Identification of the state-space model and payload mass parameter of a flexible space manipulator using a recursive subspace tracking method

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Abstract The on-orbit parameter identification of a space structure can be used for the modification of a system dynamics model and controller coefficients. This study focuses on the estimation of a system state-space model for a two-link space manipulator in the procedure of capturing an unknown object, and a recursive tracking approach based on the recursive predictor-based subspace identification (RPBSID) algorithm is proposed to identify the manipulator payload mass parameter. Structural rigid motion and elastic vibration are separated, and the dynamics model of the space manipulator is linearized at an arbitrary working point (i.e., a certain manipulator configuration). The state-space model is determined by using the RPBSID algorithm and matrix transformation. In addition, utilizing the identified system state-space model, the manipulator payload mass parameter is estimated by extracting the corresponding block matrix. In numerical simulations, the presented parameter identification method is implemented and compared with the classical algebraic algorithm and the recursive least squares method for different payload masses and manipulator configurations. Numerical results illustrate that the system state-space model and payload mass parameter of the two-link flexible space manipulator are effectively identified by the recursive subspace tracking method.

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1. Introduction

In on-orbit servicing missions, a space manipulator is an important component of a spacecraft structure that is widely applied for space object capture, space structure assembly, and even on-orbit structure identification.1–3 In order to reduce the launch mass and increase the workspace, the design of a space manipulator is generally lightweight and long-span,
so structural vibrations are caused while manipulating large payloads due to the microgravity environment. Thus, oscillations may become evident even with an extremely slow motion. Consequently, some necessary control technologies must be employed to suppress structural vibrations. In this situation, on-orbit identification technology is an effective approach to obtain new system properties after a capturing process, which has been successfully implemented in certain on-orbit spacecraft. On-orbit identification of structure parameters can not only be used for the controller design to decrease a manipulator’s oscillation during movement, but also effectively monitor system operating conditions to ensure on-orbit safety. In particular, if a target captured by using a manipulator is a non-cooperative object with an unknown property, then accurate identification of unknown object parameters may overcome system uncertainty and provide an important reference for modifications of attitude and vibration controller parameters.

A manipulator’s system model and corresponding dynamics parameters might be changed due to variations in the manipulator configuration and payload mass. Therefore, the corresponding system identification issues become more sophisticated. This study mainly focuses on the identification problems of a space manipulator’s state-space model and payload mass parameter in a fixed configuration, and subsequently the system model and dynamics parameters are only affected by changes in the payload mass. Consequently, the original nonlinear dynamics equation can be degraded as a linear model by the local linearization technology, and thus the corresponding system can be considered as linear and time-varying due to changes in the endpoint payload mass during a space capturing process.

Identification of a manipulator model and payload inertia parameter has been performed in other studies. Based on the subspace method, Liu et al. identified a state-space model of a single-link flexible manipulator through experiments, and developed a new procedure to implement model reduction and improve model accuracy. Based on the optimal experiment design, the dynamics model identification of six-degree of freedom parallel robots using the maximum-likelihood estimator was discussed in Ref. Furthermore, to deal with the strong non-linear behavior of a manipulator system, a state-space model identification approach based on a linear parameter-varying system was proposed, and a set of experiments with different configurations was performed by Mercere et al.

For the manipulator inertia parameter identification, momentum conservation and least-squares estimation are commonly applied to identify system and payload inertia parameters. Liu et al. presented a recursive differential evolution algorithm to identify the inertial parameters of an unknown target, and the friction parameters of space manipulator joints were revised simultaneously. Cambera et al. identified the payload mass parameter of a single-link flexible manipulator based on the algebraic identification method. Nevertheless, several aforementioned studies only considered the manipulator as a completely rigid structure and neglected to take into account the influence of flexible vibration.

System excitation and measurement modes are limited in the space environment, and thus many parameter identification approaches are not suitable for flexible space structures. The subspace algorithm has been proven as an effective method to identify state-space models and corresponding modal parameters. The method can determine the system order and requires little prior knowledge. However, few studies involving the subspace algorithm have investigated the identification of manipulator inertia parameters. The classical subspace method based on singular value decomposition (SVD) requires a high computation cost and is not suitable for online tracking. Therefore, a recursive subspace algorithm was proposed to track system parameters in real-time. For the space manipulator, an online identification procedure can be implemented to recalculate uncertain parameters periodically, and conveniently update the certainty equivalence controller with unexpected parameter variations. Thus, the recursive identification algorithm is highly suitable for certain control approaches that require online updating of the controller parameter such as self-adaptive control. The identified state-space model and dynamics parameters can also be used for controller correction and to improve the robustness of a control system. However, identification results using the classical recursive subspace algorithm may be affected when the noise signal is strong in a system. To reduce the influence of noise, a recursive algorithm called recursive predictor-based subspace identification (RPBSID) is employed in this study. Compared with classical recursive algorithms, adaptive filter technology is applied in the RPBSID algorithm to improve the noise immunity, and autoregressive predictors are also used to provide an asymptotically consistent estimate.

In this paper, considering a situation in which an unknown object is captured by a space manipulator, the identification of the manipulator state-space model is studied by using the RPBSID algorithm, and a payload mass parameter estimation approach based on the recursive method is proposed. By establishing the corresponding coupling nonlinear dynamics equation and linearizing at an arbitrary working point, the state-space equation is reconstructed, and system model matrices can be determined by the RPBSID algorithm. Furthermore, using the identified system state-space model, the endpoint payload mass parameter can be derived by extracting the corresponding block matrix. In practical applications, when the system state-space model at the selected working point is identified, the system modal parameter as well as the payload mass parameter can be obtained during the process of capturing the unknown object by using the proposed method, and the identified state-space model can also be applied to modify the system model and controller parameter. For different payload masses and manipulator configurations, the accuracy of the identified system state-space model is verified by a comparison of system test responses, and the payload mass identification results of the proposed method are also compared with those of the classical algebraic algorithm and recursive least squares method. Finally, a simple finite element model is also established, and the corresponding I-O signals are used to identify the payload mass parameter. Numerical results demonstrate that the proposed method is effective in the identification of the model and payload mass parameter of the flexible space manipulator.

The contents of this paper are organized as follows. Section 2 presents the dynamics model of a two-link flexible space manipulator using the Lagrange method, and then the nonlinear dynamics equation is linearized for low vibrations and can be further expressed as a block matrix form. In Section 3, the RPBSID algorithm is briefly introduced to determine the state-space model, and similarity transformations of different state-space model parameters are reviewed. In
addition, a new payload mass model and payload mass parameter of a flexible space manipulator using a recursive subspace tracking method, Chin J Aeronaut (2018), https://doi.org/10.1016/j.cja.2018.05.005
A. Eq. (7) is clearly nonlinear. In this study, only the manipulator parameter identification at a certain working point is considered, and thus the nonlinear Eq. (7) is linearized in the following section.

