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Heterogeneous coexistence between cognitive radio networks: a Markovian jump system method

Jijun Guan[1] · Meng Zheng[2] · Qingling Zhang[1]

Abstract With the increasing demand of wireless spectrum, different unlicensed wireless communication technologies have been applied in the television white space (TVWS). It is vital to understand that the mutual interference over TVWS due to incompatible protocol designs heavily degrades the quality of service of coexisting heterogeneous cognitive radio networks. In this paper, taking the activity of primary users into consideration, we formulate the heterogeneous coexistence problem over TVWS as a nonlinear Markovian jump system (NMJS) based on the Lotka–Volterra competition model. By using the local linearization method, we first obtain a linear Markovian jump system model (which approximates the NMJS linearly at the desired spectrum share) to the NMJS. Further, we obtain an effective feedback controller to the equilibrium assignment of the NMJS via solving a sufficient condition in the form of linear matrix inequalities.

Third, we propose an IEEE 802.19.1-compatible spectrum sharing algorithm which enables the NMJS to converge to the assigned spectrum share. Finally, extensive simulations are conducted to validate the effectiveness of our proposals.

Keywords Cognitive radio · Heterogeneous coexistence · IEEE 802.19.1 · Markovian jump systems

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1 Introduction

With the proliferation of wireless devices and communication traffic, the spectrum congestion problem over the industrial, scientific, and medical (ISM) bands is becoming critical. To increase the quality of service (QoS) of unlicensed wireless devices, regulators are opening up television white space (TVWS) to unlicensed wireless devices which are allowed to access the TVWS opportunistically via cognitive radio technology [1,2]. However, typical standards on cognitive radio networks (CRNs) including IEEE 802.22 [3], IEEE 802.11af [4], IEEE 802.16h [5] and ECMA392 [6], create the scenario in which different cognitive radio technologies coexist in the same TVWS spectrum, and consequently the mutual interference between heterogeneous CRNs (H-CRNs) will occur due to their incompatible MAC/PHY designs. Therefore, the harmonious coexistence between H-CRNs over TVWS (termed as heterogeneous coexistence) is essential to maintain the QoS of coexisting H-CRNs [7–9].

Existing works on heterogeneous coexistence can be summarized from two aspects. On one hand, autonomous schemes are suitable for the case in which the coordination between coexisting H-CRNs is unavailable [10–13]. New coexistence protocols [10,11] or novel decision making algo-

rithms [12,13] have been proposed to ensure fair channel access for coexisting H-CRNs. On the other hand, centralized schemes are appealing for the case in which a mediator system is available for coordinating the operations of coexisting H-CRNs. The IEEE 802.19.1 task group propose a centralized scheme [8,14] to coordinate spectrum sharing between H-CRNs. Furthermore, decision making algorithms inspired by Lotka–Volterra (L–V) competition model in the field of ecology (such as FACT [15], SHARE [16], SCHEME [17] and SSAA [18]) for an IEEE 802.19.1-compliant system are proposed. Owing to the official release of the IEEE 802.19.1 standard, centralized schemes will gain more interests. However, the existing works aforementioned assume that the TVWS spectrum is static during the spectrum sharing process. In fact, the state of the TVWS spectrum highly depends on the activity of primary users (PUs).

Motivated by above discussion, this paper removes the assumption on the static TVWS spectrum and adopts the commonly used Markov chain model [19,20] to characterize the activity of PUs, as a result of which the available bandwidth of the TVWS spectrum is also Markovian. For each operation mode, we adopt the L–V competition model [21,22] to characterize the spectrum dynamics of H-CRNs. Therefore, the heterogeneous coexistence problem is formulated as a nonlinear Markovian jump system (NMJS) model. By using the local linearization method, we obtain a linear approximation model to the NMJS and derive a sufficient condition in the form of linear matrix inequalities (LMIs) to the equilibrium assignment problem of the NMJS. We then find an effective feedback controller to the NMJS via solving the set of LMIs. Finally, an IEEE 802.19.1-compatible spectrum sharing (ICSS) algorithm is proposed to drive the NMJS to converge to a desired spectrum share. Extensive simulations demonstrate the effectiveness of this paper.

The contributions of this paper are threefold:

– The dynamic characteristic of the TVWS spectrum is for the first time considered in the heterogeneous coexistence problem and a novel NMJS model is proposed to formulate the heterogeneous coexistence problem.
– A Markovian jump system method is employed to find a feedback controller to the equilibrium assignment problem of the NMJS.
– An ICSS algorithm with the mean square convergence to any desired spectrum share is designed.

The rest of this paper is organized as follows. Section 2 reviews related works. Section 3 gives out the system model of the heterogeneous coexistence problem. Section 4 proposes an ICSS algorithm to the heterogeneous coexistence problem. Section 5 presents simulation results. Section 6 concludes this paper.

2 Related work

Self-coexistence schemes [23–25] are employed when the operations of coexisting networks can be directly coordinated. Although the direct coordination between coexisting networks in self-coexistence schemes may facilitate the spectrum sharing process, it requires the exchange of potentially sensitive information, such as traffic load and bandwidth requirement, across different networks. As a result, the issue of conflict-of-interest and the concern of customer privacy for competing service providers could be raised.

