Dynamics modelling and predictive control for 6-DOF rotorcraft aerial manipulator system

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Abstract: Rotorcraft aerial manipulator (RAM) is a new concept of aerial robots with arms, and it changes the traditional searching rotorcraft unmanned aerial vehicles (RUAVs) into operating aerial robots. The additional six degree-of-freedom (DOF) manipulator makes RUAV more flexible to accomplish ‘touching’ tasks. However, the relative force and torque disturbance, which cannot be eliminated completely in the controller, between the 6-DOF manipulator and the aerial robot, makes the operation precision of the end-effector too poor to accomplish the ‘touching’ missions. In this research, the overall dynamics model is firstly developed based on dynamic disturbance analysis between the RUAV and the joint 6 DOF robotic arm. Based on the proposed model, to compensate for the disturbance from relative dynamics with rotor system’s control delay, a predictive controller is designed to minimise the errors of positions and attitudes of the end-effector. At last, different control strategies are compared in simulated insertion tasks, and the simulation results show the effectiveness of the proposed overall dynamic model and the proposed control strategies in precise air-operation.

Keywords: rotorcraft unmanned aerial vehicle; RUAV; aerial manipulator; dynamics modelling; predictive control; flight simulation.


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1 Introduction

Typically, most researches on unmanned aerial vehicles (UAVs) have been limited to monitoring and surveillance applications in Daniel et al. (2013), Nikhil et al. (2012) and Per et al. (2012), where the objectives are limited to ‘look’ and ‘search’ rather than ‘touch’. However, with the improvement of flight control performance as well as the requirements of practical missions, the interaction between UAV and surroundings is necessary for precise operation. To solve the ‘touch’ problem, physical interaction becomes a research trend that has recently received great attention in the field of aerial robotics (Marin, 2014), such as rotorcraft unmanned aerial vehicles (RUAV) (Konstantin et al., 2013), which can expand the workspace from two-dimensions to three-dimensions by hovering operation. For this reason, RUAV with a robotic arm has greatly enhanced the utility of aerial manipulator (AM).

Several examples of RUAV physically ‘touching’ objects have been demonstrated in recent years. DLR aerial manipulating robot in Huber et al. (2013), composed of an autonomous helicopter and a 7-DOF industrial manipulator, is able to grasp a straight pole on the ground and the research studied the movement of centre of gravity when grasping. In University of Pennsylvania (Mellinger et al., 2011), their research focused on the estimation and
control for aerial grasping and manipulation with quadrotor. Test platform in Drexel University is composed of a gantry and two 4-DOF robotic arms and computer vision and force feedback are applied in their research (Korpela et al., 2012). In addition, there are some other researches on multi-rotor aircraft mounted with manipulator, which can grasp lightweight objects (Scholten et al., 2013; Kim et al., 2013; Ding and Yu, 2013). In these examples, the ‘touching’ objects are only grasped automatically by grippers, which are fixed directly on RUAV body frame. The attitude and position of the grippers, which are affected by the motion of RUAV, cannot be adjusted or compensated actively for applications, such as peg-in-hole or insertion tasks. To solve this problem, remotely controllable AM, which consists of UAV and multi degree-of-freedom (DOF) robotic arm, has been proposed as a new concept nowadays.

Rotorcraft aerial manipulator (RAM), a type of mobile manipulating RUAV in Song et al. (2010), which consists of rotocraft flying robot and a 6-DOF robotic arm with an end-effector shown in Figure 1, is designed as the new concept robotic system proposed for requirement of autonomous operation under hovering mode, in Chinese Academy of Sciences (CAS). In practical applications, this aerial 6-DOF manipulator is able to reach any arbitrary position in workspace with specified attitudes, The RAM project in CAS includes system modelling, coordinated control and planning of motion and operation, as well as construction and verification of experiment platform. We have just completed the mechanical structure design, and are building dynamics model and designing controller for future flight experiments.

Figure 1 The concept of RAM in CAS (see online version for colours)

Different from traditional RUAV control problems, which have been solved in decades (Song et al., 2013b; Du et al., 2015), the new problems, such as relative dynamics among rotocraft, manipulator and object, make the modelling and control of RAM extremely difficult during the precise operation. In few publications by now, although the load stability of RUAV and manipulator control method are analysed and demonstrated partially, the dynamics of RAM is simplified without consideration of the relative dynamics in dynamic operation in Habibur et al. (2014) and Song et al. (2013a). The dynamics of rotocraft and manipulator are built separately, and the reaction torque is only considered as external disturbance to each other and compensated by the robustness of model-free controllers, such as PID. The omitted relative dynamics, between RUAV and AM, such as the reaction torque from the motion of AM, may result in the fluctuating position error of end-effectors in precise operation, such as aiming the hole in insertion tasks in Matko et al. (2014) and Christopher et al. (2013). Hence, besides stabilisation, overall dynamics model-based control strategy, which can minimise the position and attitude errors of the end-effectors of RAM system, is also very important for real missions, which has not been solved for RAM by far.

