Utility-Based Resource Allocation in Wireless-Powered Communication Networks

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***Abstract*—This paper investigates the joint optimization of time allocation and transmit power in wireless-powered communication networks (WPCNs), where the downlink wireless energy transfer happens in the first phase, and then uplink wireless information transfer takes place in the second phase. Considering the rate fairness among users, we formulate the resource allocation in WPCNs as a network utility maximization (NUM) problem that aims to maximize the sum of utilities over all users subject to the energy causality constraint, the quality of service constraint, and the physical constraint. We further prove that the NUM problems with different channel access schemes (the time-division multiple access scheme, the concurrent transmission scheme, and the nonorthogonal multiple access scheme) can be equivalently transformed to convex problems by deliberated math transformations. Simulations demonstrate the efficiency of this paper.**

***Index Terms*—Harvest-then-transmit, radio frequency (RF) energy harvesting, utility, wireless-powered communication network (WPCN).**

I. Introduction

Energy harvesting is a promising solution to the energy-constrained problem in wireless networks. In particular, radio frequency (RF) energy harvesting becomes more flexible and sustainable than solar or wind energy harvesting since the RF signals radiated by ambient transmitters are consistently available. Numerous researches have exploited the RF signals for both wireless information transmission (WIT) and wireless energy transfer (WET) [1]–[3]. For different system configurations, a series of simultaneous wireless information and power transfer protocols have been proposed [1]. This line of work focusses on the fundamental tradeoff between the achievable throughput and the harvested energy.

There is another line of work on wireless-powered communication network (WPCN), which adopts the WET in traditional wireless communication systems [4]–[9]. In [4], "harvest-then-transmit" protocol was proposed, where the time allocated to the hybrid access point (H-AP) for downlink (DL) WET and the time and transmit power allocated to the users for uplink (UL) WIT were jointly optimized for system throughput maximization. Since then, related works have been extensively done in the contexts of WPCNs with relays [5], cognitive radio [6], massive multiple-input multiple-output [7], and nonorthogonal multiple access (NOMA) schemes [8]–[9]. So far, the fairness-aware resource allocation among users has not been fully addressed in the sense of global optimality for WPCNs [4]–[9]. Ju and Zhang [4] for the first time found "the doubly near-far" phenomenon in WPCNs and improved the max–min fairness of resource allocation by applying the common-throughput maximization method. In [8] and [9], the application of the NOMA scheme to a UL WPCN was studied to improve the max–min rate fairness and spectrum efficiency. However, other types of fairness except max–min fairness cannot be guaranteed by [4], [8], and [9].

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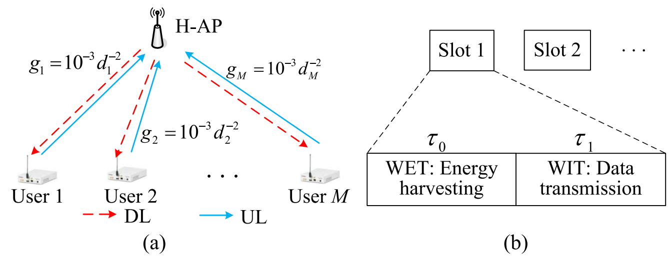


Fig. 1. System model. (a) Network architecture. (b) Slot architecture.

This paper generalizes [4], [8], and [9] by proposing a novel utility-based resource allocation method for WPCNs to achieve different types of fairness among users. We formulate the resource allocation in WPCNs as a network utility maximization (NUM) problem [10] whose objective is to maximize the sum of utilities over all users in WPCNs while simultaneously fulfilling energy causality, quality of service (QoS), and physical constraints. By using deliberated math transformations, we prove that the formulated NUM problems for time-division multiple access (TDMA) based WPCNs, concurrent transmission (CT) based WPCNs, and NOMA-based WPCNs can be equivalently transformed to convex problems. The major contributions of this paper are summarized as follows.

