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Robust Guaranteed Cost Control for Yaw Control of Helicopter

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Abstract: A new design method of robust guaranteed cost controller with adaptation mechanism was proposed. A linear time-invariant system with time-varying ellipsoidal uncertainty was considered. *First, an adaptation mechanism was introduced to establish a target model with adjustable parameters. The adjustable parameters were determined according to the designed adaptive laws to ensure the quadratic stability of the error system between the state trajectory of the plant and that of the target model. Consequently, a robust guaranteed cost tracking controller was proposed, whose gains affinely depended on the designed adjustable parameters, to guarantee the stability of the closed-loop systems.* The application of this approach to the yaw control of a small-scale helicopter mounted on an experiment platform shows the effectiveness.

Key words: guaranteed cost control; robust control; adaptation mechanism; helicopter

直升机航向鲁棒保性能控制

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摘 要：针对具有时变不确定性且不确定性界为椭球的线性系统提出了一种新的具有自适应机制的鲁棒保性能控制器设计方法。**首先，引入一个具有可由自适应律在线调整的可调参数的目标模型，通过该参数来保证由目标模型与被控模型所获得的误差系统渐近稳定。结合保证目标模型稳定性的设计，最终形成保证闭环系统稳定且控制器增益仿射依赖于可调参数的鲁棒保性能跟踪控制器。**应用于安装在试验平台上的小型直升机航向控制中，仿真试验表明了该方法的有效性。

关键词：保性能控制；鲁棒控制；自适应机制；直升机
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Introduction

In most control problems, the real challenge is to raise and maintain performance in the presence of uncertainty [1]. It is often desirable to obtain guarantee of stability and performance against uncertainty on the physical parameters including stiffness, inertia, or viscosity coefficient in mechanical systems, aero-dynamical coefficients in flight control, the values of resistors and capacitors in electrical circuits.

Helicopter as vertical take off and landing vehicles, has unique flying capabilities of hovering, low speed cruising and vertical take-off/landing. The unique flying capabilities of hovering, make helicopter ideal platform for application, such as terrain surveying, surveillance and clean-up of hazardous waste sites, aerial imaging and so on. A wide of considerable attention has been paid to analysis and synthesis about flight control system of helicopter [2-6]. The Unmanned Helicopter, called Rotorcraft Unmanned Aerial Vehicle, is very similar to manned helicopter besides the small body. Last decade, there had witnessed remarkable progress in small-scale helicopter research including modeling [7] and control [8-9]. But those control methods were based on the accurate and fixed dynamic model. The complicated dynamics of helicopter lead to both parametric and dynamic uncertainty, so the controller should be designed to robust to those effects.

Recently, a considerable amount of work has been done to design robust controllers for linear system with parameter uncertainty. Since an adequate level of performance is required in practice, recent literatures have focused on quadratic stabilizing control with some performances based on LMI or other methods [9-11]. The guaranteed cost control approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound. While a single controller with a fixed gain is considered, the resulting controllers designed by these methods inherently become conservative.

Adaptive method is one of the effective methods to deal with parameter uncertainty [12-13]. They rely on the potential of adjustments of uncertain parameters to assure stability of closed-loop systems. Most of the results in adaptive robust

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control are based on model reference adaptive control (MRAC) [14], where the outputs of closed-loop systems can track the pre-described referent outputs. Unfortunately, this adaptive method is not easily extended to treat performance tasks. It is worthwhile considering incorporating some kind of adaptation mechanism into robust control methods to enhance the performance of systems.

In this paper, an adaptation mechanism is successfully introduced into establishing a new robust tracking guaranteed cost controller to reduce conservatism inherent in traditional robust control method with fixed gains. Here both system matrix A and input matrix B have time-varying parameter uncertainty. It belongs to an ellipsoidal set, which often appears in the results of set member identification in practical systems [15]. The gains of the proposed robust tracking controller are tuned on-line based on the estimations of parameter uncertainties to guarantee the stability and improve the transient behavior of the closed-loop systems. Since the complicated dynamics of helicopter leading to both parametric and dynamic uncertainty and the parameter uncertainty are time-varying, the proposed method will be applied to the yaw control of helicopter.

The paper is organized as follows. In Section 1, the problem statement and preliminaries are formulated. Section 2 gives the adaptive robust tracking control method. The application of the proposed controller to the yaw control of small-scale helicopter is given in Section 3. Finally, Section 4 gives the conclusion.