In this paper, different from the above researches, the relative dynamics, between RUAV and AM, is considered as an inner disturbance, which is modelled accurately by force and torque analysis. The overall dynamic model of RAM is developed firstly based on Newton-Euler equation and aerial dynamics of the rotor system. Then, considering the response from the rotor system is much slower than AM’s motors, based on the proposed overall dynamics model, a predictive controller is designed to compensate for the inner disturbance and eliminate the position and attitude errors of the end-effector, which comes from relative force and torque. At last, the insert mission is simulated and the proposed control strategy is tested to verify its effectiveness in this simulation environment.

2 Model for overall dynamics of RAM

RAM dynamics obeys the Newton-Euler equation for rigid body in translational and rotational motion. Here, we consider a typical rigid RAM in/near hover flight and the dynamic equation is conveniently described with respect to the body coordinate system, which is written as:

\[
\begin{bmatrix}
    m\dot{V}_B^B \\
    \dot{\Omega}_B^B \\
    \dot{\Omega}_B^B \times V_B^B \\
    \Omega_B^B \times \Omega_B^B \\
\end{bmatrix} = \begin{bmatrix}
    \begin{bmatrix}
    \dot{V}_B^B \\
    \dot{\Omega}_B^B \\
    \dot{\Omega}_B^B \times V_B^B \\
    \Omega_B^B \times \Omega_B^B \\
\end{bmatrix} \\
    M_{ai} \\
\end{bmatrix}
\]

\[
M_{ai} = I_{ai} \dot{\theta}_i, (i = 1 \sim 6)
\]

where \(m\) is the mass of RAM; \(V_B^B \in R^{3\times1}\) is the velocity vector for RUAV body frame; \(V_B^B \in R^{3\times1}\) is the rotary speed vector for RUAV body frame; \(\theta_i, (i = 1 \sim 6)\) are the angles of six joints of AM; \(I \in R^{3\times3}\) is inertia matrix for RAM; \(I_{ai} (i = 1 \sim 6)\) are rotary inertia of the six joints of AM; \(F_{ai}^B \in R^{3\times1}\) is the force vector on RUAV body; \(M_{ai}^B \in R^{3\times1}\) is the moment vector on RUAV body; \(n_{ai} (i = 1 \sim 6)\) are the input torques, actuated by the motors on the six joints of AM. To simplify the dynamic equation, it is necessary to make some assumptions that the RUAV’s rotational principal axes coincide with the axes of the body reference system and RUAV is symmetrical about the XOZ plane and XOY plane in body-fixed frame.
The body diagram of RAM with respect to body coordinate system is as shown in Figure 2. By employing the lumped-parameter approach, in which the RUAV is considered as the composition of the main rotor, tail rotor, fuselage, horizontal stabiliser, and vertical stabiliser, these components can be considered as the source of forces and moments. The external force and moment in hovering mode can be written as:

$$
F_{ext}^B = \begin{bmatrix}
X_M & Y_M & Z_M \\
X_{AE} & Y_{AE} & Z_{AE}
\end{bmatrix} + R^B_R \begin{bmatrix}
0 \\
mg
\end{bmatrix}
$$

$$
M_{ext}^B = \begin{bmatrix}
M_M & M_Y - X_M h_M + Z_M Y_M + Y_Y h_Y \\
M_{AE} & -Y_M l_M - Y_Y l_Y
\end{bmatrix} + M_E
$$

$$
M_E = -R_{E \rightarrow B} \begin{bmatrix}
M_{A1} \\
\cdots \\
M_{A6}
\end{bmatrix} - L_{E \rightarrow B} \begin{bmatrix}
X_{AE} \\
Y_{AE} \\
Z_{AE}
\end{bmatrix}
$$

where $R^B_R \in \mathbb{R}^{3 \times 3}$ is transition matrix between navigation frame and RUAV body frame; $R_{E \rightarrow B}$ is transition matrix between the body frame of end-effector of AM and RUAV body frame; $X_{AE}, Y_{AE}, Z_{AE}$ which are considered as external disturbance, are the variable forces on the end-effector in its body frame. $L_{E \rightarrow B}$ are the position vector of end-effector in RUAV body frame.