1) We propose to use an NUM framework to study the fairness-aware resource allocation in WPCNs, which adopts utility function of rates to describe the level of satisfaction attained by a user. Different shapes of utility functions lead to different types of fairness, which include [4], [8], and [9] as special cases of this paper.
2) We show that the formulated NUM problems for WPCNs with different channel access schemes all have equivalent but convex forms despite a high degree of coupling among variables.

II. System Model

This paper considers a slotted WPCN that consists of one single-antenna H-AP and M single-antenna users. As shown in Fig. 1(a), the H-AP transmits energy to M users in DL WET phase and receives information signals from M users in UL WIT phase. The DL and UL channels for each user are assumed to be reciprocal. Both WET and

WIT are operated over the same spectrum band. All users synchronize to the H-AP and operate in a half-duplex mode. Fig. 1(b) gives the slot architecture for the WPCN, whose WET and WIT phases are, respectively, with the durations of τ_0 and τ_1. In this paper, we set the slot duration $T = 1$ s for simplicity. Obviously

$$\tau_0 + \tau_1 \leq 1. \tag{1}$$

A. WET Phase

Each user is solely powered by an energy harvester that extracts energy from the RF signals of the H-AP. Due to the leakage of supercapacitor and the absence of energy storage, the possible remaining energy after transmission cannot be used in the next slot. Therefore, the initial energy of each user is zero. Then, the available energy at user i $(i = 1, 2, \ldots, M)$ by the end of the WET phase is given by

$$E_i = \xi_i g_i P_t \tau_0 \qquad \forall i \tag{2}$$

where ξ_i denotes the energy conversion efficiency that depends on the hardware design of energy harvester [3]. P_t denotes the transmit power of the H-AP and g_i denotes the channel gain between user i and the H-AP. Recently, a nonlinear energy harvesting (NEH) model is proposed in [2]. The extension of this paper to the NEH model is a future work.

B. WIT Phase

Let P_i denote the transmit power of user i. For notational convenience, we define $\tau = (\tau_0, \tau_1)$ and $\mathbf{P} = (P_1, P_2, \ldots, P_M)$.

1) TDMA Scheme: With the TDMA scheme, users transmit data to the H-AP in sequence by consuming their harvested energies. The transmission duration for user i is denoted by τ_{1i}. Then, the totally consumed power of user i in the WIT phase is upper bounded by its available power

$$\eta_i P_i + P_i^c \leq \frac{E_i}{\tau_{1i}} \qquad \forall i \tag{3}$$

where η_i ($\eta_i \geq 1$) is the reciprocal of user is power amplifier efficiency and P_i^c is the circuit power of user i. Equation (3) is also known as the energy causality constraint.

Then, we have the maximum achievable date rate for user i with the TDMA scheme as follows:

$$R_i^{\mathrm{TD}}(\tau_{1i}, P_i) = \tau_{1i} \log\left(1 + \frac{g_i P_i}{\sigma^2}\right) \tag{4}$$

where σ^2 denotes the noise power at the receiver of the H-AP.

In order to guarantee the QoS of users, we set a QoS constraint (i.e., a minimum required rate $\bar{R}_i$) for user i

$$R_i^{\mathrm{TD}}(\tau_{1i}, P_i) \geq \bar{R}_i \qquad \forall i. \tag{5}$$

2) CT Scheme: With the CT scheme, users transmit simultaneously to the H-AP by consuming their harvested energies. At this time, the totally consumed power of user i in the WIT phase is upper bounded by its available power

$$\eta_i P_i + P_i^c \leq \frac{E_i}{\tau_1} \qquad \forall i \tag{6}$$

and the maximum achievable date rate for user i with the CT scheme is given as follows:

$$R_i^{\mathrm{CT}}(\tau, \mathbf{P}) = \tau_1 \log\left(1 + \frac{g_i P_i}{\sum_{k \neq i} g_k P_k + \sigma^2}\right). \tag{7}$$

Similar to (5), we require

$$R_i^{\mathrm{CT}}(\tau, \mathbf{P}) \geq \bar{R}_i \qquad \forall i. \tag{8}$$

3) NOMA Scheme: Recently, a few works [8], [9] have considered NOMA with a successive interference cancelation (SIC) receiver in the context of WPCNs.

