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A Track Initiation Method for the Underwater Target Tracking Environment

LI Dong-dong[a, b, *], LIN Yang[a], ZHANG Yao[a]

[a]State Key Laboratory of Robotics, Shenyang Institute of Automation, Chinese Academy of Sciences, Shenyang 110016, China

[b]University of Chinese Academy of Sciences, Beijing 100049, China

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Abstract

A novel efficient track initiation method is proposed for the harsh underwater target tracking environment (heavy clutter and large measurement errors): track splitting, evaluating, pruning and merging method (TSEPM). Track initiation demands that the method should determine the existence and initial state of a target quickly and correctly. Heavy clutter and large measurement errors certainly pose additional difficulties and challenges, which deteriorate and complicate the track initiation in the harsh underwater target tracking environment. There are three primary shortcomings for the current track initiation methods to initialize a target: (a) they cannot eliminate the turbulences of clutter effectively; (b) there may be a high false alarm probability and low detection probability of a track; (c) they cannot estimate the initial state for a new confirmed track correctly. Based on the multiple hypotheses tracking principle and modified logic-based track initiation method, in order to increase the detection probability of a track, track splitting creates a large number of tracks which include the true track originated from the target. And in order to decrease the false alarm probability, based on the evaluation mechanism, track pruning and track merging are proposed to reduce the false tracks. TSEPM method can deal with the track initiation problems derived from heavy clutter and large measurement errors, determine the target's existence and estimate its initial state with the least squares method. What's more, our method is fully automatic and does not require any kind manual input for initializing and tuning any parameter. Simulation results indicate that our new method improves significantly the performance of the track initiation in the harsh underwater target tracking environment.

Key words: track initiation, track splitting, track evaluating, track pruning, track merging, large measurement errors, heavy clutter, harsh underwater environment, TSEPM

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1 Introduction

The problem of multiple target tracking (MTT) has been an active field for many years (Isbitiren and Akan, 2011; Lee and Song, 2017; Yan et al., 2015; Raj and Krishna, 2015). Track initiation is the primary problem of the MTT system, which requires accurate estimation of the targets' initial states, and the number of existing targets (Liu et al., 2016; Mallick et al., 2015; Kural et al., 2009; Jiang et al., 2014). The similarity between the track initiation and target tracking is that both should solve the problem of data association and estimation. The distinction is that track initiation should determine the number and initial states of targets, and target tracking should improve the accuracy of estimation. Lots of novel methods were presented to solve the problem of target tracking, but little attention focuses on the track initiation in the unique underwater target tracking environment. If the track initiation method fails to initialize tracks of existing targets, trackers may miss the opportunity to track and identify the potential targets. On the other hand, in case where false tracks are initialized, limited sensor resource is wasted, and the computational burden is increased to maintain non-existing targets, resulting in the reduction of the number of true targets to be tracked. If a track is initialized in a statistically consistent manner, then it can prevent filter diverging, improve measurement-to-track association, reduce the number of false tracks, and help track management (Ristic et al., 2011; Schuhmacher et al., 2008; Kennedy, 2014).

By considering the problem of initializing a track with nearly constant velocity when position-only measurements are available, the characteristic of the active sonar for detecting the target is of low detection frequency, heavy clut-

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*Corresponding author. E-mail: lidongdong@sia.cn

ter and large measurement errors. That is why those of both conventional and modern track initiation methods cannot be applied to the underwater target tracking environment.

These conventional track initiation methods can roughly be classified as sequential methods (the heuristic method and the logic-based method) and batch processing methods (the Hough transform method and the modified Hough transform method) (Hu et al., 1997; Leung et al., 1996). While the sequential methods are widely used in radar and sonar tracking, the batch processing methods are often used in image processing and tracking. We focus only on the sequential methods in this paper in terms of the underwater sonar application.

Based on the conventional track initiation methods, more modern methods are presented to improve the performance, such as single point track initiation method (Mallick et al., 2015; Yeom et al., 2004), two points track initiation method (Musicki and Song, 2013; Musicki and La Scala, 2008), cooperative track initiation method (Liu et al., 2016), parameterized multiple model method and clutter-based test statistics method (Kennedy, 2008a, 2008b).

The single point track initiation method initializes the target's state only with a single measurement (range and azimuth), relying on the additional or auxiliary measurement information, such as amplitude, radial velocity, and range-rate measurement information to improve the association probability (Kural et al., 2006). For example, the single point track initiation method with the amplitude measurement can exclude some measurements from clutter. However, in the underwater target tracking environment, the single point track initiation method is difficult to detect the target with low amplitude (some small targets or stealth underwater vehicles).

The two points track initiation method uses information on the first two measurements alone to initialize the filter (Mallick and La Scala, 2008). Each pair of measurements from two consecutive scans satisfying the maximum target velocity constraint are used to initialize new tracks. The two consecutive measurements can initialize the position and velocity vector. It performs well in the case of less clutter, high sensor detection probability and high precision measurement. However, in the underwater target tracking environment, it is difficult to determine the initial state accurately because of large measurement errors.

Liu et al. (2016) proposed a cooperative track initiation method for distributed radar network. It aims to improve track initiation performance of invisible radar sites that have not initialized the target track, by using target track information provided by the radar sites that have tracked the target. Distributed or multiple sensors network (Lin and Huang, 2011; Charlish et al., 2012) is a hot topic in the radar field since it owns many advantages over a single radar, such as the improved detection, tracking and localization performance. However, this method is unavailable to apply to the underwater environment. First, the underwater vehicles always perform tasks itself, so it cannot obtain other information from friend forces. Second, as we all know, there is no secure and reliable communication method in the underwater environment. So even if there is sharable information, it is difficult to obtain and process shared information timely.

Parameterized multiple model method usually initializes the state of a target of interest with only one or two measurement points. Without a prior knowledge of target motion status, the parameterized multiple model method divides the possible speed interval $[v_{\min}, v_{\max}]$ and heading interval $[\varphi_{\min}, \varphi_{\max}]$ into several some subintervals. Then it establishes a filter for every subinterval (Mallick et al., 2015). All the filters run concurrently, and the estimate output is the weighting output of all the filters.