The forces and torques generated by the main rotor are controlled by $T_M, a_1, b_1$. The tail rotor is considered as a source of pure lateral force $Y_T$ and anti-torque $Q_T$, which are controlled by $T_T$. Thus, the forces and moments can be expressed as in (Christopher et al., 2013):

$$
X_M = -T_M \sin a_1 \\
Y_M = -T_M \sin b_1 \\
Z_M = -T_M \cos a_1 \cos b_1 \\
R_M = -b_1 \frac{dR_M}{dh} - Q_M \sin a_1 \\
M_M = a_1 \frac{dM_M}{dh} - Q_M \sin b_1 \\
N_M = -Q_M \cos a_1 \cos b_1 \\
Y_T = -T_T \\
M_T = -Q_T
$$

where $T_M, T_T$ are the force of main rotor and tail rotor; $a_1, b_1$ are longitudinal and lateral flapping angle; $Q_M, Q_T$ are the moment caused by main rotor and tail rotor. Based in Done
and Balmford (2001), $T_i, Q_i, i = \{T, M\}$ can be calculated as followings:

$$T_i = \frac{R_i^3 - R_0^3}{3} m_3 \theta_{i3} + \frac{m_2 m_3}{2} (R_i^3 - R_0^3)$$

$$-\frac{m_3}{8 \pi \Omega_i^2} \left[ 2 \left( \frac{3 m_1 R_i \theta_{i1}}{2} - 2 m_3 \right) \left( m_2 R_i \theta_{i3} + m_3 \right)^{3/2} 
\right]$$

$$-\frac{m_3}{15 m_2} \left( \frac{15 m_2 R_i^3 - 12 m_3 R_{i0} + 8 m_3^2}{2} \right) \left( m_4 R_i + m_5 \right)^{3/2} + 2 m_3 m_2 \left( \frac{15 m_2^2 R_i^3 - 12 m_3 m_5 R_{i0} + 8 m_3^2}{2} \right) \left( m_4 R_i + m_5 \right)^{3/2}$$

$$-(3 m_2 R_{i0} - 2 m_3 \left( m_2 R_{i0} + m_5 \right)^{3/2})$$

where

$$m_1 = \Omega_i a_i b_i c_i / 2 + 4 \pi V_c$$

$$m_2 = 8 \pi \Omega_i^2 a_i b_i c_i$$

$$m_3 = \rho \Omega_i a_i b_i c_i / 2 + 4 \pi V_c$$

$$m_4 = m_2 \theta_{i3}$$

$$m_5 = m_3 - V_c m_2 \Omega_i$$

$$m_6 = (m_1 / 8 \pi - V_c) / \Omega_i$$

$$n_1 = \rho \Omega_i^3 a_i b_i c_i / 2$$

$$n_2 = V_c - m_1 / 8 \pi$$

$$n_3 = a_i / (8 \pi \Omega_i)^2$$

$$n_4 = a_i V_c / 4 \pi \Omega_i^2$$

$$n_5 = a_i (V_c / \Omega_i)^2$$

$$n_6 = a_i (V_c / \Omega_i)^2$$

$$n_7 = n_3 n_4 - n_5 m_2 + n_6$$

$$n_8 = n_3 n_4 - n_5 m_1 + n_6$$

$$n_9 = 2 m_3 m_2 - n_3$$

where $R_{i1}$ is the radius of tail rotor, $R_{i2}$ is the radius of main rotor, $R_{i0}$ is the inner radius of tail rotor, $R_{i0}$ is the inner radius of main rotor, $\Omega_i$ is the rotation speed of tail rotor, $Q_i$ is the rotation speed of main rotor, $b_i$ is the number of main rotor’s blade, $b_f$ is the number of tail rotor’s blade, $c_j$ is the thrust coefficient of main rotor, $a_f$ is the gradient of lift curve for tail rotor, $a_m$ is the gradient of lift curve for main rotor, $\rho$ is the density of air, $c_f$ is the width of tail rotor, $c_m$ is the width of main rotor, $\theta_{i3}$, $\theta_{i6}$ is the collective pitch angle of main rotor, $\theta_{f1}$ is the collective pitch angle of tail rotor, and $V_c$ is the vertical speed of RUAV.

To simplify the analysis of the manipulator, it can also be assumed that the link velocities and angular rates change slowly; therefore, inertial forces and torques are not considered here. That is, motor torque and forces interacting with external environment are the focus of this research.

In the force and moment balance equations, each link has forces and torques exerted on it by its neighbours and in addition experiences a torque from joint actuator. When the manipulator is in contact with the target, the forces and torques due to this contact is not zero. The transformational relation about interactional forces and torques and joint actuator torques are computed recursively from the end back to the base, which is described as below:

$$f_i = \sum_{i}^{n} f_{i+1}$$

$$m_i = \sum_{i}^{n} m_{i+1} + P_{i+1} \times f_{i+1}$$

where $f_i \in R^{3 \times 1}$ is the force exerted on link $i$ by link $i-1$, which is described in frame $\{i\}$, $m_i \in R^{3 \times 1}$ is the torque exerted on link $i$ by link $i-1$, which is described in frame $\{i\}$.