Similar to the CT scheme, with the NOMA scheme, users transmit simultaneously to the H-AP, subject to the energy causality (6). For rate fairness, users' information is decoded in a sequence by an increasing order of channel gains g_i. For convenience, we denote user i as the ith user in the decoding sequence. Specifically, once user i is decoded, the reconstructed signal for user i is removed from the composite signal at the H-AP. The process continues until all users are decoded. Assuming perfect cancelation, we calculate the maximum achievable data rate for user i as follows:

$$R_i^{\mathrm{NO}}(\tau, \mathbf{P}) = \tau_1 \log\left(1 + \frac{g_i P_i}{\sum_{k=i+1}^{M} g_k P_k + \sigma^2}\right) \tag{9}$$

and $R_i^{\mathrm{NO}}(\tau, \mathbf{P})$ is required to fulfill the QoS constraint

$$R_i^{\mathrm{NO}}(\tau, \mathbf{P}) \geq \bar{R}_i \qquad \forall i. \tag{10}$$

C. Optimization Model

Next, we will formulate the utility-based optimization model in terms of τ and $\mathbf{P}$. We leverage a parameterized utility function $U_\alpha(x)$ to describe the rate level of satisfaction of users. Specifically

$$U_\alpha(x) = \begin{cases} \log x, & \alpha = 1 \\ (1-\alpha)^{-1} x^{1-\alpha}, & \alpha > 1. \end{cases} \tag{11}$$

Fairness parameter α may lead to different fairness [10], e.g., $U_\alpha(x)$ guarantees proportional, harmonic mean, and max–min fairness when α=1, 2, and $+\infty$, respectively.

Now, we are in the position to state the NUM formulation of the utility-based resource allocation problem in the WPCN

$$\text{TDMA}: \quad \max_{\tau, \mathbf{P}} \sum_{i=1}^{M} w_i U_\alpha(R_i^{\mathrm{TD}}(\tau_{1i}, P_i)) \tag{12a}$$

$$\text{s.t.} \quad (3)\text{–}(5) \tag{12b}$$

$$\tau_0 + \sum_{i=1}^{M} \tau_{1i} \leq 1 \tag{12c}$$

$$\tau_0 \geq 0, \tau_{1i} \geq 0 \qquad \forall i \tag{12d}$$

$$P_i \geq 0 \qquad \forall i \tag{12e}$$

$$P_i \leq P_i^{\max} \qquad \forall i \tag{12f}$$

or

$$\text{CT}: \quad \max_{\tau, \mathbf{P}} \sum_{i=1}^{M} w_i U_\alpha(R_i^{\mathrm{CT}}(\tau, \mathbf{P})) \tag{13b}$$

$$\text{s.t.} \quad (1) \text{ and } (6)\text{–}(8) \tag{13c}$$

$$\tau_k \geq 0, k = 0, 1 \tag{13d}$$

$$P_i \geq 0 \qquad \forall i \tag{13e}$$

$$P_i \leq P_i^{\max} \qquad \forall i \tag{13f}$$

or

$$\text{NOMA}: \quad \max_{\tau,\mathbf{P}} \sum_{i=1}^{M} w_i U_\alpha(R_i^{\text{NO}}(\tau,\mathbf{P}))$$
$$\text{s.t.} \quad (1),\ (6),\ (9),\ \text{and}\ (10)$$
$$(13c)\text{–}(13e) \tag{14}$$

where w_i ($w_i > 0$) denotes the weight associated with user i and $P_i^{\max}$ represents hardware or regulatory limitations on transmit power P_i. We assume that the channel information g_i is available for resource allocation.[1]