Different from self-coexistence schemes [23–25], coexistence schemes of H-CRNs have to modify the MAC layer of coexisting H-CRNs (autonomous schemes) or take advantage of a mediator system (centralized schemes) to facilitate the spectrum sharing process.

– *Autonomous schemes* Bian et al. [10] present a coexistence protocol utilizing beacon transmission and dynamic quiet periods to avoid packet collisions, in order to mitigate the hidden terminal problem for the coexistence between time-division-multiplexing and carrier-sense multiple-access networks. Cheng et al. [11] propose a jamming-based probing function that enhances the PU-detection ability and the fairness feature, and a decentralized MAC protocol by combing the probing function and previous CRN MAC protocols. Tseng et al. [12] study the problem of self-organized network selection in heterogeneous networks with time-varying channel availability and unknown number of secondary users by an ordinal potential game, and propose a decentralized stochastic learning based algorithm to solve it. Cacciapuoti et al. [13] design an optimal coexistence strategy that adaptively and autonomously selects the channel maximizing the expected throughput of multiple heterogeneous and independently-operated unlicensed networks.
– *Centralized schemes* Bahrak et al. [15] for the first time formulate the coexistence decision making (CDM) problem as a multi-objective combinatorial optimization problem with a set of critical constraints, each corresponding to an important prerequisite for the coexistence of H-CRNs. Based on a modified Boltzmann machine, a CDM algorithm-FACT is proposed to find a Pareto optimal solution to the CDM problem. The FACT algorithm is shown compatible with centralized coexistence enabling systems, such as the IEEE 802.19.1 system. Bian et al. [16] build an architecture SHARE for enabling harmonious heterogeneous coexistence. The SHARE leverages an IEEE 802.19.1 system to establish an indirect coordination mechanism for coexisting H-CRNs and employs an ecology-inspired spectrum sharing algorithm to achieve a weighted fair spectrum sharing allocation

among H-CRNs. Zhang et al. [17] generalize [16] by considering a QoS parameter and different mutual interference factors in the spectrum competition model. Yin et al. [18] for the first time apply nonlinear control theory to the heterogeneous coexistence problem and propose an SSAA which can allocate a feasible spectrum share with a desired fairness for each H-CRN.

Although this paper is similar to [16–18], there is one fundamental difference between this paper and [16–18]. This paper removes the assumption of static TVWS spectrum during the spectrum sharing process in [16–18], and uses the typical Markov chain model to characterize the dynamics of the TVWS spectrum due to PUs. To this end, a completely novel NMJS model and a new solution method are for the first time proposed in this paper.

Notice that, there are another branch of related works which study heterogeneous coexistence over unlicensed bands [26–28]. As we all know, coexisting networks on licensed bands are all with the same priority [26–28], while in this paper H-CRNs opportunistically access the TVWS spectrum to protect the PUs. In other words, this paper not only solves the coexistence between H-CRNs, but also takes care of PUs. Therefore, this paper investigates a more complex and challenging problem than [26–28].

3 Problem description

3.1 System model

As shown in Fig. 1, we consider N H-CRNs which coexist in the same spectrum band of TVWS. Let $\mathcal{N} = \{1, 2, \ldots, N\}$ denote the set of H-CRNs. As the QoS requirement of one CRN closely depends on its allocated bandwidth, we focus on the dynamics of the spectrum share for each H-CRN. Inspired by [16] in which the mapping between biological ecosystems and coexisting H-CRNs is given, we use the
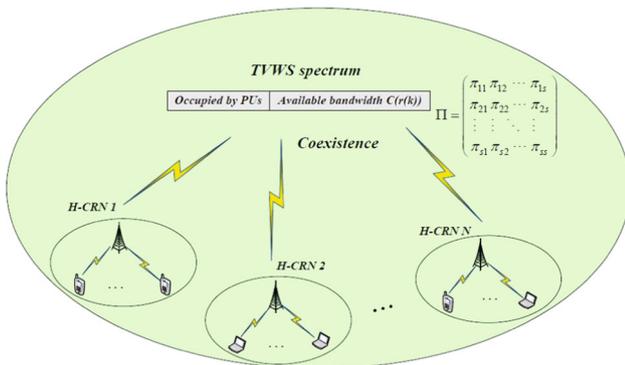