1 Problem Formulation and Preliminaries

Consider the following linear uncertainty model described by

$$\dot{x}(t) = A(\theta(t))x(t) + B(\theta(t))u(t) + B_\omega(\theta(t))\omega(t)$$
$$y(t) = C_1 x(t) \tag{1}$$

where $x(t)\in R^n$ is the state, $u(t)\in R^m$ is the control input , $y(t)\in R^p$ is the measured output and $\omega(t)\in R^l$ is an exogenous disturbance which belongs to $L_2[0,\infty)$, respectively. The system matrices have the following time-varying structure

$$A(\theta(t)) = A_0 + \sum_{i=1}^{N}\theta_i(t)A_i$$

$$B(\theta(t)) = B_0 + \sum_{i=1}^{N}\theta_i(t)B_i$$

$$B_\omega(\theta(t)) = B_{\omega 0} + \sum_{i=1}^{N}\theta_i(t)B_{\omega i}$$

where $A_0, A_1\cdots A_N, B_0, B_1\cdots B_N, B_{\omega 0}, B_{\omega 1}\cdots B_{\omega N}$ are known constant matrices. The time-varying parameter vector $\theta(t)\in R^N$ represents unknown parameters which belong to the N-dimensional ellipsoidal set expressed as

$$\Delta \equiv \{\theta\in R^N \mid \theta^T(t)\Sigma^{-2}\theta(t)\le 1\} \tag{2}$$

$$\Sigma = \mathrm{diag}(\sigma_1,\cdots\sigma_N) \tag{3}$$

where $\Sigma\in R^{N\times N}$ represents the size of the ellipsoid.

Remark 1: The ellipsoidal set can be obtained by set membership identification method. Set membership identification is one of the identification techniques that uses a priori assumptions about a parametric model to constrain the solutions to certain sets.

Control Objective: design a robust controller such that:

a) The closed-loop system is stable for all $\theta(t)\in\Delta$ with a guaranteed level of disturbance attenuation.

b) The output $y(t)$ tracks the reference signal $r_d(t)$ with zero steady-state error, that is $\lim_{t\to\infty} e(t) = 0$, where $e(t) = r_d(t) - y(t)$.

It is well known that an integral control can effectively eliminate the steady tracking error. In order to obtain a robust tracking controller with state feedback plus tracking error integral, the following augmented state-space description is introduced.

$$\dot{\bar{x}}(t) = \bar{A}(\theta(t))\bar{x}(t) + \bar{B}(\theta(t))u(t) + \bar{B}_\omega(\theta(t))\omega_a(t) \tag{4}$$

where

$$\bar{x}(t) = \begin{bmatrix} (\int_0^t e(\tau)\mathrm{d}\tau)^T & x^T(t)\end{bmatrix}^T,\ \omega_a(t) = [r_d{}^T(t) \quad \omega^T(t)]^T$$

and

$$\bar{A}(\theta(t)) = \begin{bmatrix} 0 & -C_1 \\ 0 & A(\theta(t))\end{bmatrix} \qquad \bar{B}(\theta(t)) = \begin{bmatrix} 0 \\ B(\theta(t))\end{bmatrix}$$

$$\bar{B}_\omega(\theta(t)) = \begin{bmatrix} I & 0 \\ 0 & B_\omega(\theta(t))\end{bmatrix}$$

Then the design problem can be reduced to the following: find a robust controller $u(t)$ such that:

a) The augmented closed-loop system is robust for stable for all $\theta(t)\in\Delta$.

b) Transient performance improves in time-response.

2 Adaptive Robust Tracking Controller Design

In this section, we introduce a target model with adjustable parameters which is determined so as to ensure quadratic stability of the error system between the state trajectory of the plant and that of the target model. Then a controller for the target model is designed. Consequently, an adaptive robust controller to improve transient behaviour in time-response is established.