III. Resource Allocation for TDMA-Based WPCNs

We will first transform problem (12) into an equivalent problem, and then prove its convexity despite the tight coupling in constraints (3)–(5). To this end, we introduce a set of slack variables x_i and define $e_i = \tau_{1i} P_i$. Due to the increasing monotonicity of $U_\alpha(.)$, we find an equivalent form of problem (12)

$$\text{TDMA}: \quad \max_{\hat{\tau},\mathbf{x},\mathbf{e}} \sum_{i=1}^{M} w_i U_\alpha(x_i) \tag{15a}$$
$$\text{s.t.} \quad \tau_0 + \sum_{i=1}^{M} \tau_{1i} \le 1 \tag{15b}$$
$$\eta_i e_i + \tau_{1i} P_i^c \le \xi_i g_i P_t \tau_0 \quad \forall i \tag{15c}$$
$$\tau_0 \ge 0,\ \tau_{1i} \ge 0 \quad \forall i \tag{15d}$$
$$e_i \ge 0 \quad \forall i \tag{15e}$$
$$e_i \le \tau_{1i} P_i^{\max} \quad \forall i \tag{15f}$$
$$\tau_{1i} \log\left(1 + \frac{g_i e_i}{\sigma^2 \tau_{1i}}\right) \ge x_i \quad \forall i \tag{15g}$$
$$\tau_{1i} \log\left(1 + \frac{g_i e_i}{\sigma^2 \tau_{1i}}\right) \ge \bar{R}_i \quad \forall i \tag{15h}$$

where $\hat{\tau} = (\tau_0, \tau_{11}, \dots, \tau_{1M})$, $\mathbf{x} = (x_1, x_2, \dots, x_M)$, and $\mathbf{e} = (e_1, e_2, \dots, e_M)$.

Theorem 1: Problem (15) is convex.

Proof: It can be easily verified that the objective function (15a) is concave and the constraints (15b)–(15f) are affine. The convexity of constraints (15g) and (15h) can also be shown since the left-hand side of (15g) and (15h) is the perspective of a concave function $\log\left(1 + \frac{g_i e_i}{\sigma^2}\right)$ [10]. ■

IV. Resource Allocation for CT-Based WPCNs

It is clearly shown in problem (13) that variables τ and $\mathbf{P}$ are tightly coupled in the objective function (13a) and constraints (6) and (8). In order to address the nonconvexity, we find an equivalent but convex problem to problem (13).

Lemma 1: Suppose a feasible solution $\{\tau, \mathbf{P}\}$ to problem (13) exists. Then, we have $\tau \succ 0$ and $\mathbf{P} \succ 0$, where "$\succ$" denotes the componentwise inequality.

Proof: From the definition of $R_i^{\text{CT}}(\tau,\mathbf{P})$ in (7), we know that $R_i^{\text{CT}}(\tau,\mathbf{P})$ will be 0 if either $\tau_k = 0$ or $P_i = 0$. Consequently, the constraint (8) will be violated for user i. This completes the proof of Lemma 1. ■

[1] g_i can be perfectly obtained by the H-AP via sending pilot signals to user i and collecting channel estimation feedback from user i.

Considering Lemma 1 and the increasing monotonicity of $U_\alpha(.)$, we find an equivalent problem to problem (13) by introducing a set of slack variables e^{z_i} and defining $\tau_k = e^{t_k}$ and $P_i = e^{y_i}$

$$\text{CT}: \quad \max_{\mathbf{z},\mathbf{t},\mathbf{y}} \sum_{i=1}^{M} w_i U_\alpha(e^{z_i}) \tag{16a}$$
$$\text{s.t.} \quad e^{t_0} + e^{t_1} \le 1 \tag{16b}$$
$$e^{y_i} \le P_i^{\max} \quad \forall i \tag{16c}$$
$$\eta_i e^{y_i} + p_i^c \le \xi_i g_i P_t e^{(t_0 - t_1)} \quad \forall i \tag{16d}$$
$$e^{t_1} \log\left(1 + \frac{g_i e^{y_i}}{\sum_{k\ne i} g_k e^{y_k} + \sigma^2}\right) \ge \bar{R}_i \quad \forall i \tag{16e}$$
$$e^{t_1} \log\left(1 + \frac{g_i e^{y_i}}{\sum_{k\ne i} g_k e^{y_k} + \sigma^2}\right) \ge e^{z_i} \quad \forall i \tag{16f}$$

where $\mathbf{z} = (z_1, z_2, \dots, z_M)$, $\mathbf{t} = (t_0, t_1)$, and $\mathbf{y} = (y_1, y_2, \dots, y_M)$.