Fig. 1 System architecture

discrete L–V competition model to characterize the dynamics of spectrum share for each H-CRN. Let $s_m(k)$ denote the spectrum share of H-CRN m ($m \in \mathcal{N}$) and $s(k) = (s_1(k), s_2(k), \ldots, s_N(k))^T$ denote the spectrum share vector of H-CRNs. Specifically,

$$
\begin{aligned}
s_m(k+1) &= s_m(k) + f_m(s(k); r(k)) \\
&= s_m(k) + \gamma_m s_m(k) \left(1 - \frac{s_m(k) + \sum_{n \neq m} \alpha_{mn} s_n(k)}{C(r(k))} \right),
\end{aligned}
\tag{1}
$$

where $C(r(k))$ is the available spectrum bandwidth in the k-th iteration and $\{r(k), k \in \mathbb{Z}^+\}$ represents a discrete time, finite-state Markov chain taking values in a finite set $\Xi = \{1, 2, \ldots, s\}$ with transition probability matrix $\Pi \triangleq (\pi_{ij})_{i,j \in \Xi}$. For all $i, j \in \Xi$ and $k \in \mathbb{Z}^+$, we have $\pi_{ij} \triangleq \Pr\{r(k+1) = j | r(k) = i\} \geq 0$ and $\sum_{j=1}^{s} \pi_{ij} = 1$ for each i. For simplicity, we assume Π is perfectly available for H-CRNs. γ_m and α_{mn} are two parameters respectively termed as intrinsic rate of increase for spices m and competition coefficients between spices n and spices m in the L–V competition model.

For concise presentation, we can rewrite (1) in a more compact form

$$
s(k+1) = s(k) + f_i(s(k)), \ i \in \Xi,
\tag{2}
$$

where $f_i(s(k)) \triangleq f(s(k); r(k) = i)$ and $f(s(k); r(k) = i) = (f_1(s(k); i), f_2(s(k); i), \ldots, f_N(s(k); i))^T$.

3.2 Equilibrium assignment

This paper employs the network utility maximization (NUM) framework to allocate spectrum between H-CRNs, where the family of utility functions parameterized by $a \geq 0$ is defined [29]:

$$
U_a(x) = \begin{cases} \frac{x^{1-a}}{1-a}, & a \neq 1 \\ \log x, & a = 1. \end{cases}
\tag{3}
$$

We can achieve different types of fairness by selecting different values of a. For example, when $a = 1, 2$, and ∞, the utility function is guaranteed to achieve proportional fairness, harmonic mean fairness and max–min fairness, respectively.

Additionally, we have to consider the capacity constraint, i.e.,

$$
\sum_{m \in \mathcal{N}} s_m(k) \leq C_{min},
\tag{4}
$$

where $C_{min} > 0$ is the minimum of the available TVWS bandwidth, i.e., $C_{min} = \min_{i \in \Xi} C_i$.

Finally, the equilibrium $s^* = \left(s_1^*, \ldots, s_N^*\right)^{\mathrm{T}}$ with the desired fairness can be obtained by solving the following NUM problem

$$
\begin{aligned}
&\max_{s_m} \sum_{m \in \mathcal{N}} w_m U_a(s_m) \\
&s.t. \sum_{m \in \mathcal{N}} s_m \leq C_{min},
\end{aligned}
\tag{5}
$$

where w_m is the weight associated with $U_a(s_m)$. Problem (5) is convex and thus can be efficiently solved by solution methods such as interior-point methods in polynomial-time.

4 Spectrum sharing algorithm

Obviously, the autonomous NMJS (2) cannot converge to an arbitrarily assigned spectrum share s^*. First, we design an effective feedback controller to the equilibrium assignment problem of the NMJS (2), and then propose a spectrum sharing allocation algorithm based on a deliberately designed controller which drives the NMJS (2) to converge to the pre-assigned spectrum share s^*.

4.1 Controller design

This section aims to obtain a mode dependent feedback controller $u_i(k) = -L_i s(k)$ ($L_i \in \mathbb{R}^{N \times N}$) to the equilibrium

For convenience, we define $G_i(s(k))$ as follows:

$$
G_i(s(k)) = s(k+1) - s(k) = f_i(s(k)) - L_i s(k).
\tag{8}
$$

Obviously, at the equilibrium s^* we have

$$
G_i(s^*) = s^* - s^* = f_i(s^*) - L_i s^* = 0.
\tag{9}
$$

By using the local linearization method, we find a linear Markovian jump system (LMJS) model which is a linear approximation to the NMJS (6) by calculating the first order Taylor series expansion of $G_i(s(k))$ at the target equilibrium s^*

$$
s(k+1) - s(k) = G_i(s^*) + \frac{\partial G_i(s(k))}{\partial s^{\mathrm{T}}(k)}|_{s(k)=s^*}(s(k) - s^*).
\tag{10}
$$

Combining (9) and (10), we arrive the state-space model for $e(k)$

$$
e(k+1) = (A_i + I - L_i)e(k), \ i \in \Xi,
\tag{11}
$$

where A_i is defined in (12). System (11) is in fact a discrete-time LMJS with s operation modes. Next, we will apply Markovian jump system theory [30] to derive a sufficient condition to the mean square stability of the LMJS (11).