Kennedy (2008a) proposed the clutter-based test method that a tentative track is confirmed when the track-is-on-clutter hypothesis is rejected. Based on the clutter density information, the null hypothesis is rejected when the expected distance to the nearest target peak is less than the expected distance to the nearest clutter peak, which results in a high target detection probability of the track initiation method.

Considering the situation where only the position measurements are available, the parameterized multiple model method and clutter-based test method can be used to solve the problem of track initiation in the underwater target tracking environment.

In order to deal with the large measurement errors and heavy clutter environment in the harsh underwater environment, based on the principle of multiple hypotheses tracking (MHT) (Lin and Huang, 2011), modified logic-based track initiation method, and statistical hypotheses test, we propose the track splitting, evaluating, pruning, merging method. The analyses and simulations demonstrate that the proposed method has the superior performance, compared with the parameterized multiple model method and clutter-based test method.

The structure of this paper is organized as follows. Section 2 states a brief description of the problems of track initiation in the presence of heavy clutter and large measurement errors in the harsh underwater environment. Section 3 describes the important issues of our novel method. Section 4 describes the flow diagram of the method. The performance analyses and simulations of the novel method are introduced in Section 5, and show a notable improvement in the performance of track initiation. Finally, Section 6 summarizes the main conclusions and results of the paper.

For simplicity, $p_{\rm d}$ is the sensor detection probability of a target, $p_{\rm D}$ is the algorithm detection probability of a track, and $p_{\rm F}$ is the false alarm probability.

2 Problems of track initiation in the harsh underwater environment

The purpose of track initiation is to obtain high, low $p_{\rm F}$

and relatively accurate initial state and covariance matrix. High p_D guarantees that the true track of a target can be formed as soon as possible. Low p_F reduces the disturbances of clutter, and computational burden. The relatively accurate initial state and covariance matrix guarantee the stability during the target tracking stage and avoid the divergence of the filter by use of the nonlinear measurement model and heavy clutter.

For the sake of completeness, in Subsection 2.1, we sketch briefly the unique underwater environment: heavy clutter and large measurement errors. In Subsection 2.2, we introduce the new idea about the TSEPM method.

2.1 Heavy clutter and large measurement errors

The clutter density reflects that the statistical characteristics of the background clutter (Hu et al., 1997). The clutter density λ is equal to the ratio of the expected number of clutter $\overline{m}$over the total observation area A, i.e.

$$\lambda = \frac{\overline{m}}{A}. \tag{1}$$

Both the number and the position of clutter points are assumed to be random and statistically independent from scan to scan. In particular, the number of clutter points n in each scan is assumed to have Poisson distribution, i.e.

$$p_{\overline{m}} = \frac{\overline{m}^n}{n!}\mathrm{e}^{-\overline{m}}, \tag{2}$$

where $p_{\overline{m}}$ is the Poisson function. The location of these clutter points (x_i, y_i) is assumed to be uniformly distributed in the total observation area.

Heavy clutter implies that the clutter plays nonnegligible influence on the track initiation process. For example, in Fig. 1, Track 1 is a false alarm whose measurements are derived from the clutter. Track 1 will influence the judgment of the target tracking process, and increase the computational burden. Some measurements of Track 2 are derived from the clutter, and the others are derived from a target. If a confirmed track includes some clutter, then it will decrease the accuracy of initial state. Track 3 is the true track of a target, which is our desired track. Heavy clutter will increase the probability of forming Track 1 and Track 2, and decrease the probability of forming Track 3.

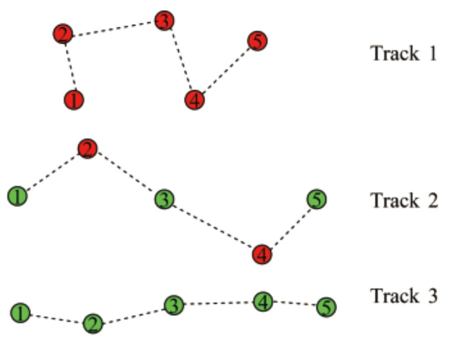

Fig. 1. Nonnegligible influence on the track initiation process generated by heavy clutter. Red and green circles indicate the clutter and measurement derived from targets, respectively. The number means time. Thus, Track 1 is a false alarm, Track 3 is a true track, and Track 2 is a semi-correct track.

The measurement errors reflect the measurement accuracy of a sensor. The measurement errors are relatively large, namely the sensor measurement factors should not be neglected compared with the target movement factors. Li et al. (2016) introduced the relationship between the target movement factors and sensor measurement factors, and that the measurement factors also affect the performance of track initiation. Moreover, we noticed two errors in Li et al. (2016): (1) defining the parameter to denote the ratio of the sensor measurement error variable to target movement variable; (2) defining the gate threshold (Aslan and Saranli, 2011). Here we present corrections to this algorithm.

$$\frac{\sigma_{\mathrm{R\,min}}}{0.5v_{\max}T} \geqslant 1; \tag{3}$$

$$\gamma_1 = v_{\max}kT + \alpha\sigma_{\mathrm{R\,max}}, \qquad k = 1,2,..., \tag{4}$$

where T is the sampling period, $v_{\max}$ is the maximum speed of the target, γ_1 is the gate threshold for associating the two measurements of different scans, $\sigma_{\mathrm{R\,min}}$ is the minor axis of the measurement covariance matrix, $\sigma_{\mathrm{R\,max}}$ is the major axis of the measurement covariance matrix, k is the scan interval number between two continuous association measurements, and α is a tunable parameter corresponding to the probability P or the confidence coefficient. If Eq. (3) holds, it denotes large measurement error environments. The follow equation provides the principle to tune the parameter α corresponding to the probability P.

$$\alpha = \sqrt{-2\ln(1-P)}. \tag{5}$$

With Eqs. (4) and (5), the probability (gate probability) is larger than P that the area determined by threshold γ_1 includes the target's measurement point of next scan. For example, if P is 0.9, then α is 2.146, and Eq. (4) can ensure that the confidence coefficient is larger than 0.90, which it would be about 90% confident that the target measurement point falls into the validation predicted area, if the target has been detected. Namely, in a bivariate Gaussian distribution, the 2.146–0.90($\alpha - P$) ellipse contains 90% of the probability mass.