2.2. Local linearization of the nonlinear model at a selected working point

As mentioned in the previous section, this study only considers the space manipulator parameter identification problem at a certain working point. Therefore, based on the local linearization theory, Eq. (7) can be linearized at a selected working point as follows:

\[
M_{\theta\theta}(q_0) \cdot \delta q(t) + C_{E} \delta q(t) + (K + K_{M} + K_{b}) \delta q(t) = \mathbf{Q}(t)
\]  

In Eq. (8), the arbitrary working point is defined as \([q_0]_0 \cdot \left[q_0\right]_0 \cdot \left[q_0\right]_0\), where ‘(.)’ denotes the value at the working point. Then, the linearized state vector \(\delta q(t) = q(t) - [q]_0\) and matrices \(C_{E}, K_{M},\) and \(K_{b}\) are expressed as follows:

\[
C_{E} = \frac{\partial \mathbf{E}}{\partial \left[q\right]_0} = \left[\begin{array}{c}
\frac{\partial \mathbf{E}}{\partial \left[\dot{q}\right]_0} \\
\frac{\partial \mathbf{E}}{\partial \left[\ddot{q}\right]_0} \\
\frac{\partial \mathbf{E}}{\partial \left[\dddot{q}\right]_0}
\end{array}\right]_0
\]

\[
K_{M} = \frac{\partial \mathbf{M}}{\partial \left[q\right]_0} = \left[\begin{array}{c}
\frac{\partial \mathbf{M}}{\partial \left[\dot{q}\right]_0} \\
\frac{\partial \mathbf{M}}{\partial \left[\ddot{q}\right]_0} \\
\frac{\partial \mathbf{M}}{\partial \left[\dddot{q}\right]_0}
\end{array}\right]_0
\]

\[
K_{b} = \frac{\partial \mathbf{E}}{\partial \left[q\right]_0} = \left[\begin{array}{c}
\frac{\partial \mathbf{E}}{\partial \left[\dot{q}\right]_0} \\
\frac{\partial \mathbf{E}}{\partial \left[\ddot{q}\right]_0} \\
\frac{\partial \mathbf{E}}{\partial \left[\dddot{q}\right]_0}
\end{array}\right]_0
\]

Thus, the linearized dynamics equation of the space manipulator at the working point is determined and can be used for modal and payload mass identification. It should be noted that the linearization equation is only used near the selected working point.

2.3. Block description of the linearized dynamics equation

The linearized dynamics Eq. (8) includes the rigid body motion with angle \(\delta \theta\) and flexible vibration with modal coordinate \(\delta \eta\). It is assumed that the modal coordinates \([\eta(t)]_0 \cdot \left[\eta(t)\right]_0 \cdot \left[\eta(t)\right]_0 \cdot \left[\eta(t)\right]_0\) at the working point are extremely low, and thus the corresponding items can be removed from matrix \(M_{\theta\theta}(q_0)\). In addition, based on this assumption, the values of matrices \(C_{E}, K_{M},\) and \(K_{b}\) are also significantly low, and subsequently these matrices are omitted in Eq. (8). Let \(\delta \theta = [\delta \theta_1, \delta \theta_2]^T\) and \(\delta \eta = [\delta \eta_{11}, \delta \eta_{21}]^T\), and thus the mass matrix \(M_{\delta\theta\delta\theta}(q_0)\) and the stiff matrix \(K\) can be simplified and partitioned as follows:

\[
M_{\delta\theta\delta\theta}(q_0) = \begin{bmatrix}
M_{\delta\theta\delta\theta} & \mathbf{0} \\
\mathbf{0} & \mathbf{0}
\end{bmatrix}_{2 \times 2}
\]

\[
K = \begin{bmatrix}
K_{\delta\theta} \\
K_{\delta\eta}\end{bmatrix}_{2 \times 2}
\]

where

\[
M_{\delta\theta\delta\theta}(q_0) = \begin{bmatrix}
\frac{1}{2} \rho_1 L_1^2 + \frac{1}{2} \rho_1 L_1^2 + m_2 L_2^2 + m_3 L_2^2 + \frac{1}{2} \rho_1 L_1^2 + m_2 L_2^2 + m_3 L_2^2 + \frac{1}{2} \rho_1 L_1^2 + m_2 L_2^2 + m_3 L_2^2 \\
\frac{1}{2} \rho_2 L_2^2 + m_2 L_2^2 + m_3 L_2^2 + \frac{1}{2} \rho_2 L_2^2 + m_2 L_2^2 + m_3 L_2^2 + \frac{1}{2} \rho_2 L_2^2 + m_2 L_2^2 + m_3 L_2^2
\end{bmatrix}
\]

Then, Eq. (8) can be simplified and rewritten as the following two equations:

\[
M_{\delta\theta\delta\theta}(q_0) \delta \theta(t) + M_{\delta\theta\delta\theta}(q_0) \delta \eta(t) = u(t)
\]

\[
M_{\delta\theta\delta\theta}(q_0) \delta \theta(t) + M_{\delta\eta\delta\theta}(q_0) \delta \eta(t) = 0
\]

where \(u(t) = [\tau_0(t), \tau_0(t)]^T\) is the input vector. Substituting Eq. (14) into Eq. (15), the manipulator vibration equation by using the block matrix form is expressed as follows:

\[
\delta \mathbf{M}(t) \delta \mathbf{q}(t) + \delta \mathbf{K} \delta \mathbf{q}(t) = -M_{\delta\theta\delta\theta}^{-1} \mathbf{u}(t)
\]

\[
M = M_{\delta\theta\delta\theta} - M_{\delta\theta\delta\eta} M_{\delta\eta\delta\theta}^{-1} M_{\delta\eta\delta\eta}
\]

For a manipulator system such as that in Eqs. (14), (15), based on the definition in Section 2.1, when a payload mass \(\Delta m\) of an unknown object is captured at an arbitrary time instant (defined as \(t < t_{\text{cap}}\)) at a fixed working point, the total endpoint mass \(m_0\) following the capture is defined as \(m_0 = m_0 + \Delta m\), where \(m_0\) is the original endpoint mass. From the form of mass matrix \(M_{\delta\theta\delta\theta}(q_0)\) in Eq. (12), it should be noted that only matrix \(M_{\delta\theta\delta\theta}\) includes the payload term \(\Delta m\), i.e. only the value of \(M_{\delta\theta\delta\theta}\) changes for the fixed working point during the entire capturing procedure. Therefore, matrix \(M_{\delta\theta\delta\theta}\) before and after capturing can be defined as follows:

\[
M_{\delta\theta\delta\theta} = \begin{cases}
M_{\delta\theta\delta\theta \text{ before}} & t < t_{\text{cap}} \\
M_{\delta\theta\delta\theta \text{ after}} & t \geq t_{\text{cap}}
\end{cases}
\]

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Matrix $M_{\text{after}}$ before the capturing process does not include the unknown payload parameter $\Delta m$, which must be identified, and thus can be considered as a priori knowledge. If matrix $M_{\text{after}}$ at each time instant after the capture process is determined, then the payload mass $\Delta m$ can be estimated by calculating the difference between $M_{\text{before}}$ and $M_{\text{after}}$. In the following section, how to identify the mass matrix $M_{\text{after}}$ for each time instant will be discussed.

