j^4\\
z_i^3 g_{i2} &\le |z_i^3|(G_{i1}+G_{i2}) = |z_i^3|\left(\sum_{j=1}^{i} z_jG_{ij1}+\sum_{j=1}^{i} z_jG_{ij2}\right)\\
&\le \frac{3i}{4}z_i^4\sum_{j=1}^{i}\left(G_{ij1}^{4/3}+G_{ij2}^{4/3}\right)+\sum_{j=1}^{i}\frac{1}{2}z_j^4\\
z_i^3 z_{i+1} &\le \frac{3}{4}z_i^4+\frac{1}{4}z_{i+1}^4
\end{aligned}
$$

and

$$
\begin{aligned}
&-\frac{1}{2}z_i^3\sum_{j,k=1}^{i-1}\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}(h_{j1}+h_{j2})^T(h_{k1}+h_{k2})\\
&\le \sum_{j,k=1}^{i-1}\left|z_i^3\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\right|(h_{j1}^Th_{j1}+h_{k2}^Th_{k2})\\
&\le \sum_{j,k=1}^{i-1}\left|z_i^3\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\right|(\bar{h}_j\sum_{l=1}^{j}|z_j||\bar{h}_{jl}|\\
&+ 2H_{k_1}\sum_{l=1}^{k}|z_k||H_{kl1}|+2H_{k2}\sum_{l=1}^{k}|z_k||H_{kl2}|)\\
&\le \sum_{j,k=1}^{i-1}(\frac{15}{4}z_i^4\left|\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\right|^{4/3}\\
&+ \frac{1}{4}\sum_{l=1}^{j} z_j^4\bar{h}_{jl}^4\bar{h}_j^4+\frac{1}{2}\sum_{l=1}^{k} z_k^4H_{kl1}^4H_{k_1}^4+\frac{1}{2}\sum_{l=1}^{k} z_k^4H_{kl2}^4H_{k_2}^4\\
&= \frac{15}{4}z_i^4\sum_{j,k=1}^{i-1}\left|\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\right|^{4/3}\\
&+ \frac{1}{4}(i-1)\sum_{j=1}^{i-1} z_j^4\left(\sum_{l=1}^{j}\left(\bar{h}_{jl}^4\bar{h}_j^4+2H_{jl1}^4H_{j_1}^4+2H_{jl2}^4H_{j_2}^4\right)\right)
\end{aligned}
$$

and

$$
\begin{aligned}
&-z_i^3\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(g_{j1}+g_{j2})\\
&\le \sum_{j=1}^{i-1}|z_i^3|\left|\frac{\partial\alpha_{i-1}}{\partial x_j}\right|(\bar{g}_j+G_{j1}+G_{j2})\\
&\le \sum_{j=1}^{i-1}\frac{9(i-1)}{4}z_i^4+\frac{1}{4}\sum_{j=1}^{i-1}\sum_{k=1}^{j} z_j^4\left(\left(\bar{g}_{jk}^{4/3}\right)+\left(G_{jk1}^{4/3}+G_{jk2}^{4/3}\right)\right)
\end{aligned}
$$