What is more, the combined effect of heavy clutter and large measurement errors will further make it harder of track initiation method to increase p_D, decrease p_F. If measurement errors are relatively small, the gating ellipse will be small, and a track is much easier to associate with the target's measurements than clutter. So if the sonar measurements, typically ranging and bearing from the senor location, have no error in them, then we can simply "connect the dots" to estimate the target's track, even though there are plenty of clutter. But as the errors in prediction or measurement increase, the gating region will increase rapidly, the tracks is more inclined to associate with clutter, and the method of "connect the dots" does not work.

2.2 New idea about the TSEPM method

The principle of logic-based track initiation method, multiple hypothesis tracking principle, hypotheses test statistics method (Tang et al., 2017) and least squares method, provide a feasible solution increase p_D, decrease p_F, and obtain the relatively accurate initial state.

If we transplant the MHT principle to track initiation process, we can accurately determine the true potential track originated from a target, and then its initial state and initial covariance matrix can be obtained by the least squares method. There are still some drawbacks for the MHT principle, since it also creates a large number of false tracks, which is unacceptable. Then if an evaluation mechanism is created for the tracks, then we can evaluate all the tracks and distinguish them by a number which can indicate a track originated from a target or clutter. If this evaluation mechanism performs effectively, then we can prune the tracks which seem more like originated from clutter, and retain the tracks which seem more like originated from targets. Then, based on the evaluation value after the track pruning process, the remaining tracks can be viewed as originated from targets. Moreover we can further decrease the number of tracks by the track merging process, if evaluation values and initial states of these tracks are close to each other, and most of their measurement points overlap, namely these tracks meet the conditions of track merging.

2.3 Important issues of the TSEPM method

In this section, some important issues about the TSEPM method will be discussed in detail. To make a long story short, firstly, based on the MHT principle and modified logic-based track initiation method, we obtain the true track of a target, and at the same time, this method also creates a large number of false tracks. In order to decrease the number of false tracks, secondly, evaluation mechanism is used, and every track is attached with a reasonable evaluation value. Thirdly, based on the evaluation values, we prune all the tracks which seem more likely originated from clutter, and retain the tracks which seem more likely originated from targets. Fourthly, we merge those tracks into one track, which meets the conditions of track merging. This is our novel TSEPM method. To some extent, the track pruning and merging process makes up the shortcomings of track splitting, and reduces the computational burden. So the TSEPM method can increase p_D without increasing p_F.

There are additional factors to be considered for track initiation problems. In real-world sonar tracking problems, the measurements are the range and azimuth in the 2D case, and the measurement information also includes the clutter from the background. No additional information can be used to distinguish the measurements from targets or clutter. The measurements originated from the target only depend on the current target state, and the state variable is assumed to obey the Gaussian distribution, shown as follows:

$$z_k = h(\boldsymbol{x}_k) + \boldsymbol{v}_k; \tag{6}$$

$$h(\boldsymbol{x}_k) = \begin{bmatrix} \sqrt{(x_{k,x} - ox_{k,x})^2 + (x_{k,y} - ox_{k,y})^2} \\ \arctan\left(\dfrac{x_{k,x} - ox_{k,x}}{x_{k,y} - ox_{k,y}}\right) \end{bmatrix}; \tag{7}$$

$$E(\boldsymbol{v}_k\, \boldsymbol{v}_k^{\mathrm{T}}) = \boldsymbol{R}_k, \tag{8}$$

where $\boldsymbol{x}_k = \left[x_{k,x}, x_{k,vx}, x_{k,y}, x_{k,vy}\right]^{\mathrm{T}}$ is the target state, $x_{k,x}$ and $x_{k,y}$ are the Cartesian coordinates position of the target, and $x_{k,vx}$ and $x_{k,vy}$ are the speed of the target at time k . v_k is the measured Gaussian white noise with zero mean and covariance matrix $\boldsymbol{R}_k$. z_k is the measurement information (position measurements). $h(\cdot)$ is the measurement model. $E(\cdot)$ is the second-order statistical properties of the noise $\boldsymbol{v}_k \boldsymbol{ox}_k = \left[ox_{k,x}, ox_{k,vx}, ox_{k,y}, ox_{k,vy}\right]^{\mathrm{T}}$ is the state of the active sonar. As for active sonar, the measurement error matrix $\boldsymbol{R}_k$ is known, and we assume that p_D is also known.

In the tracking application, the target position measurements are provided in the polar coordinates in terms of the range and azimuth with respect to the sensor location. Unfortunately, in most systems, the target motion is usually best modeled in a simple fashion using the Cartesian coordinates. This situation requires either converting the measurements into a Cartesian frame of reference and working directly on converted measurements or using the Extended Kalman Filter in the mixed coordinates. For simplicity, the former method is chosen for our novel track initiation method. Namely, an accurate approach of target positions with unbiased consistent converted measurements is presented (Zhao et al., 2004). With the unbiased consistently converted process, Eqs. (6)–(8) can be converted as the linear equations, shown as follows:

$$z_k = \boldsymbol{H}_k \boldsymbol{x}_k + \hat{\boldsymbol{v}}_k; \tag{9}$$

$$\boldsymbol{H}(\boldsymbol{x}_k) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}; \tag{10}$$

$$E(\hat{\boldsymbol{v}}_k\, \hat{\boldsymbol{v}}_k^{\mathrm{T}}) = \hat{\boldsymbol{R}}_k, \tag{11}$$

where $\hat{\boldsymbol{v}}_k$ is the converted measurement noise, $\boldsymbol{H}_k$ is the measurement matrix in the Cartesian coordinates, and $\hat{\boldsymbol{R}}_k$ is the converted measurement covariance error matrix.