Table 1  Link parameters of manipulator

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\alpha_{i-1}$</th>
<th>$a_{i-1}$</th>
<th>$b_i$</th>
<th>$\theta_{i}\left(\theta_{0}\right)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0°</td>
<td>0</td>
<td>$b_1$</td>
<td>$\theta_{1}\left(0°\right)$</td>
</tr>
<tr>
<td>2</td>
<td>-90°</td>
<td>0</td>
<td>0</td>
<td>$\theta_{2}\left(90°\right)$</td>
</tr>
<tr>
<td>3</td>
<td>0°</td>
<td>$a_2$</td>
<td>$b_3$</td>
<td>$\theta_{3}\left(90°\right)$</td>
</tr>
<tr>
<td>4</td>
<td>-90°</td>
<td>$a_3$</td>
<td>$b_4$</td>
<td>$\theta_{4}\left(0°\right)$</td>
</tr>
<tr>
<td>5</td>
<td>90°</td>
<td>0</td>
<td>0</td>
<td>$\theta_{5}\left(90°\right)$</td>
</tr>
<tr>
<td>6</td>
<td>90°</td>
<td>0</td>
<td>0</td>
<td>$\theta_{6}\left(90°\right)$</td>
</tr>
<tr>
<td>7</td>
<td>0°</td>
<td>0</td>
<td>$b_7$</td>
<td></td>
</tr>
</tbody>
</table>
the torque exerted on link \( i \) by joint \( i \) actuator torque, which is described in frame \( \{ i \} \), where \( m_{ii} = \dot{m}_{ii} \), \( i \in R^{6 \times 3} \) is the rotation matrix between frame \( \{ i \} \) and frame \( \{ i + 1 \} \), \( P_i \) \( i \in R^{6 \times 1} \) is the position vector between frame \( \{ i \} \) and frame \( \{ i + 1 \} \).

When manipulator is interacting with the environment, the forces and torques due to this contact are \( \tau = (f, f, f)^T \), \( \tau_n = (0, 0, 0)^T \), where the torque is considered as zero for conciseness here. For \( i \in R, i \in P_i \) above, their general forms are:

\[
\begin{pmatrix}
\text{c} \theta_i & -s \theta_i & 0 \\
 s \theta_i \text{c} \alpha_{i-1} & \text{c} \theta_i & -s \alpha_{i-1} \\
 s \theta_i \text{s} \alpha_{i-1} & \text{c} \theta_i \text{c} \alpha_{i-1} & \text{c} \alpha_{i-1}
\end{pmatrix}
\]

\[
\begin{pmatrix}
 a_{i-1} \\
-s \alpha_{i-1} \theta_i \\
-c \alpha_{i-1} \theta_i
\end{pmatrix}
\]

(6)

where \( \text{c} \theta_i \) is shorthand for \( \cos \theta_i \), \( s \theta_i \) for \( \sin \theta_i \) and so on, and \( \text{c} R \) is a unit matrix. The cross product is understood to be the matrix operator:

\[
P \times = \begin{pmatrix}
0 & -p_z & p_y \\
p_z & 0 & -p_x \\
-p_y & p_x & 0
\end{pmatrix}
\]

(7)

According to Newton’s third law of motion (law of action and reaction), it is easily to obtain the force and torque acted on the base as described below:

\[
F_0 = - \text{c} R \text{f}_i
\]

\[
N_0 = - \text{c} R \text{m}_i
\]

\[
\times \text{c} R \text{f}_i
\]

(8)

where \( F_0 \) is the force exerted on base by the manipulator, \( N_0 \) is the torque exerted on base by the manipulator. Note that negative signs in both equations above have the same meanings in (2).

Therefore, those corresponding matrices in (2) can be presented as follows, some of which are shown by block matrices:

\[
R_{AE} = \begin{pmatrix}
\text{c} R & \text{c} R & \text{c} R \\
\text{c} R & \text{c} R & \text{c} R \\
\text{c} R & \text{c} R & \text{c} R
\end{pmatrix}
\]

(9)

\[
R_{A-B} = \begin{pmatrix}
\text{c} R & \text{c} R & \text{c} R & \text{c} R \\
\text{c} R & \text{c} R & \text{c} R & \text{c} R \\
\text{c} R & \text{c} R & \text{c} R & \text{c} R \\
\text{c} R & \text{c} R & \text{c} R & \text{c} R
\end{pmatrix}
\]

(10)

\[
L_{A-E} = \begin{pmatrix}
L_x, L_y, L_z
\end{pmatrix}
\]