3. Identification of the system state-space model and payload mass parameter

In this section, the block dynamics equation is rewritten as a state-space form, and the recursive procedures of the RPBSID algorithm are briefly introduced to determine the system state-space model and corresponding modal parameters. Moreover, a new method is proposed to estimate the payload mass parameter. Finally, the identification procedures of the state-space model and payload mass parameter are summarized.

3.1. Identification of system state-space model parameters by the RPBSID algorithm

We define a new state vector $x(t) = [\delta q^T(t), \Delta q^T(t)]^T$ and rewrite Eq. (16) as the following state-space form:

$$
\begin{align*}
\dot{x}(t) &= A_x x(t) + B_x u(t) \\
y(t) &= C x(t) + D u(t)
\end{align*}
$$

(19)

(20)

where the vector $y(t)$ is an $m \times 1$ system output that contains angle $\theta_i$ and lateral deflection $w_i$ for each link. Furthermore, $A_x$, $B_x$, $C$, and $D$ are the $n \times n$ system, $n \times r$ input, $m \times n$ output, and $m \times r$ direct transmission matrices, respectively, of the continuous system as follows:

$$
\begin{align*}
A_x &= \begin{bmatrix}
0 & I \\
-M^{-1} K_x & 0
\end{bmatrix} \\
B_x &= \begin{bmatrix}
0 \\
-M^{-1} M_u^{-1}
\end{bmatrix} \\
C &= \begin{bmatrix}
I & 0 \\
0 & I
\end{bmatrix} \\
D &= 0
\end{align*}
$$

(21)

where $I$ is a unit matrix. To simplify the computation procedure, the output matrix $C$ in Eq. (21) is directly given as a unit matrix, that is, the modal displacement $\delta q_i$ ($i = 1, 2$) in the state vector $x(t)$ is considered as the output signal in the following simulations. However, in practical applications, the modal displacement $\delta q_i$ must be multiplied with the corresponding modal matrix (defined as $\Phi$) to obtain the actual displacement $w_i(x, t)$ of each link in which matrix $\Phi$ can be obtained by finite element analysis.

The manipulator's frequency values are derived from the eigenvalues of matrix $A_x$. Furthermore, considering the influence of noise signal, the discretized innovation forms of Eqs. (19) and (20) can be described as follows:

$$
\begin{align*}
x(k + 1) &= A(k) x(k) + B(k) u(k) + K(k) e(k) \\
y(k) &= C(k) x(k) + D(k) u(k) + e(k)
\end{align*}
$$

(22)

(23)

where $e(k)$ and $K(k)$ denote the white innovation sequence and the Kalman gain matrix, respectively. In addition, $A$ and $B$ are the discretized system and input matrices, respectively.

For the system described in Eqs. (22) and (23), we define a past window denoted by $p$ and a future window denoted by $f$, in which $p \geq f \geq n/m$ is required to ensure that vector autoregressive with exogenous (VARX) parameters can be solved by using the least squares method. In this case, the stacked vectors for input $u(k)$ are defined as follows:

$$
\begin{align*}
\bar{u}(k-p) &= [u^T(k-p), u^T(k-p+1), \ldots, u^T(k-1)]^T \\
\bar{u}(k-f) &= [u^T(k), u^T(k+1), \ldots, u^T(k+f-1)]^T
\end{align*}
$$

(24)

and the stacked vectors $\bar{y}(k-p), \bar{y}(k-f), \bar{e}(k-p)$, and $\bar{e}(k-f)$ are defined in a similar manner. Furthermore, the stacked matrices $U(k)$ and $\bar{U}(k)$ for input $u(k)$ can be defined as follows:

$$
U(k) = \begin{bmatrix}
u(k) & u(k+1) & \ldots & u(k+p) \\
u(k+1) & u(k+2) & \ldots & u(k+p+1) \\
\vdots & \vdots & \ddots & \vdots \\
u(k+p) & u(k+p+1) & \ldots & u(k+2p)
\end{bmatrix}
$$

(25)

and the stacked matrices $Y(k)$ and $\bar{Y}(k)$ are defined in a similar manner. Subsequently, we define a one-step-ahead VARX predictor as follows:

$$
\bar{y}(k) = \sum_{i=0}^{p} \varepsilon^{(i)}(k-i) u(k-i) + \sum_{i=0}^{p} \varepsilon^{(i)}(k-i) y(k-i)
$$

(26)

where superscripts $(\cdot)^{(i)}$ and $(\cdot)^{(i)}$ denote the relations with input $u$ and output $y$, respectively. The VARX parameter matrices $\varepsilon^{(i)}$ and $\varepsilon^{(i)}$ in Eq. (26) are described as follows:

$$
\varepsilon^{(i)}(k-i) = \begin{bmatrix}
D(k) \\
C(k) A(k-1) \ldots A(k-i+1) B(k-i)
\end{bmatrix}
$$

(27)

and

$$
\varepsilon^{(i)}(k-i) = \begin{bmatrix}
D(k) \\
C(k) A(k-1) \ldots A(k-i+1) K(k-i)
\end{bmatrix}
$$

(28)

Based on Eqs. (27) and (28), a VARX parameter matrix $\varepsilon$ can be defined as follows:

$$
\varepsilon = \begin{bmatrix}
\varepsilon^{(i)}(k-p) & \ldots & \varepsilon^{(i)}(k) & \varepsilon^{(f)}(k-p) & \ldots & \varepsilon^{(f)}(k-1)
\end{bmatrix}
$$

(29)