3 Methodology

3.1 Rule of forming a new confirmed track

In consideration of the real-time requirements, the modified logic-based track initiation method (*M/N* method) is used to form a new confirmed track. We modify the gate threshold rule for the harsh underwater environment, as shown in Eq. (4). Then we should determine the values of *M* and *N*. *M* denotes the number of all valid measurements which would be used to estimate the initial state. *N* affects

p_D and p_F. And many factors affect the values of M and N. In Section 5, we will introduce how to determine the values of M and N in detail.

For example, if M is 5, and N is 8, two confirmed tracks are created respectively shown in Fig. 2, in which Track 1 costs a relatively short time to form a confirmed track and Track 2 costs a relatively long time to form a confirmed track. The number means time, and there are no measurement points updating Track 2 at time 3, 6 and 7.

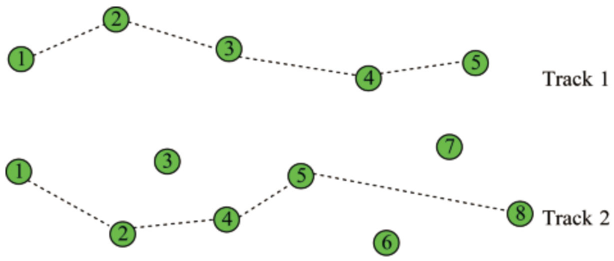

Fig. 2. Two confirmed tracks.

3.2 Track splitting

If the clutter density is large (there are multiple measurements in the validation predicted area), the track splitting method is needed for this scenario. Based on the MHT principle, we need to create a potential track for every possible association.

As shown in Fig. 3, at time 4, there are two measurement points in the validation predicted area of measurement point 3. The measurement information is not enough to distinguish and eliminate the false track at time 4. Therefore, we have to reserve these two tracks. After we have obtained enough measurements at time 5, we may distinguish them by evaluating the value information. Since there are two measurement points in the validation predicted area of the potential track at time 4, and then we should create two potential tracks for the two possible associations. Even though one of them is a false track, current measurement information is not enough to distinguish these two tracks. Track splitting increases the number of tracks, but one of them must be the true track, which is the main purpose of track splitting. As for the false tracks, we can eliminate them by track pruning and track merging.

3.3 Estimating the initial state and covariance matrix

After determining a new confirmed track by the modified logic-based method, the least squares method is used to estimate its initial state and covariance matrix.

Assume that a target moves with a nearly constant velocity and the measurements of the target position are detected in the less than or equal N scans. And further assume that state $\boldsymbol{X}_{tM}$ is needed for the estimation at time tM, and its dimension is n_x. In general, we cannot get the true value of $\boldsymbol{X}_{tM}$ odirectly at time tM, but we can obtain the noisy measurement information of the vector $\boldsymbol{X}_{tM}$. Then assume

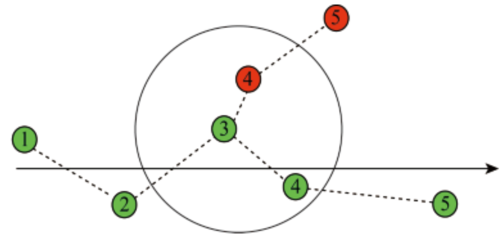

Fig. 3. A track splitting with two tracks.

that $\boldsymbol{Z}_{ti}$ $(1 \leqslant i \leqslant M)$ is the converted measurement at time ti, based on Eqs. (9)–(11), we have

$$\boldsymbol{Z}_{ti} = \boldsymbol{H}_{ti}\boldsymbol{X}_{tM} + \boldsymbol{v}_{ti} \quad (1 \leqslant ti \leqslant tM \leqslant N); \tag{12}$$

$$E(\boldsymbol{v}_{ti}\boldsymbol{v}_{ti}{}^{\mathrm{T}}) = \hat{\boldsymbol{R}}_{ti}, \tag{13}$$

where $\boldsymbol{Z}_{ti}$ is the n_z dimension measurement vector, $\boldsymbol{H}_{ti}$ is the measurement matrix, and $\boldsymbol{v}_{ti}$ is the noise vector at time ti. If there are M measurements, then

$$\begin{aligned} &\boldsymbol{Z}_{t1} = \boldsymbol{H}_{t1}\boldsymbol{X}_{tM} + \boldsymbol{v}_{t1};\\ &\boldsymbol{Z}_{t2} = \boldsymbol{H}_{t2}\boldsymbol{X}_{tM} + \boldsymbol{v}_{t2};\\ &\cdots\\ &\boldsymbol{Z}_{tM} = \boldsymbol{H}_{tM}\boldsymbol{X}_{tM} + \boldsymbol{v}_{tM}. \end{aligned} \tag{14}$$

Then we obtain the measurement formula

$$\boldsymbol{Z} = \boldsymbol{H}\boldsymbol{X}_{tM} + \boldsymbol{V}, \tag{15}$$

where, $\boldsymbol{Z} = \begin{bmatrix} Z_{t1} \\ Z_{t2} \\ \cdots \\ Z_{tM} \end{bmatrix}$, $\boldsymbol{H} = \begin{bmatrix} H_{t1} \\ H_{t2} \\ \cdots \\ H_{tM} \end{bmatrix}$, $\boldsymbol{V} = \begin{bmatrix} v_{t1} \\ v_{t2} \\ \cdots \\ v_{tM} \end{bmatrix}$ and $E(\boldsymbol{V}\boldsymbol{V}^{\mathrm{T}})$ $= \boldsymbol{R} = diag(\hat{\boldsymbol{R}}_{ti}, \hat{\boldsymbol{R}}_{t2}, ..., \hat{\boldsymbol{R}}_{tM})$. $\boldsymbol{X}_{tM}$ is the state information of a target at time tM. $\boldsymbol{Z}$ and $\boldsymbol{V}$ are the expanded vectors whose dimension is Mn_z, and $\boldsymbol{H}$ is $Mn_z \times n_x$ matrix. Assume that the transition matrix is $\boldsymbol{A}$ from the time step i to time step i+1 in the discrete system, and then we have

$$\boldsymbol{H}_{ti} = \boldsymbol{A}^{ti-tM}\boldsymbol{H}_{tM}; \tag{16}$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}; \tag{17}$$

$$\boldsymbol{H}_{tM} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{18}$$

where T is the sample period. If the rank of $\boldsymbol{H}$ is n_x, then $\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H}$ is a positive definite matrix. With the information above, we estimate $\boldsymbol{X}_{tM}$ by the least squares method as follows:

$$\hat{\boldsymbol{X}}_{tM} = (\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{Z}. \tag{19}$$