(11)

\[
L_x = [c_1 c_2 a_2 - s_1 b_3 + c_1 c_3 a_3 - c_1 s_2 s_4 b_4 - [c_1 (c_2 c_4 s_5) + c_2 s_3 s_4 s_5] b_7
\]

\[
L_y = [s_1 c_2 a_2 + c_1 b_3 + s_1 c_3 a_3 - s_1 s_2 s_4 b_4 - [s_1 (c_2 c_4 s_5) + c_2 s_3 s_4 s_5] b_7
\]

\[
L_z = [h - s_2 a_2 - s_3 a_3 - c_2 b_4 + (c_2 c_4 s_5 - c_2 s_3 c_5) b_7
\]

Let control input

\[
U = (u_1, u_2)^T
\]

\[
u_1 = (\theta_{\text{MA}}, \theta_{\text{FA}}, a_1, b_1)^T
\]

\[
u_2 = (m_{i1}, m_{i2}, m_{i3}, m_{i4}, m_{i5}, m_{i6})^T
\]

(12)

where \( a_1, b_1 \) above are selected as control inputs rather than states of system as shown in He and Han (2010), for which the mean values of flapping angles are determined by lateral and longitudinal cyclic pitch control. In practice, flapping angles can be observed from attitude measurement using state observer.

Then, we can obtain RAM overall dynamics model based on (1–11), which has the following state-space structure:

\[
\dot{X} = \begin{pmatrix}\dot{x}_1 \dot{x}_2 \dot{x}_3 \dot{x}_4 \dot{x}_5 \dot{x}_6 \end{pmatrix}
\]

\[
F_T (x_1, x_2, u_1)
\]

\[
M_T (x_1, x_2, u_1) - R_{A-B} (x_6) R_{u2}
\]

\[
I_u u_2
\]

\[
R_B (x_7) x_1 + \frac{\partial P_{d-A-B} (x_6, x_7)}{\partial x_6}
\]

\[
+ W
\]

(13)

where \((x_1^T, x_2^T, x_3^T) \equiv (V \text{g}, \Omega \text{g}, \rho^T), \text{g}^T = (u_1, v, w)^T, \Omega^T = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)^T, \rho = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)^T, R_t = \text{diag}(l_1, l_2, l_3, l_4, l_5, l_6).\)

\(R_s\) is just a conversion matrix, \(x_s \in R^{6 \times 1}\) is the position of the end-effector of AM in navigation frame, \(x_s \in R^{6 \times 1}\) is the attitude of end-effector in navigation frame, \(x_6 = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5, \theta_6)^T\) is the joint angle vector of AM, \(x_s \in R^{6 \times 1}\) is the RUAV attitude vector, \(0_{6 \times 6}\) stands for zero matrix, \(P_{d-A} (x_s, x_0) = R_B (x_7) \cdot L\)-AM’s position in RUAV fixed navigation frame, \(W = \frac{1}{m} T T^{T} \cdot R^{T} \cdot M^{T} \cdot I^{T} \cdot 0, \text{g} = \text{g}, \text{g}^{T} = (X_{d-E}, Y_{d-E}, Z_{d-E}) \in R^{6 \times 1}\) is external force disturbance vector on end-effector in its body frame, \(M = L\times R_{A-B} \cdot (X_{d-E}, Y_{d-E}, Z_{d-E}) \in R^{6 \times 1}\) is external torque disturbance vector on end-effector in RUAV body-fixed frame, \(F_T (x_1, x_2, u_1)\) are nonlinear mapping functions, where

\[
F_T (x_1, x_2, u_1) = 1/m
\]

\[
\begin{pmatrix}
X_M \\
Y_M + Y_f \\
Z_M
\end{pmatrix}
\]

\[
\begin{pmatrix}
0 \\
mg - x_2 \times m x_1
\end{pmatrix}
\]
For RAM system’s applications, such as peg-in-hole or insertion tasks, the reference input is given as the reference position \( X_{d,n-E}, Y_{d,n-E}, Z_{d,n-E} \) and reference attitude \( \phi_{d,n-E}, \theta_{d,n-E}, \psi_{d,n-E} \) for the end-effector of AM in navigation frame. Hence, based on dynamics (12), the controller will be designed and tested to track reference input accurately in the following parts.

### 3 Controller design

A proposed predictive controller is designed to compensate for the inner disturbance \( R_{d,n}(x_k)u_2 \) and eliminate the position and attitude errors of the end-effector, which comes from external disturbance \( W \).