2.1 Adjustable Target Model and Parameter Adjustment Law

In order to obtain on-line information on the parameter uncertainty, we introduce the following target model described by

$$\dot{\bar{x}}_v(t) = \bar{A}(\hat{\theta}(t))\bar{x}_v(t) + \bar{B}(\hat{\theta}(t))v(t),\ \ \bar{x}_v(0) = \bar{x}_0 \tag{5}$$

where $\hat{\theta}(t)\in R^N$ denotes the adjustable parameter vector, and let the matrices $\bar{A}(\hat{\theta})$ and $\bar{B}(\hat{\theta})$ have the same structure as the system matrices of (4). If we define the error vector as $\bar{e}=\bar{x}-\bar{x}_v$, then the error equation between (4) and (5) is written as

$$\begin{aligned}\dot{\bar{e}}&=\bar{A}(\theta)\bar{e}+\bar{B}(\theta)(u-v)+\sum_{i=1}^{N}(\theta_i-\hat{\theta}_i)(\bar{A}_i\bar{x}_v+\bar{B}_iv)+\bar{B}_\omega(\theta)\omega_a(t)\\&=\bar{A}(\theta)\bar{e}+\bar{B}(\theta)(u-v)+(E(\bar{x}_v)+E(v))(\theta-\hat{\theta})+\bar{B}_\omega(\theta)\omega_a(t)\end{aligned}\quad(6)$$

where $E(\bar{x}_v)\in R^{(n+p)\times N}$, $E(v)\in R^{(n+p)\times N}$ is given by

$$E(\bar{x}_v)=[\bar{A}_1\bar{x}_v\,|\cdots|\,\bar{A}_N\bar{x}_v],\ E(v)=[\bar{B}_1v\,|\cdots|\,\bar{B}_Nv]$$

By considering the control input

$$u(t)=v(t)+F(\hat{\theta})\bar{e}(t)\quad(7)$$

where $F(\hat{\theta})=F_0+\sum_{i=1}^{N}\hat{\theta}_iF_i$, $F_0,F_1\cdots F_N$ are the error feedback to be designed. Then (6) can be written as

$$\dot{\bar{e}}=(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))\bar{e}+(E(\bar{x}_v)+E(v))(\theta-\hat{\theta})+\bar{B}_\omega\omega_a(t)\quad(8)$$

In the following design, we choose the performance indexes

$$J_t=\int_0^t\left[\bar{e}^TQ\bar{e}+(u-v)^TR(u-v)\right]dt\quad(9)$$

where, $Q=diag[Q_1\ \ Q_2]$, $Q_1\in\mathrm{R}^{l\times l}$ and $Q_2\in\mathrm{R}^{n\times n}$ are symmetric positive semi-definite and $R\in\mathrm{R}^{m\times m}$ is symmetric positive definite.

Here, for the error system (8), we determine the parameter vector $\hat{\theta}(t)$ and the gain matrix $F(\hat{\theta})$ so as to ensure quadratic stability and performance index.

Theorem 1: Consider the error system (8) and the performance index J_t, for a given positive constant γ, if there exists symmetric matrix M and matrices H_0, H_i, $i=1\cdots N$, the following linear matrix inequality holds

$$\begin{bmatrix}M\bar{A}^T(\theta)+H^T(\hat{\theta})B^T(\theta)+\bar{A}(\theta)M+\bar{B}(\theta)H(\hat{\theta}) & \bar{B}_\omega(\theta) & H^T(\hat{\theta})R^{1/2} & MQ^{1/2}\\ * & -\gamma^2I & 0 & 0\\ * & * & -I & 0\\ * & * & * & -I\end{bmatrix}<0$$

for all $\theta(t),\ \hat{\theta}(t)\in\Delta$　(10)

where, * denotes the symmetric part ,

$$M=P^{-1},\ H(\hat{\theta})=H_0+\sum_{i=1}^{N}H_i\hat{\theta}_i,$$
$$H_0=F_0M,\ \ H_i=F_iM,\ \ i=1\cdots N.$$

And also if $\hat{\theta}(t)$ is determined according to the adaptive law

$$\hat{\theta}(t)=\begin{cases}\dfrac{\Sigma^2[E^T(\bar{x}_v)+E^T(v)]P\bar{e}}{\left\|\Sigma[E^T(\bar{x}_v)+E^T(v)]P\bar{e}\right\|} & \text{if }[E^T(\bar{x}_v)+E^T(v)]P\bar{e}\neq0\\ \hat{\theta}(t^-) & \text{if }[E^T(\bar{x}_v)+E^T(v)]P\bar{e}=0\end{cases}\quad(11)$$

where $t^-=\lim_{\tau>0,v\to0}(t-\tau)$ and the initial guess of the parameter $\hat{\theta}(t)$, denoted by $\hat{\theta}(0)$, is supposed to be chosen from on the boundary surface of the ellipsoidal set Δ. Then the error system (8) is stablized and the upper bound of performance index is given by

$$J_t\le\gamma^2\int_0^t\omega_a^T(t)\omega_a(t)dt+\bar{e}^T(0)M^{-1}\bar{e}(0)$$