$$
A_i = \gamma_m \begin{pmatrix}
1 - \frac{2s_1^* + \sum_{n \neq 1} \alpha_{1n} s_n^*}{C_i} & -\frac{\alpha_{12} s_1^*}{C_i} & \cdots & -\frac{\alpha_{1N} s_1^*}{C_i} \\
-\frac{\alpha_{21} s_2^*}{C_i} & 1 - \frac{2s_2^* + \sum_{n \neq 2} \alpha_{2n} s_n^*}{C_i} & \cdots & -\frac{\alpha_{2N} s_2^*}{C_i} \\
\vdots & \vdots & \ddots & \vdots \\
-\frac{\alpha_{N1} s_N^*}{C_i} & -\frac{\alpha_{N2} s_N^*}{C_i} & \cdots & 1 - \frac{2s_N^* + \sum_{n \neq N} \alpha_{Nn} s_n^*}{C_i}
\end{pmatrix}_{N \times N}.
\tag{12}
$$

s^* assignment of the NMJS (2). Specifically, we will design a gain matrix L_i for the NMJS (2)

$$
s(k+1) = s(k) + f_i(s(k)) - L_i s(k), \ i \in \Xi,
\tag{6}
$$

such that the tracking error $e(k) = s(k) - s^*$ converges to zero as $k \to \infty$ in the mean square sense, i.e.,

$$
\lim_{k \to \infty} \mathbb{E} \| e(k; e(0)) \|^2 = 0,
\tag{7}
$$

for any initial $e(0) \in \mathbb{R}^N$.

Theorem 1 *The tracking error $e(k)$ in (11) converges to zero in the mean square sense if there exist positive-definite matrices P_i and matrices $R_{ji} \{i, j \in \Xi\}$ of appropriate dimensions such that the following LMIs are feasible:*

$$
\begin{pmatrix}
-P_1 & \mathbf{0} & \cdots & \mathbf{0} & \sqrt{\pi_{i1}}(P_1(A_i + I) - R_{1i}) \\
* & -P_2 & \cdots & \mathbf{0} & \sqrt{\pi_{i2}}(P_2(A_i + I) - R_{2i}) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
* & * & \cdots & -P_s & \sqrt{\pi_{is}}(P_s(A_i + I) - R_{si}) \\
* & * & \cdots & * & -P_i
\end{pmatrix} < 0
\tag{13}
$$

and

$$P_j f(s^*) - R_{ji} s^* = 0 \qquad (14)$$

where the symbol $*$ in (13) denotes the matrix transpose of the upper-triangular matrix blocks.

Proof Consider the following mode-dependent candidate Lyapunov function:

$$V_i(k) = e^{\mathrm{T}}(k) P_i e(k), \ P_i > 0, \ i \in \Xi. \qquad (15)$$

According to the Markovian jump system theory [30], $e(k)$ is mean square stable if $V_i(k) > 0$ and $\Delta V_i(k) < 0$ for all $e(k) \neq 0$. Obviously, $V_i(k) > 0$ holds for all $e(k) \neq 0$ since P_i is positive-definite.

Next, we calculate $\Delta V_i(k)$

$$
\begin{aligned}
&\Delta V_i(k) \\
&= E[V(e(k+1), r(k+1)) \mid e(k), r(k) = i] - V_i(k) \\
&= e^{\mathrm{T}}(k+1)\overline{P}_i e(k+1) - e^{\mathrm{T}}(k) P_i e(k),
\end{aligned}
\qquad (16)
$$

where $E(\cdot)$ stands for the mathematical expectation operator and $\overline{P}_i = \sum_{j=1}^{s} \pi_{ij} P_j$.

Further, by plugging (11) into (16), we conclude $\Delta V_i(k) < 0$ if and only if

$$\sum_{j=1}^{s} \pi_{ij} (A_i + I - L_i)^{\mathrm{T}} P_j (A_i + I - L_i) - P_i < 0. \qquad (17)$$

Applying Schur complement lemma to (17), we have

$$
\begin{pmatrix}
-P_1 & \mathbf{0} & \cdots & \mathbf{0} & \sqrt{\pi_{i1}} P_1(A_i + I - L_i) \\
* & -P_2 & \cdots & \mathbf{0} & \sqrt{\pi_{i2}} P_2(A_i + I - L_i) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
* & * & \cdots & -P_s & \sqrt{\pi_{is}} P_s(A_i + I - L_i) \\
* & * & \cdots & * & -P_i
\end{pmatrix} < 0.
\qquad (18)
$$

Additionally, multiplying (9) by P_j yields

$$P_j f(s^*) - P_j L_i s^* = 0, \ i, j \in \Xi. \qquad (19)$$

By using the notation $R_{ji} = P_j L_i$, we arrive at LMIs (13) and (14). This completes the proof of Theorem 1. □

When conditions (13) and (14) are feasible, we can calculate the mode dependent feedback gain L_i as $L_i = (l_{mn}^{(i)})_{m,n \in \mathcal{N}} = \left(\sum_{j=1}^{s} P_j P_j\right)^{-1} \left(\sum_{j=1}^{s} P_j R_{ji}\right)$. Then, the
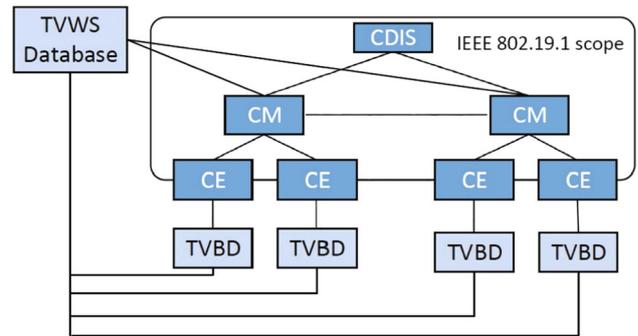