Considering that the response of RUAV’s rotor system is much slower than AM’s motors, the control input \( u_1 \) must be computed based on the future possible disturbance from \( u_2 \), hence, predictive controller is applied to overall dynamics. To design an online predictive controller, the RAM model (12) is simplified as a linear control reference model at hovering flight mode, where velocity \( x_1 \) and rotation rate \( x_2 \), \( x_3 \) are close to zeros, as

\[
\dot{X} = \begin{bmatrix} \dot{x}_1 & \dot{x}_2 & \dot{x}_3 \end{bmatrix}^T
\]

\[
A = \begin{bmatrix} A_{T1} & A_{T2} & 0_{3 \times 6} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ A_{M1} & A_{M2} & 0_{3 \times 6} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 6} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} \\ A_{P1} & A_{P2} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} & 0_{3 \times 3} \\ 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 6} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} \\ 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 6} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} \\ 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 6} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3} \\ \end{bmatrix} + W
\]

\[
Y = CX
\]

\[
A_{F1} = \frac{\partial F}{\partial x_1} \bigg|_{x_1,x_2,0} = 0, \quad A_{F2} = \frac{\partial F}{\partial x_2} \bigg|_{x_1,x_2,0} = 0
\]

\[
A_{M1} = \frac{\partial F}{\partial x_1} \bigg|_{x_1,x_2,0} = 0, \quad A_{M2} = \frac{\partial F}{\partial x_2} \bigg|_{x_1,x_2,0} = 0
\]

\[
A_{p0} = R_0^a (x_7,0)
\]

\[
A_{p1} = \frac{\partial P_{d \rightarrow N} (x_6, x_7)}{\partial x_7} \bigg|_{x_1,x_7,x_6,y_7} \quad A_{p2} = \frac{\partial P_{d \rightarrow N} (x_6, x_7)}{\partial x_6} \bigg|_{x_1,x_7,x_6,y_7}
\]

\[
B_{F1} = \frac{\partial F}{\partial u_1} \bigg|_{x_1,x_2,0} = 0, \quad B_{M1} = \frac{\partial F}{\partial u_2} \bigg|_{x_1,x_2,0} = 0
\]

\[
B_{M2} = R_{d \rightarrow B} (x_6, 0)
\]

where \( I_{m \times m} \) is a \( m \times m \) unit matrix, \( x_{i,0}, u_{j,0} \) stand for the balance value of state \( x_i \) and control input \( u_j \), respectively, and \( Y \) is the measurable output.

The overall dynamics (14) can be expressed in discrete form with considering the control delay for rotor system as

\[
\begin{bmatrix} X_{k+1} \\ Y_k \end{bmatrix} = \begin{bmatrix} A_d & B_d & U_{k-d} & W_k \\ C_x & X_k \end{bmatrix}
\]

where \( k \) is the sampling time, \( X_k \) is the sampling value of \( X \), \( W_k \) is the sampling value of \( W \), \( U_k = (u_{1,k-d} \quad u_{2,k}) \) is the sampling value of \( U \), \( u_{1,k} \) is the sampling value of \( u_{1,k} \), \( u_{2,k} \) is the sampling value of \( u_{2,k} \), \( d \in R \) is the time delay, \( Y_k \) is the sampling value of system output \( Y \), and \( \{A_d, B_d, C_x\} \) is the discrete expression of system \( \{A, B, C\} \). RAM model (14) is measurable, and the model predictive control scheme (Amalm and Moez, 2014; Bilal and Hemanshu, 2011) can be designed, based on (14), for the tracking control of state \( X_1 \) as:

#### 3.1 Step 1: make prediction

First, based on (14), for the case that prediction step \( i \) is less than time-delay \( d \) (i.e., the time instant that system behaviour cannot be regulated through the current and future control action), prediction is carried out as follows:

\[
\hat{X}_{k+i} = A_d \hat{X}_{k+i-1} + B_d U_{k+i-1-d} \quad A_d = A_d^1
\]

where \( 1 \leq i \leq d-1 \), \( \hat{X}_{k+i} \) is the predicted state for \( X_{k+i} \) at time \( k + i \), the superscript 1 denotes the part of predicted variable, which is independent of the current and future control actions.

Secondly, for the case that prediction step is larger than the time delay \( d \),

\[
\hat{X}_{k+i} = A_d \hat{X}_{k+i-1} + B_d U_{k+i} = \hat{X}_{k+i} + \sum_{n=0}^{i} A_d^n B_d U_{k+n}
\]

where
where $0 \leq i \leq p - 1$, $p$ is the prediction range; similarly, $\hat{X}_{k+d+r+i|\hat{k}}$ denotes the sub-variable of $\hat{X}_{k+d+r+i|\hat{k}}$ that is independent of the current and future control actions.