Theorem 2: Problem (16) is convex.

Proof: First, we prove that $U_\alpha(e^{z_i})$ is concave in z_i. For notational convenience, we define $\psi_i = e^{z_i}$ and take the second derivative of $U_\alpha(\psi_i)$ with respect to z_i

$$\frac{d^2 U_\alpha(\psi_i)}{dz_i^2} = \left(\frac{d^2 U_\alpha(\psi_i)}{d\psi_i^2}\psi_i + \frac{dU_\alpha(\psi_i)}{d\psi_i}\right)\psi_i. \tag{17}$$

It is trivial to verify that $\frac{d^2 U_\alpha(\psi_i)}{dz_i^2} \le 0$ for $\alpha \ge 1$, which implies the concavity of $U_\alpha(e^{z_i})$ in z_i. Further, we can declare the concavity of objective function (16a), which is the weighted sum of all $U_\alpha(e^{z_i})$.

Then, we focus on the proof of convex constraints. Obviously, constraints (16b) and (16c) are convex since exponential functions are convex. Taking the natural logarithm function on both sides of (16d), we have

$$\log(\eta_i e^{y_i} + p_i^c) + t_1 - t_0 \le \log(\xi_i g_i P_t)$$

which implies the convexity of constraint (16d). To prove the convexity of constraint (16e), we reformulate it as follows:

$$\underbrace{\log\left(\frac{\sigma^2}{g_i} e^{-y_i} + \sum_{k\ne i} \frac{g_k}{g_i} e^{(y_k - y_i)}\right)}_{\Delta(\mathbf{y})} + \underbrace{\log\left(e^{\bar{R}_i e^{-t_1}} - 1\right)}_{f(t_1)} \le 0. \tag{18}$$

As $\log(\Delta(\mathbf{y}))$ is the convex log-sum-exp function of $\mathbf{y}$ [10], the convexity of constraint (18) depends on $f(t_1)$. Taking the first derivative of $f(t_1)$ with respect to t_1 yields

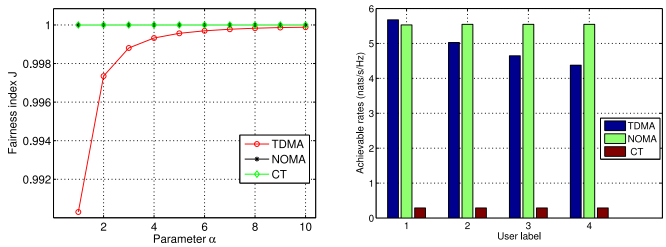
$$\frac{df(t_1)}{dt_1} = \frac{-\bar{R}_i e^{-t_1} e^{\bar{R}_i e^{-t_1}}}{e^{\bar{R}_i e^{-t_1}} - 1}. \tag{19}$$

It is clear from (19) that $\frac{df(t_1)}{dt_1}$ is monotonic increasing in t_1 (i.e., $\frac{d^2 f(t_1)}{dt_1^2} > 0$), which proves the convexity of (16e).

Constraint (16f) has an equivalent form similar to (18)

$$\log(\Delta(\mathbf{y})) + \underbrace{\log\left(e^{e^{z_i - t_1}} - 1\right)}_{F(z_i, t_1)} \le 0. \tag{20}$$

According to convex theory, the convexity of $F(z_i, t_1)$ in (z_i, t_1) can be established if the Hessian matrix of $F(z_i, t_1)$ is positive semidefinite. Skipping the tedious math deduction, we obtain the Hessian matrix of $F(z_i, t_1)$ as $H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}$, where $H_{11} = H_{22} = \frac{\Xi e^{\Xi}(e^{\Xi} - \Xi - 1)}{(e^{\Xi} - 1)^2}$,



(a) (b)

Fig. 2. Performance comparison. (a) Rate fairness. (b) Spectrum efficiency.