Fig. 2 The system architecture of IEEE 802.19.1

spectrum dynamics of H-CRN m in mode i is described as follows:

$$
\begin{aligned}
s_m(k+1) = &\ s_m(k) - \sum_{n \in \mathcal{N}} l_{mn}^{(i)} s_n(k) \\
&+ \gamma_m s_m(k) \left(1 - \frac{s_m(k) + \sum_{n \neq m} \alpha_{mn} s_n(k)}{C_i} \right).
\end{aligned}
\qquad (20)
$$

4.2 Algorithm description

This paper employs the IEEE 802.19.1 system to coordinate the interaction between H-CRNs. The architecture of an IEEE 802.19.1 system is briefly displayed in Fig. 2 [14].

- *Coexistence manager (CM)* The CM has the function of making coexistence related decisions. This includes generating and providing corresponding coexistence requests or commands and control information to CEs. The CM also assists network operators in management related to TVWS coexistence and collects adjacent information and available channel information from the coexistence database and information server (CDIS).
- *CDIS* The CDIS facilitates discovery and communication among CMs, and collects information related to the TVWS coexistence.
- *Coexistence enabler (CE)* CEs obtain information from the TV band device (TVBD) networks (e.g., H-CRNs in this paper) and enable communications between the TVBD networks and the IEEE 802.19.1 system.

This paper assumes that the available channel bandwidth information C_i from the TVWS database is accurate, and we thus only focus on the coexistence between H-CRNs, ignoring the transmission collision between PUs and H-CRNs.

In the following we design an IEEE 802.19.1 compatible spectrum sharing (ICSS) algorithm whose details are given as follows

Initialization All the coexisting H-CRNs are registered at the CDIS (i.e., $\mathcal{N}$ is available to the CDIS). Each H-CRN m starts its spectrum sharing process by reporting the bandwidth requirement c_m to the CDIS via its associated CE. The CDIS retrieves the bandwidth of the available channels C_i from the TVWS database. The competition coefficient α_{mn}, the intrinsic rate of increase γ_m and the fairness of the utility function a are publicly available within the coexistence system.

Preparation At the CDIS, for each mode i a feasible feedback gain matrix L_i is constructed as $L_i = \left(\sum_{j=1}^{s} P_j P_j \right)^{-1} \left(\sum_{j=1}^{s} P_j R_{ji} \right)$, where R_{ji} and P_i are feasible solutions to (13) and (14). The CDIS then achieves the expected spectrum share s^* by solving the optimization problem (5) with a specified fairness parameter a and weights $w_m = \frac{c_m}{\sum_{n \in \mathcal{N}} c_n}$ $(m \in \mathcal{N})$.

Main loop of ICSS

- Each H-CRN m $(m \in \mathcal{N})$ sends the spectrum share $s_m(k)$ to the CDIS periodically. The CDIS sends back the global information $\beta_m(k) = \sum_{n \in \mathcal{N}} \alpha_{mn} s_n(k)$ (defining $\alpha_{mn} = 1$) and the local information $\delta_m^{(i)} = \sum_{n \in \mathcal{N}} l_{mn}^{(i)} s_n^*$ to CM_m.
- Each CM_m $(m \in \mathcal{N})$ periodically updates the spectrum share s_m as (20), i.e.,

$$s_m(k+1) = s_m(k) + \gamma_m s_m(k) \left(1 - \frac{\beta_m(k)}{C_i} \right) - \delta_m^{(i)}. \tag{21}$$

- The above two steps repeat until all H-CRNs are with zero changing rate of spectrum share, i.e., $\| s(k+1) - s(k) \| < \varepsilon$, where ε is a sufficiently small positive number.

Output CM_m notifies H-CRN m the spectrum share s_m via its CE. The pseudo code of the ICSS algorithm is shown in Algorithm 1.

Remark 1 Notice that the whole spectrum sharing process of ICSS is completed within the IEEE 802.19.1 system. In doing so, sensitive information such as traffic load and bandwidth requirement is not shared across different H-CRNs, which avoids the issue of conflict-of-interest and the privacy concern for competing service providers.

5 Simulation results

In this section, simulations are employed to demonstrate the efficiency of the proposed results. Considering the activity of

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Algorithm 1 ICSS

Require: the competition coefficient α_{mn}, the intrinsic rate of increase γ_m, the available bandwidth C_i, the set of currently coexisting H-CRNs $\mathcal{N}$, the bandwidth requirement of H-CRN m, c_m, the fairness of the utility function a, a sufficiently small $\varepsilon > 0$.