If the system is deadbeat, Eq. (26) can be rewritten in the following linear regression form:
where $\delta(k) = [\hat{u}^T(k-p), \hat{u}^T(k), \hat{y}^T(k-p)]^T$. Based on the adaptive filter theory, the recursive update of matrix $\Xi(k)$ can be implemented as follows:

$$
\Xi(k) = \Xi(k-1) + (y(k) - \Xi(k-1)\delta(k))\delta^T(k)\Xi(k) \tag{31}
$$

where $\Xi(k)$ is the error covariance matrix. Generally, the initial value of $\Xi(k)$ is given as $\Xi(0) = (1/\xi_1)I$, and the parameter $\xi_1 > 0$ is selected to ensure that the recursive problem is well conditioned. Moreover, the following iteration update form for matrix $\Xi(k)$ is used by the recursive least squares (RLS) filter:$^{27}$

$$
\Xi(k) = \frac{1}{\beta_1} \Xi(k-1) - \frac{1}{\beta_1} \Xi(k-1)\delta(k)\delta^T(k)\Xi(k-1) + \delta^T(k)\Xi(k-1)\delta(k)^{-1}\delta^T(k)\Xi(k-1) \tag{32}
$$

where the forgetting factor $\beta_1$ satisfies $0 \ll \beta_1 \leq 1$. Subsequently, two VARX parameter matrices, $P(k)$ and $Q(k)$, are defined from matrix $\Xi$ as follows:

$$
P(k) = \begin{bmatrix}
\Xi^{(0)}(k-p) & \Xi^{(0)}(k-p+1) & \cdots & \Xi^{(0)}(k-p+f-1) & \cdots & \Xi^{(0)}(k-1) \\
0 & \Xi^{(0)}(k-p) & \cdots & \Xi^{(0)}(k-p+f-2) & \cdots & \Xi^{(0)}(k-2) \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \Xi^{(0)}(k-p) & \cdots & \Xi^{(0)}(k-f)
\end{bmatrix} \tag{33}
$$

$$
Q(k) = \begin{bmatrix}
\Xi^{(i)}(k-p) & \Xi^{(i)}(k-p+1) & \cdots & \Xi^{(i)}(k-p+f-1) & \cdots & \Xi^{(i)}(k-1) \\
0 & \Xi^{(i)}(k-p) & \cdots & \Xi^{(i)}(k-p+f-2) & \cdots & \Xi^{(i)}(k-2) \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & \Xi^{(i)}(k-p) & \cdots & \Xi^{(i)}(k-f)
\end{bmatrix} \tag{34}
$$

Thus, the state vector $x(k)$ can be obtained from the stack vectors $\bar{u}(k-p)$ and $\bar{y}(k-p)$ and the VARX parameter matrices $P(k)$ and $Q(k)$ as follows:

$$
x(k) = S(P(k)\bar{u}(k-p) + Q(k)\bar{y}(k-p)) \tag{35}
$$

where matrix $S$ should be selected as a row full rank to ensure that the recursive computational of the state vector $x(k)$ is stable and convergent. When the state vector $x(k)$ is obtained by Eq. (35), Eqs. (22) and (23) can be rewritten as

$$
x(k+1) = \Theta^{(o)}(k)\gamma(k) \tag{36}
$$

$$
y(k) = \Theta^{(i)}(k)\gamma(k) + e(k) \tag{37}
$$

where

$$
\gamma(k) = [x^T(k), u^T(k), e^T(k)]^T, \quad \bar{u}(k) = [\hat{u}^T(k), u^T(k)]^T
$$

$$
\Theta^{(o)}(k) = [A(k), B(k), K(k)], \quad \Theta^{(i)}(k) = [C(k), D(k)]
$$

The recursive forms of matrices $\Theta^{(o)}(k)$ and $\Theta^{(i)}(k)$ can be obtained by using the RLS filter, and the detailed recursive forms are directly given as follows:$^{27,29}$

$$
\Theta^{(o)}(k) = \Theta^{(o)}(k-1) + (x(k) - \Theta^{(o)}(k-1)\gamma(k-1))\gamma^T(k-1)I(k) \tag{38}
$$

$$
\Theta^{(i)}(k) = \Theta^{(i)}(k-1) + (y(k) - \Theta^{(i)}(k-1)\gamma(k-1))\gamma(k-1)A(k) \tag{39}
$$

$$
\text{where}
$$

$$
A(k) = \frac{1}{\beta_2} A(k-1) - \frac{1}{\beta_2} A(k-1)x(k-1)(\beta_2 - 1)
$$

$$
+ e^T(k-1)A(k-1)x(k-1)^{-1}e^T(k-1)A(k-1) \tag{40}
$$

$$
I(k) = \frac{1}{\beta_3} I(k-1) - \frac{1}{\beta_3} I(k-1)x(k-1)(\beta_3 - 1)
$$

$$
+ \gamma^T(k-1)I(k-1)x(k-1)^{-1}\gamma^T(k-1)I(k-1) \tag{41}
$$

$$
e(k) = y(k) - \Theta^{(o)}(k)\gamma(k) \tag{42}
$$

The initial values $A(0)$ and $I(0)$ are similar to $\Xi(0)$, and are typically selected as $A(0) = (1/\xi_2)I$ and $I(0) = (1/\xi_3)I$, respectively, where the parameters $\xi_2$ and $\xi_3$ should satisfy $\xi_2 > 0$ and $\xi_3 > 0$. In addition, the forgetting factors $\beta_2$ and $\beta_3$ in Eqs. (40) and (41) still require $0 \ll \beta_2 \leq 1$ and $0 \ll \beta_3 \leq 1$, respectively.

The detailed derivation procedures of the RPBSID algorithm are also provided in Ref.$^{27}$ The recursive procedures of the RPBSID algorithm are briefly summarized as follows:

(a) Update the VARX parameter matrix $\Xi(k)$ by using Eqs. (31) and (32), where the initial value $\Xi(0)$ of matrix $\Xi(k)$ can be selected as $\Xi(0) = Y(0)[U^T(0), U^T(1), \ldots, U^T(N)]^T$.

(b) Construct matrices $P(k)$ and $Q(k)$ from matrix $\Xi(k)$ by using Eqs. (33) and (34).

(c) Estimate the state vector $x(k)$ by using Eq. (35).

(d) Recursively compute the parameter matrices $\Theta^{(o)}(k)$ and $\Theta^{(i)}(k)$ by using Eqs. (38)-(42), where the initial values are $\Theta^{(o)}(0) = [I_n, I_{nx}, I_{nx}]$ and $\Theta^{(i)}(0) = [I_{nxn}, I_{nx}]$.