Proof: By the Schur Complement formula, (10) is equivalent to

$$M\bar{A}^T(\theta)+H^T(\hat{\theta})B^T(\theta)+\bar{A}(\theta)M+\bar{B}(\theta)H(\hat{\theta})+\frac{1}{\gamma^2}B_\omega B_\omega^T+MQM+H^T(\hat{\theta})RH(\hat{\theta})<0$$

Moreover, by pre- and post-multiplying $M=P^{-1}$ on above inequality, it follows

$$(\bar{A}^T(\theta)+B^T(\theta)F(\hat{\theta}))^TP+P(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))+\frac{1}{\gamma^2}PB_\omega B_\omega^TP+Q+F^T(\hat{\theta})RF(\hat{\theta})<0\quad(12)$$

Choose the following candidate Lyapunov function

$$V=\bar{e}^T(t)Pe(t)\quad(13)$$

Then from the derivative of V along the error system (8), we can get

$$\begin{aligned}\dot{V}&=\bar{e}^T[(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))^TP+P(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))]\bar{e}+\\&\quad 2\bar{e}^TP(E(\bar{x}_v)+E(v))(\theta-\hat{\theta})+2\bar{e}^TP\bar{B}_\omega\omega_a\\&\le-\alpha\|\bar{e}\|^2+2\bar{e}^TP(E(\bar{x}_v)+E(v))(\theta-\hat{\theta})+2\bar{e}^TP\bar{B}_\omega\omega_a\end{aligned}$$

where

$$\alpha=-\lambda_{\max\limits_{\theta\in\Delta}}[(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))^TP+P(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))]>0$$

According to Lyapunov stability theorem, it can be assumed that $\omega_a\equiv0$, when only the stability for system (8) is concerned. Setting the parameter adjustment law as (11) results in $\dot{V}\le-\alpha\|\bar{e}\|^2$, because using (2) the following relation holds

$$\begin{aligned}\left|\bar{e}^TP(E(\bar{x}_v)+E(v))\theta\right|&\le\left\|\bar{e}^TP(E(\bar{x}_v)+E(v))\Sigma\right\|\left\|\Sigma^{-1}\theta\right\|\\&\le\left\|\bar{e}^TP(E(\bar{x}_v)+E(v))\Sigma\right\|\\&=\bar{e}^TP(E(\bar{x}_v)+E(v))\hat{\theta}\end{aligned}$$

Therefore the stability of the error system (8) is ensured. Furthermore, from (12)

$$\begin{aligned}J_t&=\int_0^t\left[\bar{e}^TQ\bar{e}+(u-v)^TR(u-v)\right]dt\\&=\int_0^t\bar{e}^T[Q+F^T(\hat{\theta})RF(\hat{\theta})]\bar{e}dt\\&<-\int_0^t\bar{e}^T\left[(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))^TP+P(\bar{A}(\theta)+\bar{B}(\theta)F(\hat{\theta}))+\frac{1}{\gamma^2}PB_\omega B_\omega^TP\right]\bar{e}dt\\&=-\int_0^td[v]+\gamma^2\int_0^t\omega_a^T\omega_adt-\int_0^t(-\gamma\omega_a^T+\frac{1}{\gamma}\bar{e}^TPB_\omega)(-\gamma\omega_a^T+\frac{1}{\gamma}\bar{e}^TPB_\omega)^Tdt\\&\le-\int_0^td[v]+\gamma^2\int_0^t\omega_a^T\omega_adt\\&\le\bar{e}^T(0)P\bar{e}(0)+\gamma^2\int_0^t\omega_a^T\omega_adt\end{aligned}$$

Thus,

$$J_t\le\bar{e}^T(0)P\bar{e}(0)+\gamma^2\int_0^t\omega_a^T\omega_adt$$

Remark 2: The adjustable parameter $\hat{\theta}(t)$ satisfies $\hat{\theta}(t)\Sigma^{-2}\hat{\theta}(t)=1$, which means that $\hat{\theta}(t)$ is adjusted on the boundary surface of the prespecified ellipsoidal set Δ.