and

$$
\begin{aligned}
& z_i^2\left((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T\right)^T \\
\times & \left((h_{i1}+h_{i2})^T-\sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(h_{j1}+h_{j2})^T\right) \\
\leq & z_i^2(4h_{i1}^Th_{i1}+4h_{i2}^Th_{i2} \\
+ & \sum_{j=1}^{i-1}\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^2(4h_{j1}^Th_{j1}+4h_{j2}^Th_{j2})) \\
\leq & 4z_i^2(\left(\sum_{j=1}^{i}z_j\bar{h}_{ij}\right)^2+2\left(\sum_{j=1}^{i}z_jH_{ij1}\right)^2 \\
+ & 2\left(\sum_{j=1}^{i}z_jH_{ij2}\right)^2) \\
+ & 4z_i^2\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2(\left(\sum_{k=1}^{j}z_k\bar{h}_{jk}\right)^2+2\left(\sum_{k=1}^{j}z_kH_{jk1}\right)^2 \\
+ & 2\left(\sum_{k=1}^{j}z_kH_{jk2}\right)^2) \\
\leq & 8iz_i^2(\sum_{j=1}^{i}z_j^2\bar{h}_{ij}^2+2\sum_{j=1}^{i}z_j^2H_{ij1}^2+2\sum_{j=1}^{i}z_j^2H_{ij2}^2) \\
+ & 8iz_i^2\sum_{j=1}^{i-1}(\frac{\partial\alpha_{i-1}}{\partial x_j})^2(\sum_{k=1}^{j}z_k^2\bar{h}_{jk}^2+2\sum_{k=1}^{j}z_k^2H_{jk1}^2 \\
+ & 2\sum_{k=1}^{j}z_k^2H_{jk2}^2) \\
\leq & 4i\sum_{j=1}^{i}\left(5z_j^4+z_i^4\bar{h}_{ij}^4+2z_i^4H_{ij1}^4+2z_i^4H_{ij2}^4\right) \\
+ & 4i\sum_{j=1}^{i-1}\left(5jz_j^4+z_i^4\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4\right. \\
& \left.\left(\sum_{k=1}^{j}\bar{h}_{jk}^4+2\sum_{k=1}^{j}H_{jk1}^4+2\sum_{k=1}^{j}H_{jk2}^4\right)\right)
\end{aligned}
$$

If we choose q, α_i and u as

$$
\begin{aligned}
q = & \frac{1}{1-\eta}\sum_{i=1}^{n}\left(\frac{3i}{4}z_i^4\sum_{j=1}^{i}G_{ij2}^{4/3}\right. \\
& +\frac{1}{4}(i-1)\sum_{j=1}^{i-1}z_j^4\sum_{l=1}^{j}2H_{jl2}^4H_{j_2}^4 \\
+ & \frac{1}{4}\sum_{j=1}^{i-1}\sum_{k=1}^{j}z_j^4G_{jk2}^{4/3}+4i\sum_{j=1}^{i}2z_i^4H_{ij2}^4 \\
+ & \left.8i\sum_{j=1}^{i-1}z_i^4\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4\sum_{k=1}^{j}H_{jk2}^4\right)
\end{aligned}
$$

$$
\begin{aligned}
\alpha_i = & -z_i\left(c_i+\frac{3i}{4}\bar{g}_{ij}^{4/3}+\frac{3i}{4}\sum_{j=1}^{i}\left(G_{ij1}^{4/3}+\frac{1}{1-\eta}G_{ij2}^{4/3}\right)\right. \\
+ & \frac{15}{4}\sum_{j,k=1}^{i-1}\left|\frac{\partial^2\alpha_{i-1}}{\partial x_j\partial x_k}\right|^{4/3}+\frac{9}{4}(i-1)(n-1)+1 \\
+ & 20i(2n+1-2i)+\frac{3}{4}(n+1-i) \\
+ & \frac{1}{4}(i-1)(n-i)\sum_{j=1}^{i}\left(\bar{h}_{ij}^4\bar{h}_i^4+2H_{ij1}^4H_{i1}^4+\frac{2}{1-\eta}H_{ij2}^4H_{i2}^4\right) \\
+ & \frac{1}{4}(i-1)(n-i)\sum_{j=1}^{i}\left(\bar{g}_{ij}^{4/3}+G_{ij1}^{4/3}+\frac{1}{1-\eta}G_{ij2}^{4/3}\right) \\
+ & 4i(n+1-i)\sum_{j=1}^{i}\left(\bar{h}_{ij}^4+2H_{ij1}^4+\frac{2}{1-\eta}H_{ij2}^4\right) \\
+ & 4i(n-i)\sum_{j=1}^{i-1}\left(\left(\frac{\partial\alpha_{i-1}}{\partial x_j}\right)^4\sum_{k=1}^{j}\bar{h}_{jk}^4+2\sum_{k=1}^{j}H_{jk1}^4\right. \\
+ & \left.\left.\frac{2}{1-\eta}\sum_{k=1}^{j}H_{jk2}^4\right)\right)
\end{aligned}
$$