The initial covariance matrix is shown as follows:

$$\boldsymbol{P}\boldsymbol{X}_{tM} = (\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})^{-1}\boldsymbol{H}^{\mathrm{T}}\boldsymbol{R}\boldsymbol{H}(\boldsymbol{H}^{\mathrm{T}}\boldsymbol{H})^{-1}. \tag{20}$$

3.4 Track evaluation

Here we provide two track evaluating methods. The first is proposed by the authors with the normalized distance testing method, and the second is proposed by Kennedy

(2008a) with the hypothesis test principle.

After determining a confirmed track and estimating its initial state and covariance matrix, we need to obtain its evaluation value, too. By the similar process of subsection 3.3, the state sequence of a track can also be estimated at time $1 \leqslant t1 \leqslant ... \leqslant tM \leqslant N$. Namely, at time tM, if there is a confirmed track formed, the initial state sequence $\hat{\boldsymbol{X}}_k$ and covariance sequence $\boldsymbol{PX}_k(k = t1,\ \ldots,\ tM)$ are also estimated subsequently based on Eqs. (12)–(20). And the converted measurement covariance matrix of the sensor is known as $\hat{\boldsymbol{R}}_k$. Assume that the estimated states are accurate for a target, and the measurements $\boldsymbol{Z}_k$ from the target are assumed to follow Gaussian distribution, then

$$\begin{aligned}&\boldsymbol{SC}_k = \boldsymbol{H}_k \cdot \boldsymbol{PX}_k \cdot \boldsymbol{H}_k^{\mathrm{T}} + \hat{\boldsymbol{R}}_k;\\&\boldsymbol{Z}_k \sim \mathcal{N}(\boldsymbol{H}_k \cdot \hat{\boldsymbol{X}}_k, \boldsymbol{SC}_k).\end{aligned} \tag{21}$$

At time k, the evaluation function (similar to the cost function) is chosen as follows:

$$f(k) = (\boldsymbol{Z}_k - \boldsymbol{H}_k\hat{\boldsymbol{X}}_k)^{\mathrm{T}} \cdot \boldsymbol{SC}_k^{-1} \cdot (\boldsymbol{Z}_k - \boldsymbol{H}_k\hat{\boldsymbol{X}}_k). \tag{22}$$

Eq. (22) is also the normalized distance function between the predicted and actual measurements. If the value of $f(k)$ is small, it means that the distance between the predicted and actual measurements is small, and the estimation accuracy is high. Then the evaluation value of the confirmed track during the track initiation stage is

$$f_1(Eva) = \max[f(k)],\ k = t1, ..., tM. \tag{23}$$

The evaluation value $f_1(Eva)$ reflects the maximum distance of all the normalized distances.

The second track evaluation method is proposed by Kennedy (2008a). If a track originates from clutter, the distance between two adjacent points is obey the chi-squared distribution Kennedy (2008a).

$$\begin{aligned}&2\lambda V \sim \chi^2(2);\\&V = \pi\gamma_1^2,\end{aligned} \tag{24}$$

where, V is the gate volume. If a confirmed track is initiated at tM, then from the reproductive property of chi-squared variables, the following relationship (the second evaluation mechanism) holds if the track is only updated by the clutter peaks.

$$f_2(Eva) = 2\sum_{k=1}^{M-1} \lambda V(k) \sim \chi^2(2M-2). \tag{25}$$

3.5 Track pruning

Compared with the random distribution of clutter, the Gaussian distribution of measurements from a target is relatively more regular, which provides the principle of selecting the confirmed tracks. Track pruning is to prune the tracks which seem like originated from clutter. It includes two rules: (a) the potential tracks whose number of the measurement points does not meet the requirement of M/N rules should be terminated in advance; (b) the confirmed tracks whose evaluation values are exceptional should be deleted.

We can understand that there are two aspects of distinction between the track originated from a target and from clutter. Typically, the first of the meanings is obvious, that the measurements originated from a target are easier to form a track. The other is that the distance between the predicted and actual measurement originated from a target is smaller. The first rule eliminates the false tracks in the viewpoint of the quantity requirement of measurement points, and the second rule eliminates that in the viewpoint of the quantity requirement of measurement points. As for the potential tracks, the first rule is used to prune the false tracks. As for the confirmed tracks, the second rule is used to prune the false tracks.

As for the proposed track evaluating method (the first evaluating method), the second rule of track pruning is that: if the evaluation value of a new confirmed track is smaller than the threshold γ_{TP}, this track should be retained; and if the value is larger than the threshold, then this track should be deleted, because its distribution seems more like the distribution from clutter. Another important issue of the track pruning is how to determine the threshold. If the threshold is set too large, then many false tracks cannot be eliminated; if the threshold is set too small, it is possible to eliminate the true target track. The above mentioned Eq. (5) can also be used to determine the threshold corresponding to the known probability P_{TP}.

$$\gamma_{\mathrm{TP}} = \sqrt{-2\ln(1 - P_{\mathrm{TP}})}. \tag{26}$$

The value of P_{TP} is recommended as 0.99 for determining the threshold γ_{TP} based on Eq. (26).

For the threshold of the second track evaluating method, the null hypothesis (it is false track) is rejected if it exceeds a specified confirmation threshold, where the specified confirmation threshold is selected to give the desired size, using the inverse Chi-Squared Cumulative Density Function (CDF) (Kennedy, 2008a).

3.6 Track merging

After the track pruning, these confirmed tracks should be merged into one track, if these confirmed tracks meet the three conditions: (a) these confirmed tracks are formed simultaneously; (b) their most measurements overlap (more than $M/2$); (c) their initial states are close to each other. Namely, firstly, we should determine which tracks can be merged, and secondly, we should determine how to merge those tracks.