When matrices $\Theta^{(o)}(k)$ and $\Theta^{(i)}(k)$ are determined for each time instant, matrices $A(k), B(k), C(k), D(k)$, and $K(k)$ can be computed by extracting the corresponding block matrix of $\Theta^{(o)}(k)$ and $\Theta^{(i)}(k)$. Comparing with the frequently-used recursive method based on projection subspace estimation,
the adaptive filter is applied in the RPBSID algorithm to overcome the noise influence, and the VARX predictor is also employed to provide an asymptotically consistent estimate. Therefore, the RPBSID method not only provides an unbiased state estimation, but also increases the noise resistance ability.

If the system matrix $A(k)$ is derived, then the modal parameter identification of the system can be implemented. The eigenvalue decomposition of the system matrix $A(k)$ at sampling time $k$ is as follows:

$$A(k) = \Sigma(k)A(k)\Sigma^{-1}(k)$$ (43)

where $A(k)$ is the diagonal eigenvalue matrix, and $\Sigma(k)$ is the corresponding time-varying matrix of eigenvectors. Specifically, $A(k) = \text{diag}(\lambda_1(k), \lambda_2(k), \ldots, \lambda_n(k))$, in which $\lambda_j(k) \ (j = 1, 2, \ldots, n)$ contains time-varying conjugate complex eigenvalues. The $j$th pseudo-eigenvalue is $\lambda_j(k) = \exp(-\zeta_j(k)\Delta t \pm io_j(k)\Delta t)$, in which $-\zeta_j(k)$ and $o_j(k)$ are referred to as the $j$th pseudo-damping ratio and the pseudo-damped natural frequency, respectively; $\Delta t$ is the sampling time, and $\sqrt{-1}$.

3.2. Similarity transformation of the identified state-space model

The RPBSID algorithm is used to obtain a set of the discrete state-space model $\{A(k), B(k), C(k)\}$ in which superscript ‘n’ denotes the identified values that are distinguished from the original values $\{A(k), B(k), C(k)\}$. Although the two state-space models $\{A, B, C\}$ and $\{A, B, C\}$ have the same system input and output (I-O) relationship, their detailed element values, such as those of matrices $A$ and $A$, differ. In other words, a system includes an infinite set of the state-space model $\{A(i), B(i), C(i)\} \ (i = 1, 2, \ldots, \infty)$. Therefore, when a set of the state-space model is identified by the RPBSID algorithm, if one needs to estimate the detailed elements for the original state-space model, then a similarity transformation should be firstly performed between the original and identified state-space models by using a transformation matrix to obtain the original model parameters $\{A, B, C\}$ (some references also refer to this transformation as ‘topologically equivalent’ or ‘coordinate transformation’). 30, 31 In this case, the identified and original state-space models satisfy the following relationships:

\[
\begin{align*}
\dot{A} &= T^\dagger AT \\
B &= T^\dagger B \\
\dot{C} &= CT
\end{align*}
\] (44)

in which $T$ is the transformation matrix. To obtain matrix $T$, we assume that the original input matrix $C$ is known (since $C$ can generally be obtained by certain prior knowledge or finite element analysis). Subsequently, $T = C^\dagger C$, in which superscript “$\dagger$” denotes the Moore-Penrose inverse (pseudo inverse). Thus, from Eq. (44), the original matrix parameters $\{A, B, C\}$ can be obtained and transformed into the continuous system model $\{A, B, C\}$.

3.3. Payload mass estimation by using the identified state-space model

When the state-space Eqs. (22) and (23) are established, the identified state-space model $\{A, B, C\}$ can be obtained using the RPBSID algorithm, as discussed in Section 3.1. Furthermore, based on the theory in Section 3.2, the original state-space model $\{A, B, C\}$ can be determined by using the similarity transformation of Eq. (44), and the corresponding continuous-system model $\{A, B, C\}$ is obtained.

When the original matrices $A_0$ and $B_0$ are identified, the corresponding block matrices $-M_1^{-1}K_0$ and $-M_1^{-1}M_1^{-1}c_0$ can be extracted from $A_0$ and $B_0$, respectively, as follows:

$$-M_1^{-1}K_0 = A_0(n/2 + 1 : n, 1 : n/2)$$ (45)

$$-M_1^{-1}M_1^{-1}c_0 = B_0(:,:, n/2 + 1 : n)$$ (46)

where $A_0(:,:, n : n)$ (or $A_0(:,:, 1:n)$) denotes the first $n$ rows (or $n$ columns) of matrix $A_0$. The elements of $K_0$ only include the Young modulus $E$, second moment of area $I$, and length $L$, of the $i$th link, and thus these relevant values are known and remain unchanged during the capture process of an unknown object. Consequently, matrix $K_0$ can be regarded as a priori knowledge. In addition, matrices $M$ and $A_0(n/2 + 1 : n, 1 : n/2)$ are both square and full rank. Therefore, matrices $\dot{M}$ and $M_1^{-1}M_1^{-1}c_0$ can be computed as follows:

$$\dot{M} = -K_0A_0^{-1}(n/2 + 1 : n, 1 : n/2)$$ (47)

$$M_1^{-1}M_1^{-1}c_0 = K_0A_0^{-1}(n/2 + 1 : n, 1 : n/2)B_0(:,:, n/2 + 1 : n)$$ (48)

When matrix $\dot{M}$ is determined in a manner similar to that used for matrix $K_0$, matrix $M_1^{-1} c_0$ in Eq. (17) only includes the inherent parameters of the links, and thus can also be considered as known. Consequently, the block matrix $M_1^{-1}M_1^{-1}c_0$ is determined as follows:

$$M_1^{-1}M_1^{-1}c_0 = M_0 - \dot{M}$$ (49)

Substituting Eq. (48) into Eq. (49), matrix $M_0$ can be expressed as follows:

$$M_0 = (M_0^{-1}M_0^{-1})(M_0 - \dot{M})$$

$$= (K_0A_0^{-1}(n/2 + 1 : n, 1 : n/2)B_0(:,:, n/2 + 1 : n)\dot{M})$$

$$= (B_0(:,:, n/2 + 1 : n)A_0(n/2 + 1 : n, 1 : n/2)K_0^{-1})(M_0 - \dot{M})$$ (50)

where the block matrix $M_0^{-1}M_0^{-1}$ should satisfy the condition in which the number of rows exceeds the number of columns to ensure that its pseudo-inverse matrix is unique.

When matrix $M_0$ is computed, matrix $M_0^{-1}$ (i.e. matrix $M_0$-after) can be obtained from Eq. (17) as follows:

$$M_0 = M_0^{-1}M_0$$ (51)

where the dimensions of $M_0$ should also satisfy the condition that its pseudo-inverse is unique. Subsequently, matrix $M_0$-after in Eq. (18) at each time instant can be computed by Eq. (51), and thus the payload mass parameter $\Delta m$ is determined as follows:

$$\Delta m = \begin{vmatrix} M_0^{-1}M_0 \end{vmatrix}$$ (52)

where notation “$\begin{vmatrix}$” denotes the trace of a matrix.