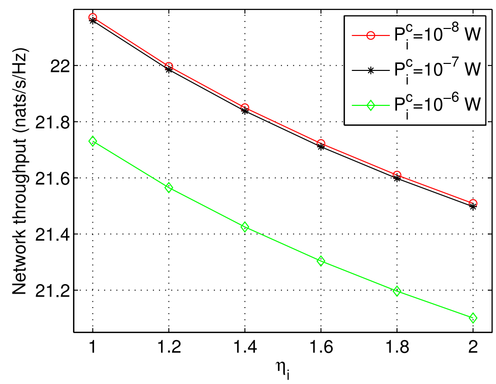


Fig. 3. Performance analysis.

$H_{12} = H_{21} = -\frac{\Xi e^{\Xi}(e^{\Xi}-\Xi-1)}{(e^{\Xi}-1)^2}$, and $\Xi = e^{z_i - t_1}$. As function $\hat{f}(t) = e^t - t - 1$ is monotone increasing, we have $\hat{f}(\Xi) > \hat{f}(0) = 0$ since $\Xi > 0$. Therefore, $H_{11} > 0$. Furthermore, we calculate the determinant of H, $|H| = H_{11}H_{22} - H_{12}H_{21} = 0$. So far, we have proved the positive semidefiniteness of H, which implies the convexity of constraint (16f).

Finally, we conclude Theorem 2 since (16a) is concave and the feasible domain defined by (16b)–(16f) is convex. ■

Due to the convex nature, problems (15) and (16) can be centrally solved at the H-AP via efficient convex solvers [10]. The resource allocation problem (14) can be similarly shown convex as problem (13) and thus omitted here.

V. Numerical Results

A WPCN with one H-AP and four users is simulated. The distance between user i ($i = 1, 2, 3, 4$) and the H-AP is set $d_i = 2.5i$ (Unit: meter). Simulation parameters are set as in [4], [8], and [9]: $M = 4$, $P_t = 1$ W, $\sigma^2 = -160$ dBm/Hz, $P_i^{\max} = 1$ W, $\bar{R}_i = 0.5$ nats/s/Hz, $g_i = 10^{-3} d_i^{-2}$, $\eta_i = 1$, and $w_i = \frac{1}{M}$.

Fig. 2(a) shows the rate fairness comparison of WPCNs with different channel access schemes ("TDMA," "CT," and "NOMA"). We set $\eta_i = 1$ and $P_i^c = 0$ W for fair comparison with [4], [8], and [9], in which η_i and P_i^c are not considered. In order to fairly compare the three schemes, we use Jain's fairness index [9]. A larger J implies a more fair rate allocation. It is clearly shown in Fig. 2(a) that Jain's fairness indexes of "NOMA" and "CT" are always close to 1 for different values of α, whereas "TDMA" is less fair due to "the doubly near-far" phenomenon. We also observe that Jain's fairness index of "TDMA" increases with the increase of α, which implies that we can achieve a more fair rate allocation for "TDMA" by setting a large α in problem (15).

Fig. 2(b) shows the spectrum efficiency comparison of three schemes when $\alpha = 1$. Due to the strictly limited transmit power and multiple-access interference, the spectrum efficiency of "CT" is so poor that the best effort rate for each user is 0.2887 nats/s/Hz, which is lower than $\bar{R}_i$. In contrast, "TDMA" has an improved rate owing to the orthogonal transmission in the WIT phase. "NOMA" further improves the spectrum efficiency of "TDMA" by adopting the SIC technique and large transmission time. As a result, the rate gain of "NOMA" to "CT" and "TDMA" are 19.2673 and 1.1239, respectively.

Fig. 3 analyzes the network throughput of "NOMA" under different values of (η_i, P_i^c) when $\alpha = 1$. It is obvious that the network throughput decreases with respect to η_i and P_i^c. This is due to the fact that increasing η_i and P_i^c aggravates the power consumption of user i, which further shrinks the feasible domain of problem (16) and finally degrades the network throughput.

VI. Conclusion

This paper has studied the joint optimization of time allocation and transmit power in WPCNs with the harvest-then-transmit protocol. Simulations have been performed to validate the efficiency of this work.

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