Remark 3: In order to transform (5) to a convex problem, a substitute set for the ellipsoidal set Δ can be used in Theorem 1, that is

$$\bar{\Delta}=\{\theta(t)\in R^N\mid\theta_i(t)\le\sigma_i,i=1\cdots N\}$$

Then since $\theta(t),\ \hat{\theta}(t)$ appear affinely in (5), the problem can be reduced to check (10) for all $\theta(t),\ \hat{\theta}(t)\in\bar{\Delta}_{vex}$, where $\bar{\Delta}_{vex}=\{\theta\in R^N\mid\theta_i=\sigma_i\text{ or }\theta_i=-\sigma_i\}$ denotes the set of 2^N vertices of $\bar{\Delta}$.

Remark 4: The error feedback gain $F_0,F_1\cdots F_N$ can be chosen to minimize the upper bound of the performance index

J_t according to the following optimization problem

min *Trace* (T)

s.t. (10) and $\begin{bmatrix} T & I \\ I & M \end{bmatrix} > 0$

2.2 Adjustable Controller Design for Target Model

Since the parameter $\hat{\theta}(t)$ is available on-line, we will establish $v(t)$ in a state feedback form with a parameter-dependent gain $K(\hat{\theta})$

$$v(t) = K(\hat{\theta}(t))\overline{x}_v(t) \tag{14}$$

where

$$K(\hat{\theta}(t)) = K_0 + \sum_{i=1}^{N}\hat{\theta}_i(t)K_i \tag{15}$$

Substituting (14) and (15) into (5) results in the close-loop form

$$\dot{\overline{x}}_v(t) = (\overline{A}(\hat{\theta}) + \overline{B}(\hat{\theta})K(\hat{\theta}))\overline{x}_v(t) \tag{16}$$

Choose cost function

$$J = \int_0^\infty (\overline{x}_v^T Q\overline{x}_v + v^T Rv)\mathrm{d}t \tag{17}$$

Theorem 2: The target model (5) with the adjustable controller (14) is stable if there exists a positive definite matrix S and matrices $W_0, W_i, M_i (i = 1\cdots N)$ such that for all $\hat{\theta} \in \overline{\Delta}_{vex}$

$$\begin{bmatrix} S\overline{A}^T(\hat{\theta}) + W^T(\hat{\theta})\overline{B}^T(\hat{\theta}) + A(\hat{\theta})S + \overline{B}(\hat{\theta})W(\hat{\theta}) + \sum_{i=1}^{N}\hat{\theta}_i^2 M_i & S & W^T(\hat{\theta}) \\ * & -Q^{-1} & 0 \\ * & * & -R^{-1} \end{bmatrix} < 0 \tag{18}$$

$$W_i^T\overline{B}_i^T + \overline{B}_i W_i + M_i \ge 0,\ M_i \ge 0,\ \ i = 1\cdots N \tag{19}$$

where $S = P^{*-1}$, $W(\hat{\theta}) = W_0 + \sum_{i=1}^{m}\hat{\theta}_i(t)W_i$, $W_0 = K_0 S$ $W_i = K_i S$, $i = 1\cdots N$.

Moreover, the upper bound of the cost function (17) with respect to $\hat{\theta}(t) \in \overline{\Delta}$ is given as

$$J \le \overline{x}_v^T(0)P^*\overline{x}_v(0) = \overline{x}_0^T S^{-1}\overline{x}_0 \quad \text{for all } \hat{\theta}(t) \in \overline{\Delta} \tag{20}$$

Proof: Choose the following Lyapunov function

$$V = \overline{x}_v{}^T(t)P^*\overline{x}_v(t)$$

Then if follows

$$J \le \int_0^\infty (x_v^T Qx_v + v^T Rv + \frac{\mathrm{d}}{\mathrm{d}t}V)\mathrm{d}t + \overline{x}_v{}^T(0)P^*\overline{x}_v(0)$$
$$\le \int_0^\infty x_v^T\Psi x_v + \overline{x}_v{}^T(0)P^*\overline{x}_v(0)$$

where

$$\Psi = (\overline{A}(\hat{\theta}) + \overline{B}(\hat{\theta})K(\hat{\theta}))^T P^* + P^*(\overline{A}(\hat{\theta}) + \overline{B}(\hat{\theta})K(\hat{\theta})) + Q + K^T(\hat{\theta})RK(\hat{\theta})$$