It is easy to understand the first and second conditions. For the third condition, their initial states are close, which means that their initial states overlap with the probability P_{TM} (P_{TM}<1, we always set the value of P_{TM} as 0.9 with respect to the threshold γ_{TM} mentioned below). Assume that the initial states and initial covariance matrixes of two confirmed tracks are $\hat{\boldsymbol{X}}_{tM}^{1}$, $\hat{\boldsymbol{X}}_{tM}^{2}$ and $\boldsymbol{P}_{tM}^{1}$, $\boldsymbol{P}_{tM}^{2}$, then the state distribution information X_i of track i is assumed to be the Gaussian distribution.

$$\boldsymbol{X}_{tM}^{i} \sim \mathcal{N}(\hat{\boldsymbol{X}}_{tM}^{i}, \boldsymbol{PX}_{tM}^{i}), \ \ i = 1,2. \tag{27}$$

These two tracks can be merged into one track, which means that their measurements are originated from a same target, so X_{tM}^{i} can be considered as the i-th estimation for the target ($i = 1,2$). We set

$$\hat{\boldsymbol{X}}_{tM} = \boldsymbol{X}_{tM}^{1} - \boldsymbol{X}_{tM}^{2}. \tag{28}$$

Then,

$$\hat{\boldsymbol{X}}_{tM} \sim \mathcal{N}(0, \boldsymbol{PX}_{tM}^{1} + \boldsymbol{PX}_{tM}^{2}). \tag{29}$$

For simplicity, set $\boldsymbol{PX}_{tM} = \boldsymbol{PX}_{tM}^{1} + \boldsymbol{PX}_{tM}^{2}$, then based on the distribution information of $\tilde{\boldsymbol{X}}_{tM}$, we can determine the third condition for the track merging.

$$\tilde{\boldsymbol{X}}_{tM}^{\mathrm{T}} \cdot \boldsymbol{PX}_{tM}^{-1} \cdot \tilde{\boldsymbol{X}}_{tM} \leqslant \boldsymbol{\gamma}_{tM}^{2} \tag{30}$$
,

where γ_{tM} is the threshold corresponding to the normalized distance with the probability coefficient P_{tM}. These two tracks overlap, or are originated from the same target with P_{tM} probability, if Eq. (30) holds. The value of P_{tM} is recommended to be 0.9 for determining the threshold γ_{tM} based on Eq. (5). If the two tracks have met the three conditions simultaneously, they can be merged into one track.

Assume that there are n targets ($\pi_{tM}^{i} : \hat{\boldsymbol{X}}_{tM}^{i}, \boldsymbol{PX}_{tM}^{i}, p_i$) ($i$ = 1, ..., n) should be merged into one target, $\hat{\boldsymbol{X}}_{tM}^{i}$ and $\boldsymbol{PX}_{tM}^{i}$ are the mean and covariance matrix of the i-th track, and p_i is a probability associated with the i-th track such that $p_i > 0$ and $\sum p_i = 1$. Then

$$p_i = \frac{f_1(i)}{\sum\limits_{i=1}^{n} f_1(i)}, \ \ i = 1, \ldots n; \tag{31}$$

$$\boldsymbol{X}_{tM} = \sum_{i=1}^{n} p_i \boldsymbol{X}_{tM}^{i}; \tag{32}$$

$$\boldsymbol{PX}_{tM} = \sum_{i=1}^{n} p_i \boldsymbol{PX}_{tM}^{i}, \tag{33}$$

where, $\boldsymbol{X}_{tM}$ and $\boldsymbol{PX}_{tM}$ are the initial state estimation and state covariance matrix respectively after track merging process, and $f_1(i)$ is the first track evaluating value of i-th track.

4 TSEPM method

In this section, we summarize the major steps of the proposed TSEPM method for the track initiation problems.

Step 1: Obtain and convert measurements. After obtaining the measurements, the new reconstruction measurements are converted from the polar coordinates to the Cartesian coordinates. Determine whether there are potential tracks. If not, skip to Step 8, and create new potential tracks. If yes, skip to Step 2, and update the potential tracks.

Step 2: Update measurements. Based on the MHT principle, we associate every potential track and every measurement in its validation predicted area, and update the potential tracks. If a potential track is updated by measurements (it is associated with any measurement in its validation predicted area), we should update some parameters about this potential track: the number of all measurements association $N_{\mathrm{a}} = N_{\mathrm{a}} + 1$, the number of validation measurements association $N_{\mathrm{v}} = N_{\mathrm{v}} + 1$, and determine the gate threshold based on Eq. (4). If a potential track is not updated (there is no measurement in its validation predicted area), then $N_{\mathrm{a}} = N_{\mathrm{a}} + 1$, and determine the gate threshold again, but the value of N_{v} remains invariable. Skip to Step 3.

Step 3: Confirm the track. If $N_{\mathrm{a}} \leqslant N$ and $N_{\mathrm{v}} \geqslant M$, a new confirmed track is formed. And if $N_{\mathrm{a}} \geqslant N$ and $N_{\mathrm{v}} < M$, then those measurements cannot make up a confirmed track (they do not meet the quantity requirements for a new confirmed track), and we should terminate this potential track in advance. Skip to Step 4.

Step 4: Estimate the initial state and covariance matrix. If a new confirmed track is formed after Step 3, the least squares method is used to estimate the initial state and covariance matrix for a new confirmed track. Skip to Step 5.

Step 5: Determine the evaluation value. For every new confirmed track, we should determine its evaluation value. Skip to Step 6.

Step 6: Prune track. After determining all the evaluation values of confirmed tracks, we prune all the tracks whose evaluation values are larger than the threshold, and retain the true tracks. Skip to Step 7.

Step 7: Merge track. For the true tracks, we merge those tracks that seem like originated from the same target. Then we need to estimate the initial state and covariance matrix for the new track again, after merging. Skip to Step 8.