### 3.2 Step 2: receding horizon optimisation

Following the prediction, the control vector can be obtained by minimising the following cost function:

$$ J = (R_k - C^\top X_k) \hat{U}_k + \sum_{i=0}^{p-1} U_k^i \gamma U_k $$

and the optimal control inputs can be obtained by

$$ U_k^* = (G_0^T G_0 + \gamma) G_0 (R_k - C^\top X_k) $$

where $G_0$ is the prediction matrix,

$$ X_k^* = \left[ \hat{X}_{k+d+r|\hat{k}}, \ldots, \hat{X}_{k+d+p-r|\hat{k}} \right]^\top $$

is the predicted state vector,

$$ X_k^* = \left[ \hat{X}_{k+d+r|\hat{k}}, \ldots, \hat{X}_{k+d+p-r|\hat{k}} \right]^\top $$

is the known vector inside $X_k^*$, which is without the term $U_k$, $\gamma \in R^{4p \times 4p}$ is the weight of control input, $\hat{U}_k = (U_1^T, U_2^T, \ldots, U_{k+p-r}^T)^T$ is future control vector to be calculated, and

$$ C^\top = \text{diag}\left[ C_d, C_d, \ldots, C_d \right], R_k^i = \begin{bmatrix} R_{1,k} & \cdots & R_{p,k} \end{bmatrix}^\top, $$

$$ R_{x,k} \begin{bmatrix} x_{1,k}^T \cdots \ x_{p,k}^T \end{bmatrix}^\top $$

where $x_{1,k}$ and $x_{p,k}$ are the desired position and attitude states vector of the end-effector of AM at time $k$, respectively. The detailed definition of these matrices can be found in Song et al. (2013a).

### 3.3 Step 3: control implementation

The first element of $U_k^*$ is used as the control to the actual plant, and go back to Step 1 at time $k+1$.

The control structure can be described by Figure 3, and is simulated in the next part.

**Figure 4** Control structure for simulation (see online version for colours)

### 4 Simulation

#### 4.1 Simulated environment

The HIL virtual simulation environment in Wang et al. (2013) is used to simulate the RAM model for peg-in-hole insertion $I$ as shown in Figure 5. In this mission, a hose was grasped and inserted into a replacement pump by the manipulator. There is 1 cm of clearance for the insertion. The reference position is a line from point $(0, 0, 12)$ to point $(0, 0, 10)$ in navigation frame, and the reference attitude angles of end-effector are all set as $0^\circ$ for pitch, roll and yaw. The proposed overall model-based predictive control is applied and compared with $H_\infty$ controller (Lima and Heinz, 2015), which is designed based on the same model (14) without consideration of the inner torque disturbance, to verify the necessary of the prediction procedure for precise air operations for RAM system. The simulation parameters are listed in Table 2.

**Table 2** Dynamics parameters for simulation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{1T}$</td>
<td>0.17 m</td>
<td>$\Omega_T$</td>
<td>4,000 rpm</td>
</tr>
<tr>
<td>$R_{1M}$</td>
<td>2.16 m</td>
<td>$\Omega_M$</td>
<td>1,400 rpm</td>
</tr>
<tr>
<td>$R_{0T}$</td>
<td>0.08 m</td>
<td>$c_d$</td>
<td>1.37</td>
</tr>
<tr>
<td>$R_{0M}$</td>
<td>0.21 m</td>
<td>$a_T$</td>
<td>0.28</td>
</tr>
<tr>
<td>$l_M$</td>
<td>0.12 m</td>
<td>$a_M$</td>
<td>0.57</td>
</tr>
<tr>
<td>$l_T$</td>
<td>1.82 m</td>
<td>$b_T$</td>
<td>2.00</td>
</tr>
<tr>
<td>$b_M$</td>
<td>2.00</td>
<td>$c_T$</td>
<td>0.045 m</td>
</tr>
<tr>
<td>$c_M$</td>
<td>0.097 m</td>
<td>$\rho$</td>
<td>1.29 kg/m$^3$</td>
</tr>
<tr>
<td>$h_M$</td>
<td>0.47 m</td>
<td>$h_T$</td>
<td>0.19 m</td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.5 m</td>
<td>$b_3$</td>
<td>0.15 m</td>
</tr>
<tr>
<td>$a_2$</td>
<td>0.3 m</td>
<td>$a_3$</td>
<td>0.15 m</td>
</tr>
<tr>
<td>$b_4$</td>
<td>0.3 m</td>
<td>$b_7$</td>
<td>0.4 m</td>
</tr>
<tr>
<td>$I_{11}$</td>
<td>0.97 kgm$^2$</td>
<td>$I_{12}$</td>
<td>0.57 kgm$^2$</td>
</tr>
<tr>
<td>$I_{33}$</td>
<td>0.45 kgm$^2$</td>
<td>$I_{34}$</td>
<td>0.16 kgm$^2$</td>
</tr>
<tr>
<td>$I_{55}$</td>
<td>0.27 kgm$^2$</td>
<td>$I_{66}$</td>
<td>0.1 kgm$^2$</td>
</tr>
</tbody>
</table>