If (18) and (19) hold, by Lemma 3.1 and Remark 3.6 of [16] and we have for all $\hat{\theta}(t) \in \overline{\Delta}$

$$S\overline{A}^T(\hat{\theta}) + W^T(\hat{\theta})\overline{B}^T(\hat{\theta}) + A(\hat{\theta})S + SQS + \overline{B}(\hat{\theta})W(\hat{\theta}) + W^T(\hat{\theta})RW(\hat{\theta}) < 0$$

which implies for all $\hat{\theta}(t) \in \overline{\Delta}$, $\Psi < 0$. Then

$$J \le \overline{x}_v^T(0)P^*\overline{x}_v(0) = \overline{x}_0^T S^{-1}\overline{x}_0.$$

The following is algorithm to optimize the cost function (17) of target model.

Algorithm: The cost function J is minimized if the following optimization problem is solvable

$$\min_{Z,S,W_0,W_1\cdots W_N} \mathrm{Tr}(Z)$$

s.t. (18), (19) and $\begin{bmatrix} Z & I \\ I & S \end{bmatrix} \ge 0$

After designing adjustable controller $v(t)$ for target model, now the controller $u(t)$ has the following form

$$u(t) = K(\hat{\theta}(t))\overline{x}_v(t) + F(\hat{\theta}(t))\overline{e}(t)$$

where $\hat{\theta}(t)$ is determined by (11), K_0, K_i , F_0 , F_i , $i = 1\cdots N$ can be obtained from Theorem 1 and Theorem 2, respectively.

Total system is guaranteed to be stable because both target model and error system have been stabilized.

Remark 5: The adaptive robust tracking controller design method in Section 2 is investigated for linear systems with affine time-varying ellipsoidal uncertainty (21). Once some practical systems can be described by the general model (22), the presented controller design method can be applied to the practical systems. Next, the application to the yaw control of a small-scale helicopter will show the effectiveness of the proposed method.

3 Small–scale Helicopter Control

3.1 Modelling Yaw Dynamic

In this paper a framework of the simulation model for the helicopter-platform (see Fig. 1) is set up using rigid body equations of motion of the helicopter fuselage. In hovering and low-velocity flight, the torque generated by main and force generated by tail rotor are dominant [17]. By simplifying the fuselage and vertical fin damping, the yaw dynamics can be rewritten as:

$$\begin{cases} \dot{\varphi} = r \\ I_{zz}\dot{r} = -Q_{mr} + T_{tr}l_{tr} + b_1 r + b_2\varphi \end{cases} \tag{21}$$

where Q_{mr} is the torque of main rotor, T_{tr} is the thrust of tail rotor, l_{tr} is the distance between the tail rotor and z-axis, b_1 and b_2 are damping constants. The expressions of T_{tr} and Q_{mr} has been given in [18]:

$$T_{tr} = C_1\theta_{tr} + \tfrac{1}{2}C_2(C_2 + \sqrt{C_2^2 + 4C_1\theta_{tr}}) \tag{22}$$

with

$$C_1 = \tfrac{1}{6}\rho a_{tr}b_{tr}c_{tr}\Omega_{tr}{}^2(R_{tr}{}^3 - R_{\mathrm{tr}0}^3)$$

$$C_2 = \tfrac{1}{8}\rho a_{tr}b_{tr}c_{tr}\Omega_{tr}\sqrt{2/\rho\pi R_{tr}{}^2}(R_{tr}{}^2 - R_{tr0}^2)$$

where ρ, a_{tr}, b_{tr}, c_{tr}, Ω_{tr}, θ_{tr}, r_{tr}, v_{tr1} A_{tr} are respectively, density of air, slope of the lift curve, number of the rotor, chord of the blade, speed of the tail rotor, pitch angle, radial distance, induced speed of the tail rotor and area of the tail rotor disc.