Step 8: Creating new potential tracks. If there are residual measurements, we should create a new potential track for every residual measurement. And initialize some parameters for a new potential track: the value of N_{a} is 1, N_{v} is 1, and determine the gate threshold. Skip to Step 9.

Step 9: Wait for next measurements. Skip to Step 1.

As shown in Fig. 4, it is the flow diagram.

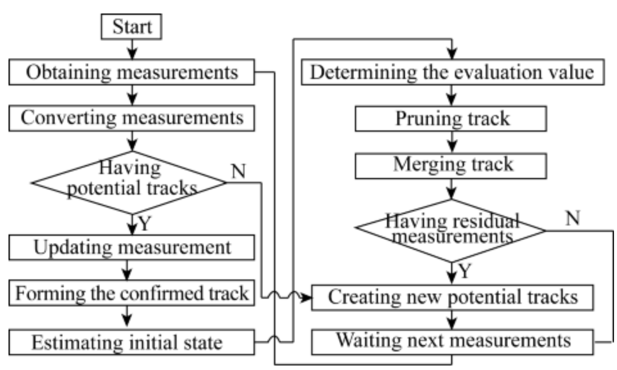

Fig. 4. Flow diagram of the TSEPM method for the track initiation problems.

5 Analyses and simulations

In this section, the effectiveness and advantages of our proposed novel method are analyzed and demonstrated. Firstly, we will determine the parameters (M and N). Secondly, we will compare the performance of three track initiation methods: the TSEPM method, the parameterized multiple model method and the clutter-based test method.

5.1 Determining the parameters M and N

Assume that a target moves along a straight line in the two-dimensional plane. This simulation covers a time span of N scans ($N \times T$ seconds). The parameters of this simulation environment are shown below.

The sensor: active sonar. The active sonar can obtain the noisy positions (range and azimuth) of a target.

The sampling period: 12 s.

The measurement errors: the standard deviation of the range measurement is 0.015 multiplied by the real distance; the standard deviation of the azimuth measurement is 1°.

The detection area: a circle area with the radius of 9000 m.

The probability corresponding to the gate threshold: 0.9.

The target's initial state: [5000 5000] (m).

The target's speed: 5 m/s, uniform rectilinear motion.

The maximum speed of the target: 10 m/s.

The experiment consists of 500 simulation runs.

We list P_F and P_D with respect to the changeable parameters M, N and p_d in Table 1.

From Table 1, different values of M, N and p_d correspond to different P_D and P_F. Considering the practical application where p_d is known and expected P_D has been determined in advance, and then we can determine the values of M and N with Table 1. For example, if (a) p_d is 0.9, (b) we expect that P_D is larger than 0.85 and (c) P_F is smaller than 0.05, then we can choose the rule (5/8) or (6/9). From Table 1, for a constant value N, if M equals N/2 or (N+1)/2, P_D is larger than other values of M; as for P_F, the larger the value M is, the smaller P_F is. Moreover, the larger value M is, the higher the accuracy of the initial estimate state is. After comprehensively compromising, the pair M/(M+3) is a relatively better choice, such as 4/7, 5/8, and 6/9, if M+3=N is smaller than or equal to 9. Here, we select M/N to be 5/8.

5.2 Performance analyses of three track initiation methods

In this subsection, the TSEPM track initiation method is compared with the parameterized multiple model method and the clutter-based test method, considering the simulation scenario of Subsection 5.1.

As shown in Fig. 5, it denotes P_D of three track initiation methods with variable clutter density. P_D of the TSEPM method is approximately equal to that of the clutter-based test method, because both of them adopt the track splitting method, which makes certain that the true track of a target is detected. Therefore, no matter what the clutter density is, their P_D do not decrease. As for the parameterized multiple model method, its P_D decreases along with the increasing clutter density. As the ordering of the measurements is arbitrary and measurement errors are relatively large, if the clutter density increases, it is easier to be associated with the clutter measurements, if it does not adopt the track splitting method. Thus, the parameterized multiple model method cannot retain the true track of a target. From the analyses of Fig. 5, the track splitting process is necessary for the track initiation in the heavy clutter environment, if we want to increase P_D.

As shown Fig. 6, it denotes P_F of three track initiation methods with variable clutter density. P_F of the TSEPM

Table 1 P_F and P_D with respect to the changeable track initiation rules and P_d

M/N	(*1)	(*2)	p_d = 1.0	p_d = 0.9	p_d = 0.8
	P_F (1)	P_F (2)	P_D	P_D	P_D
3/4	0.1822	0.1004	0.9720	0.8149	0.6470
3/5	0.3953	0.1253	0.9963	0.8788	0.7445
4/5	0.0545	0.0276	0.9477	0.7509	0.5494
4/6	0.1554	0.0618	0.9871	0.8348	0.6506
5/6	0.0151	0.0048	0.9185	0.6819	0.4558
4/7	0.2621	0.1007	0.9944	0.8676	0.7161
5/7	0.0536	0.0174	0.9842	0.8217	0.6243
6/7	0.0040	0.0015	0.8857	0.6119	0.3715
5/8	0.1319	0.0418	0.9973	0.8749	0.7187
6/8	0.0170	0.0059	0.9743	0.7819	0.5535
7/8	0.0010	0.0003	0.8503	0.5440	0.2987
5/9	0.3446	0.0997	0.9996	0.8925	0.7650
6/9	0.0497	0.0194	0.9950	0.8572	0.6725
7/9	0.0025	0.0083	0.9619	0.7367	0.4822
8/9	0.0002	0.0000	0.8131	0.4797	0.2375

* Clutter density: 0.000001 m^{-2}. '–' means that it lacks a meaning physical interpretation or that the track initiation rules do not apply to the practical scenery. (*1) denotes that P_F is a theoretical calculated value before the track pruning and merging process, and (*2) denotes that P_F is obtained by simulation after the track pruning and merging process. It is difficult to predict P_F of the TSEPM method by theoretically calculation due to the track merging process. So we have to obtain P_F of the TSEPM method by statistical method.