In Eq. (2), only the first modal displacement for each link is selected in the model, i.e. $\kappa = 1$, and this system model contains two angle variables $(\theta_1, \theta_2)$ and two modal displacements $(\eta_{11}, \eta_{21})$. Consequently, the block matrices $M_{\kappa}$, $M_{\kappa}$,
and $M_r$ in Eq. (12) are all non-singular square matrices, and the corresponding inverse matrices always exist. However, the number of modal displacements $k$ must be $k > 1$ in most situations, and thus $M_r$ and $M_r^kM_r^{-1}$ typically correspond to non-square matrices, so the corresponding pseudo inverse matrices must be solved.

For the two-link manipulator, if the number of modal displacements is $k > 1$ for each link, then the number of columns in matrix $M_r$ evidently exceeds the number of rows. Therefore, for the procedures which require computation of the pseudo inverse, such as Eqs. (50) and (51), the pseudo inverse matrix must be unique. In general, if the number of links is higher (such as three or more), because the dimensions of the identified matrix $M_r$ (namely, $M_{r, after}$) are determined by the number of angle variables $\theta$, then the corresponding pseudo inverse matrices for $M_r$ and $M_r^kM_r^{-1}$ are ensured as unique only if $k \geq \theta$. Therefore, this method can be applied to uniquely determine matrix $M_r$ for different $\theta$ and $\eta$.

3.4. Summary of identification procedures

The identification procedures for the system state-space model and payload mass parameter in this paper can be briefly summarized as follows.

Step 1: The block matrix form Eq. (16) of Eq. (8) can be rewritten and discretized as Eqs. (22) and (23), and thus the identified state-space model parameters $\{A(k), B(k), C(k)\}$ are determined by using the RPBSID algorithm described in Table 1.

Step 2: The discrete model matrices $A(k)$ and $B(k)$ are determined by using the similarity transformation in Eq. (44). Subsequently, the corresponding original state-space model parameters $A_r$ and $B_r$ can be calculated.

Step 3: Matrices $M_r$ and $M_r^kM_r^{-1}$ are determined using the state-space model parameters $A_r$ and $B_r$ from Eqs. (47) and (48), and thus matrix $M_r$ is expressed using Eq. (50).

Step 4: Matrix $M_{r, after}$ is determined from Eq. (51), and then the payload mass $\Delta m$ can be identified by Eq. (52).

4. Numerical simulations

In simulations, a two-link space manipulator model is established, and the appropriate I-O signals are selected for identification. Subsequently, the state-space model matrices of the manipulator at a selected working point are determined by using the RPBSID algorithm. Furthermore, the payload mass parameter is identified using the presented approach, the recursive least squares method, and the algebraic algorithm.\(^{20,32,33}\) Finally, a simple finite element model is established, and the corresponding I-O signals are extracted to identify the payload mass parameter using the proposed method.

4.1. Simulation parameters

The structural parameters of the manipulator are listed in Table 2, and the system working point $\{q\}_0 = [\theta_1_0, \ldots, \theta_{k-1}_0, \eta_{11}_0, \ldots, \eta_{k-1,k-1}_0]$ is selected as $\theta_1_0 = 60^\circ$, $\theta_2_0 = 45^\circ$, and $\eta_{11}_0 = \eta_{21}_0 = 0$. In order to ensure that the manipulator’s motion is near the working point, the input torque is designed as a square signal. The input torques of joints $\tau_1$ and $\tau_2$ are shown in Fig. 2.

Assume that an unknown object $\Delta m = 20$ kg is captured when the time $\tau_{cap}$ is set as $3$ s. In simulations, the measurement noise is a stationary zero-mean Gaussian random noise, and the SNR is selected as 40 dB. Then, the corresponding system responses about rotation angles $\delta \theta$ and modal displacements $\delta \eta$ can be computed, as shown in Fig. 3, where it is observed that the rotation angle and vibration amplitude are small (the angle is less than $3^\circ$), and thus the original nonlinear dynamics equation can be linearized. Next, the designed input torques and computed system responses are employed as I-O signals to identify the system state-space model and the corresponding payload mass, in which the parameters of the RPBSID algorithm in Table 1 are selected as $p = 10, f = 5, \alpha_{1,3} = 0.1, \beta_{1,3} = 0.99$, respectively. The sampling time $\Delta t$ is selected as $\Delta t = 0.001$ s.

4.2. Simulation results

For the identification of the system state-space model and payload mass parameter, the following two situations are considered: (1) different payload masses captured at the same working point, and (2) the same payload mass captured at different working points. These situations are discussed in the following sections.

4.2.1. Simulation example 1: capturing different payload masses at the same working point

The system working point is still selected as $\theta_1_0 = 60^\circ$, $\theta_2_0 = 45^\circ$, and $\eta_{11}_0 = \eta_{21}_0 = 0$. Three different payload masses $\Delta m$ are discussed as follows: (1) Case 1: $\Delta m = 20$ kg; (2) Case 2: $\Delta m = 50$ kg; and (3) Case 3: $\Delta m = 100$ kg. The RPBSID algorithm is implemented to identify the system state-space model parameters by using the designed I-O signals as discussed in Section 4.1.

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Physical parameters of the two-link flexible manipulator for simulations.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Component</td>
<td>Symbol</td>
</tr>
<tr>
<td>Link length</td>
<td>$I_1, I_2$</td>
</tr>
<tr>
<td>Length density of arms</td>
<td>$\rho_1, \rho_2$</td>
</tr>
<tr>
<td>Elastic modulus of links</td>
<td>$E_1, E_2$</td>
</tr>
<tr>
<td>Inertia moment of links</td>
<td>$I_1, I_2$</td>
</tr>
<tr>
<td>Motor mass</td>
<td>$m_1, m_2$</td>
</tr>
<tr>
<td>Original endpoint mass</td>
<td>$m_o$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Average relative errors of frequencies at different working points using the RPBSID algorithm.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manipulator configuration</td>
<td>Frequency</td>
</tr>
<tr>
<td>Case 1: $\theta_1_0 = 0^\circ, \theta_2_0 = 0^\circ$</td>
<td>2.5030</td>
</tr>
<tr>
<td>Case 2: $\theta_1_0 = 45^\circ, \theta_2_0 = 0^\circ$</td>
<td>2.7005</td>
</tr>
<tr>
<td>Case 3: $\theta_1_0 = -45^\circ, \theta_2_0 = 45^\circ$</td>
<td>3.0885</td>
</tr>
</tbody>
</table>
Different sets of the system state-space model satisfy the same I-O relationship. Thus, to verify the accuracy of the identified model, the same test inputs are applied to the original state-space model \( A(k), B(k), C(k) \) and the identified model \( \hat{A}(k), \hat{B}(k), \hat{C}(k) \) with the initial state condition \( x(0) = \hat{x}(0) = 0 \). The test response values and the corresponding absolute errors are shown in Figs. 4 and 5, respectively. The results indicate that the estimated test responses of the state-space model are generally consistent with those of the original system.