1: **if** LMIs (13) and (14) are feasible **then**
2:     $F := 1$;
3:     The CDIS calculates L_i as $L_i = \left(\sum_{j=1}^{s} P_j P_j \right)^{-1} \left(\sum_{j=1}^{s} P_j R_{ji} \right)$ for each mode i
4:     The CDIS achieves s^* by solving problem (5) with $w_m = \frac{c_m}{\sum_{n \in \mathcal{N}} c_n}$
5:     **repeat**
6:         Each H-CRN m sends $s_m(k)$ to the CDIS periodically
7:         The CDIS sends back $\beta_m(k)$ and $\delta_m^{(i)}$ to CM_m
8:         Each CM_m periodically updates the spectrum share s_m according to (21)
9:     **until** $\| s(k+1) - s(k) \| < \varepsilon$
10: **else**
11:     $F := 0$
12: **end if**
Ensure: $F := 0$: no solution found; $F := 1$: spectrum share for H-CRN m, $s_m = s_m^*$

PUs, we adopt a discrete-time Markov chain model with three modes for the dynamic channel bandwidth. Specifically, we have $C_1 = 40M$, $C_2 = 30M$ and $C_3 = 20M$. We assume that at most three H-CRNs compete for the $C_{min} = 20M$ bandwidth and the bandwidth requirement vector of the three H-CRNs is [36M 16M 8M]. Without loss of generality, we let $\alpha = \alpha_{mn} = 1.8$ and $\gamma = \gamma_m = 1.25$ to reflect the equality of all the coexisting H-CRNs. The simulation code is developed by Matlab. The transition probability matrix Π is defined as follows:

$$\Pi = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.4 & 0.2 & 0.4 \\ 0.1 & 0.7 & 0.2 \end{pmatrix}. \tag{22}$$

When $a = 1$, a group of Markovian switching signals (as shown in Fig. 3a) are generated according to the transition probability matrix Π in (22). In order to test the dynamic adaptability of the ICSS algorithm, we vary the number of coexisting H-CRNs. Specifically, at the beginning (first 15 iterations), only H-CRN 1 and H-CRN 2 compete for the TVWS spectrum. The optimal solution to NUM (5) with the weight vector $W_1 = [0.69 \quad 0.31]$ is $s_{(1)}^* = [13.85 \quad 6.15]$. With C_i and $s_{(1)}^*$, we update each A_i according to (12)

$$A_1 = \begin{pmatrix} 0.0385 & -0.7788 \\ -0.3462 & 0.0865 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -0.3654 & -1.0385 \\ -0.4615 & -0.3013 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -1.1731 & -1.5577 \\ -0.6923 & -1.0769 \end{pmatrix}.$$
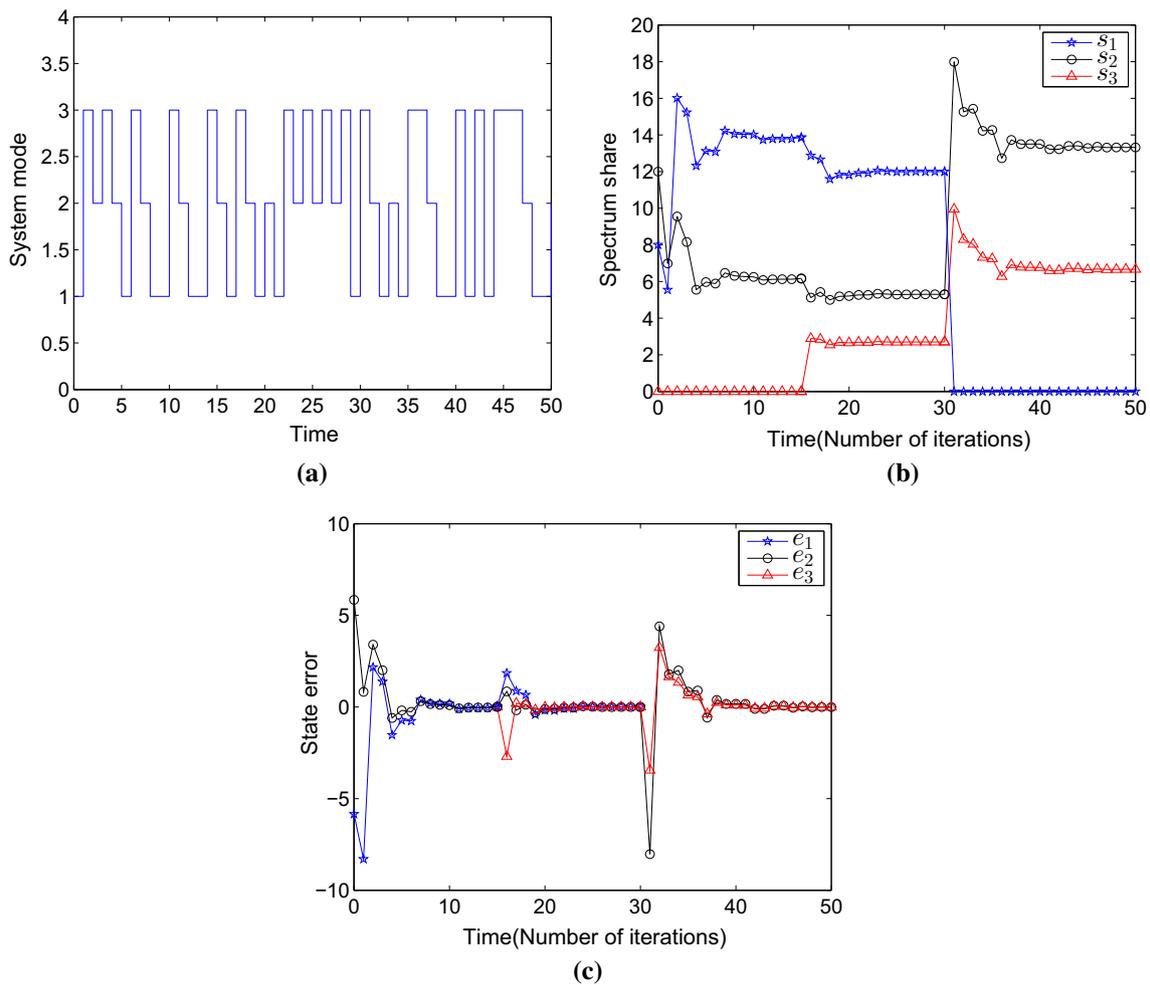
Fig. 3 Convergence to an equilibrium with proportional fairness (a = 1). **a** Switching signals, **b** states evolution, **c** errors evolution