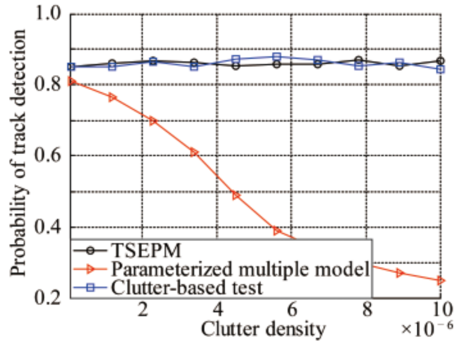

Fig. 5. Probability of the track detection (P_D) with variable clutter density.

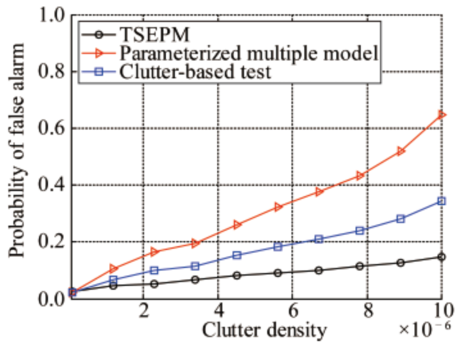

Fig. 6. Probability of the false alarm (P_F) with variable clutter density.

method is smaller than that of the clutter-based test method, and P_F of the clutter-based test method is smaller that of the parameterized multiple model method. For the former scenario, even though the TSEPM and clutter-based test methods both have the ability to delete some false tracks derived from clutter, the TSEPM has an additional step–the track merging–to delete more false tracks. From plenty of simulations, we learn that false tracks are inclined to cluster, in part because of the track splitting process. Therefore, it is necessary to merge redundant false tracks into one track in order to reduce the computational burden and satisfy the real-time requirements. For the latter scenario, even though the track splitting process increases the false alarm, the track pruning (track merging) process deletes most false tracks, and the true feasible track formed by the parameterized multiple model method is easier to be corrupted by the clutter with the clutter density increasing. Thus, from the analyses of Fig. 6, the track pruning and merging processes are necessary for the track initiation in the heavy clutter environment, if we want to decrease P_F.

As shown in Fig. 7, it denotes P_D of three track initiation methods with variable measurement errors (the clutter density is 0.000005). For the first scenario (P_D of the TSEPM method is approximately equal to that of the clutter-based test method, and P_D of the parameterized multiple model method decrease rapidly), the reasons have been analyzed, which are similar to those of Fig. 5. For the second scenario, when the measurement errors are smaller than 75,

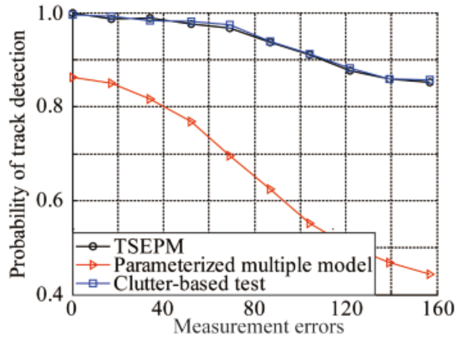

Fig. 7. Probability of the track detection (P_D) with variable measurement errors.

P_D of the TSEPM method (the clutter-based test method) decreases slowly. According to Eqs. (4) and (5), if $\alpha\sigma_{R\max}$ is smaller than $v_{\max}KT$ significantly, then the gate probability is more than 0.9, which is roughly 1. Thus, the major reason of the second scenario is target's movement factors. For the third scenario, when the measurement errors are between 75 and 120, P_D of the TSEPM method decreases relatively rapidly. Because the measurement error factors ($\alpha\sigma_{R\max}$) are close to target's movement factors ($v_{\max}KT$), and the gate probability decreases rapidly from 1 to 0.9. Therefore, P_D of the TSEPM method decreases relatively rapidly, when the measurement errors are between 75 and 120. For the fourth scenario, when the measurement errors are larger than 120, P_D of the TSEPM method does not varied, because the gate probability is almost constant, according to Eqs. (4) and (5).

As shown in Fig. 8, it denotes P_F of three track initiation methods with variable measurement errors. For the first scenario (P_F of three methods increase with the measurement errors increase), according to Eq. (4), the larger the measurement errors are, the larger the validation prediction area is, so the easier clutter is to form false track, and then P_F increases. For the second scenario (P_F of the TSEPM method is smaller than that of the clutter-based test method, and P_F of the clutter-based test method is smaller that of the parameterized multiple model method), the reasons are similar to those of Fig. 6.

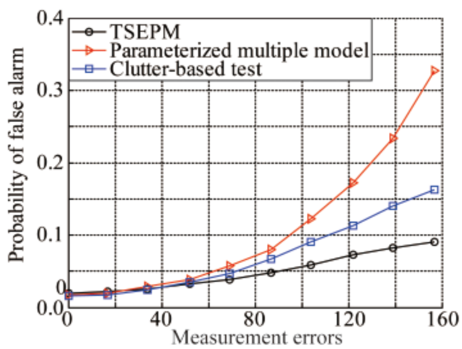

Fig. 8. Probability of the false alarm (P_F) with variable measurement errors.

6 Conclusion

Track initiation is an important issue of the target tracking. It is difficult to find the true target and determine its initial state quickly and accurately in the presence of large measurement errors and heavy clutter in the harsh underwater environment. For dealing well with these problems, based on the MHT principle and modified logic-based track initiation method, in order to increase the detection probability of a track, track splitting creates a large number of tracks which include the true track originated from the target. However, the track splitting also forms a lot of false tracks, which is not our primal purpose. Thus, in order to decrease the false alarm probability, based on the evaluation mechanism, the track pruning and track merging are proposed to reduce the false tracks. At last, simulations and analyses verify the effectiveness and performance of the novel method. For the novel method, there are still some problems. First of all, the computational burden is much larger than that of the logic-based method, because of the track splitting process, even though the track pruning and track merging processes have decreased the computational burden. Secondly, the false alarm probability should be computed and analyzed by the method of statistical theory. Therefore, for our further work, it is to reduce the computational burden and analyze the accurate false alarm probability in order to find a better compromise between performance and computational burden. And we will report our work on these topics in a future paper.

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