When the system matrix \( A_c \) is identified, the system frequency \( \omega_l \) can also be solved easily. The identified manipulator vibration frequencies \( \omega_1 \) and \( \omega_2 \) at the working point in the capturing procedure are shown in Fig. 6. It should be noted that Fig. 6 only provides the frequency identification results of Case 1 since the situations for Cases 2 and 3 are essentially consistent with those in Case 1 and therefore not shown here. In Fig. 6, the system frequencies evidently change in \( t_{cap} = 3 \) s due to the capture of the object. The results in Fig. 6 indicate that the system frequencies change with the payload mass, and the recursive algorithm can also effectively identify the manipulator frequency at the working point.

When the state-space model parameters are obtained using the RPBSID algorithm, the payload mass can be determined by using the proposed method in Section 3.3. In this simulation, in addition to the proposed algorithm, the recursive least

---

**Fig. 2** Designed input torque signals \( u_1 \) and \( u_2 \).

**Fig. 3** Response signals of rotation angles \( \delta\theta_i \) and modal displacements \( \delta\eta_i \) (for \( \Delta m = 20 \) kg and SNR = 40 dB).

Please cite this article in press as: NI Z et al. Identification of the state-space model and payload mass parameter of a flexible space manipulator using a recursive subspace tracking method, *Chin J Aeronaut* (2018), https://doi.org/10.1016/j.cja.2018.05.005
squares method and the algebraic algorithm are also employed to identify the payload mass $Dm$. The classical algebraic algorithm is commonly used for a single-link manipulator. Therefore, a dynamics model that only includes the second link and the endpoint payload is established when we apply the algebraic algorithm to identify the payload mass parameter.

Fig. 4  System test response results for the identified state-space model parameters using the RPBSID algorithm at the working point (for $Dm = 20$ kg, excluding measurement noise).

Fig. 5  Absolute errors of system test responses between the original and identified state-space model parameters using the RPBSID algorithm at the working point (for $Dm = 20$ kg).

Please cite this article in press as: NI Z et al. Identification of the state-space model and payload mass parameter of a flexible space manipulator using a recursive subspace tracking method, Chin J Aeronaut (2018), https://doi.org/10.1016/j.cja.2018.05.005
Besides, the gravity condition which is used in the algebraic algorithm is also provided in the simulation because the payload mass parameter is invariable whether it is in the space or on ground conditions. The main procedures of the algebraic algorithm for a single-link flexible manipulator can be briefly summarized as

\[
\begin{align*}
\begin{bmatrix} \Delta m(k) \\ v(k) \end{bmatrix} &= A_s(k)^{-1} B_s(k) \\
& \text{where the initial values } A_s(0) \text{ and } B_s(0) \text{ are zero matrices, } \\
\Phi_s(k) &= [L^2 \beta_s(k) + gL \zeta_s(k) \eta_s(k)], \text{ and } g \text{ is the gravity acceleration. The expressions of parameters } \\
\beta_s(t), \zeta_s(t), \eta_s(t), \text{ and } q_s(t) \text{ are given respectively as} \\
q_s(t) &= \begin{cases} \dot{z}_1 \\ \dot{z}_2 \\ \dot{r}_1 \Gamma(t) \end{cases} \\
\beta_s(t) &= \begin{cases} \dot{z}_3 + r^2 \dot{\theta}_1(t) \\ \dot{z}_4 - 4\dot{\theta}_1(t) \\ 2\dot{\theta}_1(t) \end{cases} \\
\zeta_s(t) &= \begin{cases} \dot{z}_5 \\ \dot{z}_6 \\ \dot{r} \cos(\theta_1(t)) \end{cases} \\
\end{align*}
\]

\[\begin{align*}
\begin{bmatrix} A_s(k) \end{bmatrix} &= A_s(k-1) + \Phi_s(k) \Phi_s^T(k) \Delta t \\
B_s(k) &= B_s(k-1) + \Phi_s(k) q_s(k) \Delta t
\end{align*}\]

where \( \Delta m(k) \) and \( v(k) \) are the payload mass and the viscous friction damping coefficient for each sampling time \( k \), respectively, and the recursive form of matrices \( A_s(k) \) and \( B_s(k) \) can be computed as

\[\begin{align*}
q_s(t) &= \begin{cases} \dot{z}_1 \\ \dot{z}_2 \\ \dot{r}_1 \Gamma(t) \end{cases} \\
\beta_s(t) &= \begin{cases} \dot{z}_3 + r^2 \dot{\theta}_1(t) \\ \dot{z}_4 - 4\dot{\theta}_1(t) \\ 2\dot{\theta}_1(t) \end{cases} \\
\zeta_s(t) &= \begin{cases} \dot{z}_5 \\ \dot{z}_6 \\ \dot{r} \cos(\theta_1(t)) \end{cases} \\
\end{align*}\]
\[
\begin{align*}
\eta_i(t) &= z_i \\
\dot{z}_i &= z_i + \dot{t} \theta_i(t) \\
\ddot{z}_8 &= -2\dot{t} \theta_i(t)
\end{align*}
\]  

where } \gamma(t) \text{ and } \theta_i(t) \text{ are the computed angle of the tip position and the measured torque at the base of the link, respectively. Then, the functions } \beta_i(t), \xi_i(t), \eta_i(t), \text{ and } \theta_i(t) \text{ can be sampled at discrete times } t = k\Delta t, \ k = 1, 2, 3 \ldots
\]

The detailed procedures about the algorithm are provided in Refs. \(20,32\). Now a total of 30 experiments are conducted for the three identification methods, and identified mass parameter results are shown in Fig. 7. It should be noted that Fig. 7 only displays the results from 4 s to 6 s because all the three algorithms require the iteration process to be as close to the original value between 3 s and 4 s as possible, and the identified results in the stable tracking phase using the recursive least squares method are slightly higher than the original values due to the flexible influence of the system.