$$\begin{aligned}Q_{mr}=&\frac{c\theta_{mr}}{48\pi R^2}\{8C_{d2}\Omega\sqrt{\rho\pi R^2(2C_3\theta_{mr}+C_4^2-C_4\sqrt{C_4^2+4C_3\theta_{mr}})}(R_0^3-R^3)+\\&4a\Omega\sqrt{\rho\pi R^2(2C_3\theta+C_4^2-C_4\sqrt{C_4^2+4C_3\theta_{mr}})}(R^3-R_0^3)+\\&6C_{d2}C_3(R^2-R_0^2)+6aC_3(R_0^2-R^2)+6C_{d1}\rho\pi\Omega^2R^2(R^4-R_0^4)\}+\\&\frac{c}{48\pi R^2}\{6C_{d0}\rho\pi\Omega^2R^2(R^4-R_0^4)+3aC_4^2(R_0^2-R^2)+\\&3C_{d2}C_4\sqrt{C_4^2+4C_3\theta_{mr}}(R_0^2-R^2)+3aC_4\sqrt{C_4^2+4C_3\theta}(R^2-R_0^2)+\\&4C_{d1}\Omega\sqrt{\rho\pi R^2(2C_3\theta_{mr}+C_4^2-C_4\sqrt{C_4^2+4C_3\theta_{mr}})}(R_0^3-R^3)+\\&3C_{d2}C_4^2(R^2-R_0^2)\}+\tfrac{1}{8}C_{d2}\rho c\Omega^2(R^4-R_0^4)\theta_{mr}^{\ 2}\end{aligned}\qquad(23)$$

with $C_3=\frac{1}{6}\rho abc\Omega^2(R^3-R_0^3)$

$$C_4=\tfrac{1}{8}\rho abc\Omega\sqrt{2/\rho\pi R^2}(R^2-R_0^2)$$

where R, θ_{mr} are respectively, radial and pitch angle of main rotor, a,α, r,c,ϕ,v_1,Ω are respectively slope of the lift curve, the angle of attack of the blade element, speed radial distance, chord of the blade, inflow angle, induced speed and rotor speed of the main rotor.

From (21) we can see that there exists couplings between main rotor torque Q_{mr} and tail rotor thrust T_{tr}. and (22) and (23) further demonstrate that the models are highly nonlinear and too complex to be used for control design. Instead of the dynamics described by (22) and (23), a simplified model is proposed for control design [18]:

$$\begin{cases}\dot\varphi=r\\ I_{zz}\dot r=-(k_{Q2}\theta_{mr}^2+k_{Q1}\theta_{mr}+k_{Q0})\\ \qquad+(k_{T2}\theta_{tr}^{\ 2}+k_{T1}\theta_{tr}+k_{T0})l_{tr}+b_1r+b_2\varphi\end{cases}\qquad(24)$$

The nonlinear dynamic can be presented by a state space description :

$$\dot x=f(x,u)+\zeta$$

where, ζ is the disturbance due to main rotor, wind and so on, $x=[\varphi\quad r]^T$, $u=\theta_{tr}$.

Furthermore (24) can be linearized at a trim point (x_0,u_0)

$$\dot x=Ax+Bu+\zeta\qquad(25)$$

with

$$A=\frac{\partial f}{\partial x}|_{x0,u0}=\begin{bmatrix}0&1\\a_1&a_2\end{bmatrix},\ B=\frac{\partial f}{\partial u}|_{x0,u0}=\begin{bmatrix}0\\a_3\end{bmatrix}$$

where, $a_3=2k_{T2}l_{tr}I_{zz}^{-1}\theta_{tr0}+k_{T1}l_{tr}I_{zz}^{-1}+b_3\Omega$, $a_1=b_2I_{zz}^{-1}$, $a_2=b_1I_{zz}^{-1}$.

3.2 Simulations

The proposed controller design method is verified by the simulation model obtained from the helicopter-on-arm platform, shown as Fig. 1. A small-scale electrical helicopter is mounted at the end of a two-DOF arm, and the weight of the helicopter is perfectly balanced at the other side of the arm. First, the parameters of the nonlinear yaw dynamic model are identified

$$\begin{cases}\dot\varphi=r\\ \dot r=k_1r+k_2\theta_{tr}+k_3\theta_{tr}^2+k_4\Omega\theta_{tr}+k_5\varphi\end{cases}\qquad(26)$$

with, k_1= -1.3828, k_2= 63.0923, k_3= 11.6514, k_4= -0.1380 k_5= -3.3286, $\Omega=1200$. System (26) can be linearized at trim point