By solving the LMIs (13) and (14), we can calculate the mode-dependent feedback matrix L_i as follows:

$$L_1 = \begin{pmatrix} -0.0907 & -0.4883 \\ -0.6103 & 0.6809 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 0.2157 & -1.1776 \\ -0.3146 & 0.0156 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} 0.1424 & -1.0127 \\ -0.2809 & -0.0602 \end{pmatrix}.$$

For an arbitrarily initial value $s_0 = [8\ 12]$ (i.e, $e_0 = [-5.85\ 5.85]$), Fig. 3b, c show that the convergence of the ICSS algorithm is achieved within 15 iterations. Since then, H-CRN 3 participates in the competition to the TVWS spectrum and a new equilibrium $s_{(2)}^* = [12\ 5.3\ 2.7]$ is calculated by NUM (5) with weight vector $W_2 = [0.60\ 0.27\ 0.13]$. Again, we update each A_i according to (12) and calculate L_i by solving the LMIs (13) and (14)

$$A_1 = \begin{pmatrix} 0.0500 & -0.6750 & -0.6750 \\ -0.2981 & 0.0919 & -0.2981 \\ -0.1519 & -0.1519 & 0.1081 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} -0.3500 & -0.9000 & -0.9000 \\ -0.3975 & -0.2942 & -0.3975 \\ -0.2025 & -0.2025 & -0.2725 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -1.1500 & -1.3500 & -1.3500 \\ -0.5962 & -1.0663 & -0.5962 \\ -0.3038 & -0.3038 & -1.0337 \end{pmatrix}$$

and

$$L_1 = \begin{pmatrix} 0.2418 & -1.0470 & -0.7973 \\ -0.5135 & 0.6319 & -0.4008 \\ -0.3224 & -0.2432 & 1.0454 \end{pmatrix}$$

$$L_2 = \begin{pmatrix} 0.1527 & -0.8003 & -0.8855 \\ -0.3590 & 0.3570 & -0.5479 \\ -0.2936 & -0.1589 & 0.7517 \end{pmatrix}$$
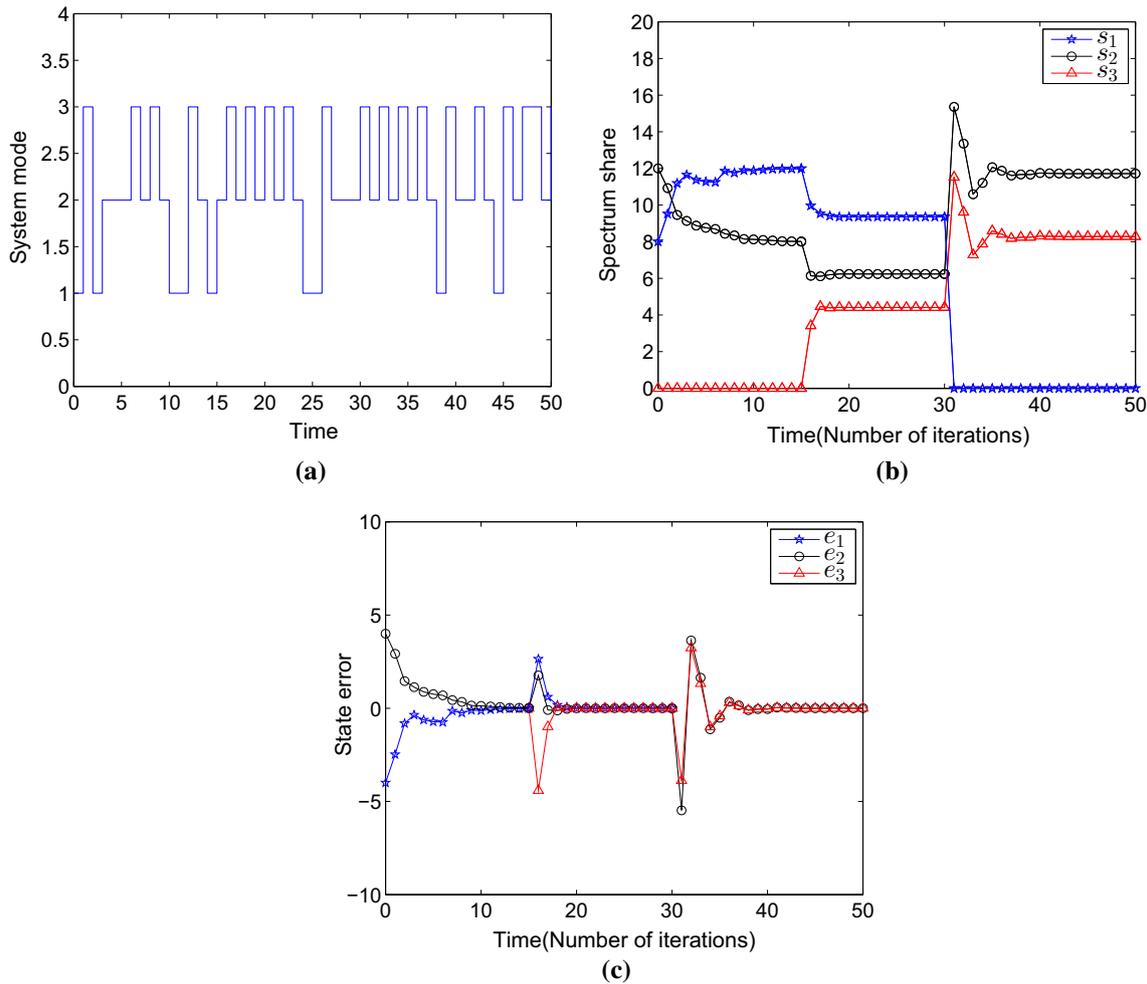
Fig. 4 Convergence to an equilibrium with proportional fairness (a = 2). **a** Switching signals, **b** states evolution, **c** errors evolution