Fig. 8 shows the absolute errors of the identified payload mass parameter } \Delta m \text{ with time for the three methods, and the corresponding average relative errors are shown in Fig. 9. The results in Figs. 8 and 9 indicate that the proposed method can identify the payload mass parameter stably. Although the average relative error of the proposed method exhibits a certain increase with a change in the payload mass, the overall identification accuracy is still acceptable.

4.2.2. Simulation example 2: capturing the same payload mass at different working points (different configurations)

In example 2, the identification of the state-space model and payload mass parameter at different working points is implemented as follows: (1) Case 1: } (\theta_1)_b = 0^\circ \text{ and } (\theta_2)_b = 0^\circ; \text{ (2) Case 2: } (\theta_1)_b = 45^\circ \text{ and } (\theta_2)_b = 0^\circ; \text{ and (3) Case 3: }

\begin{align*}
\theta_1 &= 0^\circ, \theta_2 = 0^\circ \\
\theta_1 &= 45^\circ, \theta_2 = 0^\circ \\
\theta_1 &= -45^\circ, \theta_2 = 45^\circ
\end{align*}

\[\text{Fig. 10 Three manipulator configurations.}\]
\((\theta_1)_0 = -45^\circ\) and \((\theta_2)_0 = 45^\circ\). The corresponding three manipulator configurations are shown in Fig. 10, and the payload mass is set as \(\Delta m = 100\) kg for each case. The other simulation parameters are the same as those in Section 4.1.

Test response results of rotation angles \(\delta \theta\) for the identified state-space model are shown in Fig. 11, and the corresponding absolute errors are shown in Fig. 12. As shown in Figs. 11 and 12, the results illustrate that state-space model parameters can be obtained by the RPBSID algorithm for different manipulator configurations.

Frequencies \(\omega_1\) and \(\omega_2\) for the three configurations using the RPBSID algorithm are shown in Fig. 13, and Table 2 shows the average relative errors of the identified frequencies. The results indicate that the average relative error is less than 5\%, which is acceptable for identification accuracy.
In a manner similar to that of example 1, when the state-space model parameters are obtained, the three approaches are implemented to identify the payload mass parameter $D_m$.

Fig. 14 shows identification results between 4 s and 6 s for the three configurations, and absolute errors with time and the corresponding average relative errors are shown in Figs. 15 and 16, respectively. The results in Figs. 14–16 indicate that the proposed method can achieve a fast tracking speed to approach original values and exhibit satisfactory identification accuracy.

4.3. Finite element verification for the payload mass parameter

To further verify the validity of the presented method, in this section, a simple finite element model of a two-link manipulator is established, and the proposed recursive method is used to identify the payload mass parameter using the response data of the finite element model. The defined coordinate system is shown in Fig. 1, and the working point is selected as $(\theta_1)_0 = 45^\circ$, $(\theta_2)_0 = 0^\circ$, and $(\eta_{11})_0 = (\eta_{21})_0 = 0$. The payload
mass is given as $\Delta m = 20$ kg, and the other simulation parameters are the same as those aforementioned.

Using the designed input signals as shown in Fig. 2, an explicit dynamic analysis is carried out to generate system vibration response signals. Displacement and velocity responses about the middle position of the $i$th link ($i = 1, 2$) are shown in Figs. 17 and 18, respectively. In Figs. 17 and 18, the displacement and velocity values of the 2nd link in the X-direction are almost zero because the corresponding vibration signals mainly reflect the link’s lateral deflection.

Now by extracting the input and output data of the finite element model, the proposed method is used to identify the payload mass parameter again, and the identified mass parameter result from 4 s to 6 s is shown in Fig. 19. It can be seen that now the identified error is higher than that of the theoretical computation in the

![Fig. 15](image15.png)

**Fig. 15** Absolute errors of the mass parameter $\Delta m = 100$ kg for three different configurations.

![Fig. 16](image16.png)

**Fig. 16** Average relative errors of the payload mass parameter $\Delta m = 100$ kg for three different configurations using three methods (from 4 s to 6 s).

![Fig. 17](image17.png)

**Fig. 17** Displacement responses in the middle position of the $i$th link ($i = 1, 2$).
paper because the finite element model is more sophisticated than the mathematical model, but at least the payload mass parameter can still be tracked after multiple iterations.

5. Conclusions

(1) In this study, the state-space model identification of a flexible space manipulator is studied, and a novel method based on the recursive subspace algorithm is presented to estimate the payload mass parameter. By linearizing and reconstructing the manipulator nonlinear dynamics equation at a selected working point, system state-space model parameters are determined recursively using the RPBSID algorithm.

(2) Moreover, using similarity transformation of matrices, the payload mass parameter can be calculated by extracting the corresponding block matrix of the identified state-space model. In simulations, cases with different payload masses and working points are discussed. Computation results demonstrate that the proposed algorithm can be used to identify the manipulator state-space model and payload mass parameter for capturing an unknown space object.

(3) However, certain problems still exist and need to be improved in the proposed payload mass identification method, which include the following: (a) some prior knowledge, such as the values of matrices $C$, $M_c$, and $M_a/C_0$, must be obtained before the identification, and the dynamics model of the space manipulator is established as a simply supported beam to simplify the derivation; and (b) the proposed method is based on the linearized model only at a certain working point, that is, it should satisfy the same working point before and after the capturing process. Hence, it is necessary to develop a more commonly-used identification algorithm.

(4) From identification results of the system frequency in numerical simulations, it can be found that the modal parameters of the manipulator change at a fixed working point due to the variation in the endpoint payload mass. Therefore, a future study will attempt to establish the relationship between the system frequency and the payload mass and use the frequency difference value to estimate the mass parameter. For example, some online estimator methods used for flexible structures have been proposed to estimate frequency values and corresponding vibration signal parameters. In this manner, the payload mass parameter can be determined simultaneously when the modal parameter identification procedure is implemented.

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Appendix A

Fig. 19 Identification results of the payload mass parameter $\Delta m = 20$ kg using the proposed method for the configuration $(\theta_1)_0 = 45^\circ$ and $(\theta_2)_0 = 0^\circ$.

The detailed elements of matrices $M(q)_{4\times4}$, $E(q, \dot{q})_{4\times1}$, and $K_{4\times4}$ in Eq. (7).

The detailed elements of the mass matrix $M(q)_{4\times4}$, the coupling coefficient matrix $E(q, \dot{q})_{4\times1}$, and the stiffness matrix $K_{4\times4}$.
Identification of the state-space model and payload mass parameter of a flexible space manipulator using a recursive subspace tracking method, Chin J Aeronaut (2018), https://doi.org/10.1016/j.cja.2018.05.005