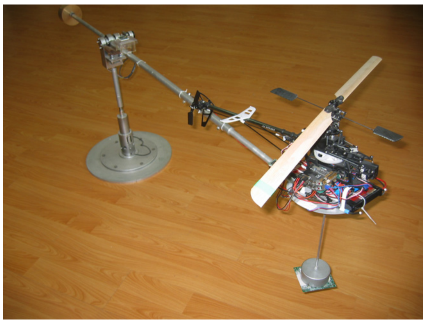

Fig.1 Helicopter on arm

(x_0,u_0), with $x_0=[30\ \ 0]^T$, $u_0=6.7$, and the corresponding system matrices are as follows:

$$A_0=\begin{bmatrix}0&1\\-3.3286&-1.3828\end{bmatrix},\ A_1=\begin{bmatrix}0&0\\-3.3286&0\end{bmatrix},$$

$$A_2=\begin{bmatrix}0&0\\0&-1.3828\end{bmatrix},\ A_3=\begin{bmatrix}0&0\\0&0\end{bmatrix},\ B_0=\begin{bmatrix}0\\72.2633\end{bmatrix},$$

$$B_1=\begin{bmatrix}0\\0\end{bmatrix},\ B_2=\begin{bmatrix}0\\0\end{bmatrix},\ B_3=\begin{bmatrix}0\\72.2633\end{bmatrix}$$

Parameter uncertainty

$$\theta^T\Sigma^{-2}\theta\le1,\Sigma=diag(0.3,0.3,0.35)$$

In the following simulations, the initial conditions are: $\varphi(0)=0$, $r(0)=0$. The tracking command of φ is $\varphi_d=0$, $0\le t\le t_{off}$. The following disturbance is used:

$$\zeta_1=\begin{cases}10 & 10\le t\le 11(s)\\0&\text{else}\end{cases}\qquad \zeta_2=\begin{cases}10&10\le t\le11(s)\\0&\text{else}\end{cases}$$

Next, the proposed adaptive robust tracking controller design method in Section 2 is applied to the yaw control of the helicopter. The corresponding gains $F_i,K_i,i=0,\cdots,3$ are the following:

F_0 = [10.0242 -8.1354 -1.1132]
F_1 = [0.0000 0.0193 -0.0000]
F_2 = [-0.0000 0.0000 0.0035]
F_3 = [-1.4375 1.1666 0.1596]
K_0 = [1.9066 -3.7309 -1.3803]
K_1 = [0 0 0]
K_2 = [0 0 0]
K_3 = [-0.1868 0.3656 0.1353]

The traditional guaranteed controller with fixed gains [19] K_f = [4.2030 -9.2801 -1.2488] has also been implemented on yaw dynamic of the helicopter.

From Fig.2 and Fig.3, it is easy to see that the closed-loop

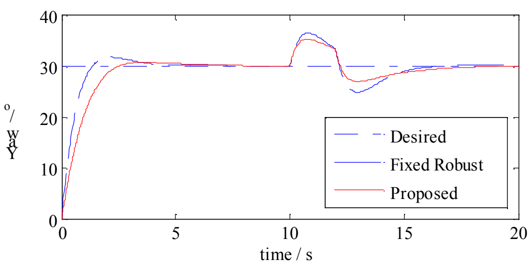

Fig. 2 Yaw behaviours with the fixed robust controller and the proposed robust controller

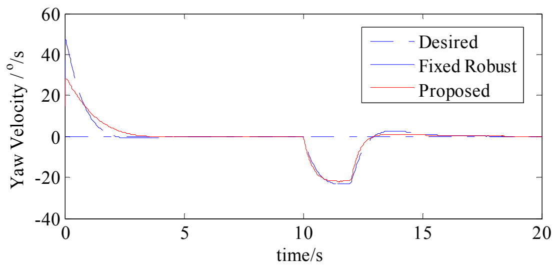

Fig.3 r behaviours with the fixed gain robust controller and the proposed robust controller

system is stable and has zero tracking error even in presence of disturbance. Moreover, the proposed adaptive robust controller can improve the system performance in the presence of disturbance and time-varying uncertainty, compared with the traditional robust controller with fixed gains.

4 Conclusions

In this paper, a new guaranteed controller design method is proposed for linear uncertain systems. The considered uncertainties are assumed to be time-varying ellipsoidal uncertainties. An adaptation mechanism is introduced to construct a variable gain guaranteed cost tracking controller to reduce conservatism inherent in traditional robust control with fixed gains and improve transient performance in time-response. The application of this approach to the yaw control of a small-scale helicopter has demonstrated the effectiveness of the proposed method.

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