$$L_3 = \begin{pmatrix} 0.4100 & -1.2517 & -1.1429 \\ -0.1560 & -0.1575 & -0.4401 \\ -0.0883 & -0.2638 & 0.0452 \end{pmatrix}.$$

After the 30-th iteration, H-CRN 1 leaves the coexistence system. The weight vector and the designed equilibrium become $W_3 = [0.67\ 0.33]$ and $s^*_{(3)} = [13.33\ 6.67]$, respectively. The A_i and L_i in this stage are updated as follows

$$A_1 = \begin{pmatrix} 0.0707 & -0.7498 \\ -0.3462 & 0.1156 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -0.3224 & -0.9997 \\ -0.4615 & -0.2626 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} -1.1086 & -1.4996 \\ -0.6923 & -1.0189 \end{pmatrix}$$

and

$$L_1 = \begin{pmatrix} 0.2740 & -1.0980 \\ -0.5562 & 0.5264 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 0.1281 & -0.8065 \\ -0.2944 & 0.0032 \end{pmatrix}$$

$$L_3 = \begin{pmatrix} 0.2223 & -0.9946 \\ -0.3186 & 0.0515 \end{pmatrix}.$$

The fast convergence of the ICSS algorithm during the last two stages is again shown in Fig. 3b, c.

When $a = 2$, we generate another group of Markovian switching signals according to the transition probability matrix (22). For the same initial value of $s(0)$ (or $e(0)$), Fig. 4b, c present the similar stable curves to Fig. 3b, c except the equilibria at each stage. To be specific, the equilibria at different stages are given as follows: $s^*_{(4)} = [12\ 8]$, $s^*_{(5)} = [9.35\ 6.24\ 4.41]$ and $s^*_{(6)} = [11.72\ 8.28]$. Due to the lack of space, we will not give the values of A_i and L_i at each stage. Due to the increase of a, $s^*_{(4)}$–$s^*_{(6)}$ are much fairer than $s^*_{(1)}$–$s^*_{(3)}$.

For other values of (α, γ, a), similar figures to Figs. 3 and 4 can be plotted and thus omitted. Despite that both [16,17] and this paper adopt the L–V competition model, in

[16,17] only the equilibrium with weighted fairness can be achieved, while from Figs. 3 and 4 we know that this paper can achieve equilibria with different fairness indexes (i.e., a). Additionally, SHARE [16] and SCHEME [17] are stable only when $0 < \alpha < 1$, while the proposed ICSS algorithm still converges for large competition coefficient (e.g., $\alpha \geq 1$).

6 Conclusion

In this paper a novel NMJS model has been proposed to formulate the heterogeneous coexistence problem. The novelty of the NMJS model lies in the consideration of the dynamics of the TVWS spectrum for the first time. A Markovian jump system method has been employed to find a feedback controller to the equilibrium assignment problem of the NMJS. Based on the achieved feedback controller, we have designed an ICSS algorithm whose fast convergence and fairness have been demonstrated by both theory and simulations.

Future work will focus on the scenario in which the sets of spectrum channels for different coexisting networks are heterogeneous.

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