**Figure 5** Insertion mission for simulation (see online version for colours)
The study in this paper highlights the simulation verification of control system in the mode of hovering near the ground. The air disturbance reflected from ground, also known as the ground disturbance, has an effect on the main rotor and its theoretical model is quite complicated. The ground disturbance is considered as the combination of sinusoidal disturbances which have uncertain boundaries (Kutz et al., 2012). Meanwhile, it is additive moment disturbance for pitch and roll attitudes and is additive force disturbance in vertical direction. Hence, ground disturbance can be regarded as the superposition of some bounded sinusoidal signals in simulation and controller parameters can be regulated manually to acquire adequate robustness to decrease corresponding impact on AM’s position and attitude tracking accuracy. Simulated ground disturbances are as follows:

\[
\begin{align*}
T_x &= \Theta_M T_{x,0} (1 + \sin (\Omega_x nt / 30)) \\
T_y &= \Theta_M T_{y,0} (1 + \cos (\Omega_y nt / 30)) \\
f_z &= \Theta_M f_{z,0} (1 + \cos (\Omega_z nt / 30))
\end{align*}
\]

where \(T_x, T_y\) respectively, are roll and pitch additive moment disturbance, \(f_z\) is additive force disturbance in vertical direction. \(T_{x,0}, T_{y,0}, f_{z,0}\) are random numbers in (0, 1), respectively.

### 4.2 Simulation and analysis of stability

The simplified linear model (13) is used to approximate the nonlinear dynamics model in the hover mode, and 1 Hz sinusoidal low frequency signals are used to set as control inputs, each of which has an amplitude of 20% for the control variable’s extreme value. Comparing horizontal positions and pitch attitudes in simplified linear model with those in nonlinear model, the results are shown in Figure 6.

Figure 6 shows that the simplified model (13) has a high fitting degree over 70% comparing to the high-fidelity model (12). Hence, based on the reference model (13), robust controller can be designed to achieve high-precision control with model (12).

Comparing the proposed predictive controller with \(H_\infty\) controller to acquire their respective robust stability of model uncertainty in Lima and Heinz (2015), \(a_M, b_M\) change between 5% and 20% of perturbation in simulation and RUAV tracks the flight with speed from 1 m/s to 10 m/s. The stabilised process of end-effector’s pitch attitude is shown in Figure 7. When the kinetic parameters of the disturbance are over 15%, \(H_\infty\) controller has poor stability, which end-effector’s pitch attitude appears oscillation with constant amplitude or even divergence. However, the pitch attitude under model predictive control only present 8% of relative fluctuation before convergence. In conclusion, parameter perturbation stability of the predictive control is better than that of \(H_\infty\) control for manipulation of the robotic arm in hover mode. A detailed comparison of the two control methods is presented in the next section, which is about the tracking control precision for RAM precise operation in the absence of parameter perturbation.

### 4.3 Tracking control simulation and analysis

This simulation was intended to verify the accurate end-effector control capability, based on the proposed overall dynamics-based predictive control strategy, while performing the insertion task within the defined performance metric (hose to pump insertion). The ground effect is also considered as a sine disturbance. The simulation results are shown in Figure 8 and the corresponding comparisons of control inputs are in Figure 9.

The results showed that the RUAV’s controller cannot guarantee its position error below 1 cm because of the external disturbance, and the AM must be actuated properly to eliminate the position and attitude errors to accomplish the precise insertion. However, the anti-torque from the motors of AM is also a disturbance to the RUAV, which also enlarges the position and attitude error if it is not
Figure 8 shows the cyclic pitches and three principal axis control variables $m_{\mu3}$, $m_{\mu4}$, $m_{\mu5}$. The other three axis control variables are locked in the simulation for simplification of analysis. According to Figure 9, predictive control input is five control periods ahead of $H_\infty$ control and corresponding control absolute values are significantly larger than those in $H_\infty$ control. Based on the tracking results in Figure 8, the conservative of $H_\infty$ controller is not conducive to RAM’s accurate operation in hover mode. The proposed predictive control method in this paper applies receding horizon optimisation method to solving the conservative and dynamics control lag problems, and is more suitable for control applications of RAM.

Figure 8  Control performance comparisons for end-effector, (a) position tracking for the end-effector (b) the attitude control error for end-effector (see online version for colours)
5 Conclusions and future work

The dynamics of RAM system was analysed, and an overall dynamics model was developed in this paper. The relative disturbance between RUAV and robotic arm was predicted based on the proposed overall dynamics. Based on the proposed predictive controller, the relative force and torque disturbance was compensated completely for precise air operations. Compared with traditional controller, the efficiency of the proposed model and controller had been verified in insertion mission by simulations.

In the future, the platform of RAM will be constructed and modelled based on. The proposed model will be used to design a predictive controller, which will be also tested on the real flights.

References