$$
\begin{aligned}
u = & -z_n\left(c_n+\frac{3n}{4}\bar{g}_{nj}^{4/3}+\frac{3n}{4}\sum_{j=1}^{n}\left(G_{nj1}^{4/3}+\frac{1}{1-\eta}G_{nj2}^{4/3}\right)\right. \\
+ & \frac{15}{4}\sum_{j,k=1}^{n-1}\left|\frac{\partial^2\alpha_{n-1}}{\partial x_j\partial x_k}\right|^{4/3}+\frac{9}{4}(n-1)^2+20n \\
+ & \left.\frac{7}{4}+4n\sum_{j=1}^{n}\left(\bar{h}_{nj}^4+2H_{nj1}^4+\frac{2}{1-\eta}H_{nj2}^4\right)\right)
\end{aligned}
$$

where $c_i > 0$. We can see that α_i is only related to $\bar{g}_j, \bar{h}_j, G_{jk}, H_{jk}, j = 1, \cdots, i, k = 1, 2$ and those are only related to $\alpha_l, l = 1, \cdots, i-1$. And thus we conclude that the recursive design is effective. Then the infinitesimal generator of the closed-loop system is negative definite

$$\mathcal{L}V \leq -\sum_{i=1}^{n}c_iz_i^4.$$

Therefore, the closed-loop system is almost surely asymptotically stable [8].

4 Example

Now we give an example to show the result, we consider the following Markovian jump time-delay nonlinear system

$$
\begin{aligned}
dx_1 = & \left(x_1+x_1^2\times sinr(t)+x_1^2+(x_1\times(t-d(t))^2+sin(t)\right)d \\
+ & (x_1^2\times sinr(t)+x_1^2+[x_1\times(t-dt)]^2+sin(t))dw
\end{aligned}
$$

$$
\begin{aligned}
dx_2 = & (u+x_1^2\times x_2^2\times sinr(t)+(x_1+\times x_2)^2 \\
+ & (x_1\times x_2\times(t-d(t))^2)+sinr(t))dt \\
+ & \left(x_1^2\times x_2^2\times sinr(t)+(x_1\times x_2)^2\right. \\
+ & \left.(x_1\times x_2\times(t-d(t)))^2+sinr(t)\right)dw \quad (3)
\end{aligned}
$$

Let us suppose the state space of Markov process is $1, 2, 3$ and the state transition problem matrix is

$$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.1 & 0.5 & 0.4 \\ 0.05 & 0.35 & 0.6 \end{bmatrix} \quad (4)$$

it is easy to proof the system (3) satisfy $Assumption$ 1 and $Assumption$ 2. Through compute the system, we can get the following result : from the figure $Fig.1$, we can know the fact the state of the system x_1 and x_2 both become 0 after 15 seconds. Thus the system is asymptotically stable.

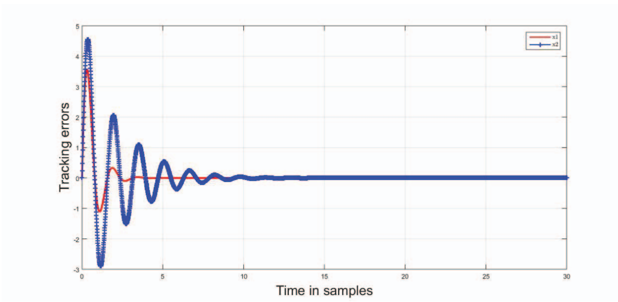

Fig. 1: Example figure

5 Conclusion

In this paper, we discuss the problem of robust stabilization of time-delay nonlinear systems with Markovian jump with a triangular structure by a Lyapunov-base recursive design approach. We proved that the closed-loop system is almost surely asymptotically stable. The method in this note can be extended to the output feedback stabilization of a similar system.

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