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–Brief Paper–

L_∞ OBSERVER FOR UNCERTAIN LINEAR SYSTEMS

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ABSTRACT

In this paper, an L_∞ observer design method is proposed for linear system subject to parameter uncertainty and bounded disturbance. The proposed L_∞ observer, which satisfies a peak-to-peak disturbances attenuation performance, is designed to overbound the estimation error. Moreover, sufficient conditions for the design of L_∞ observer are derived and expressed in terms of linear matrix inequalities (LMIs). The novelty of the proposed method is that we develop an L_∞ observer that not only can attenuate bounded disturbance but also provides an upper bound of estimation error norm. Simulation results are presented to illustrate the effectiveness of the proposed method.

Key Words: L_∞ observer, parameter uncertainty, peak-to-peak, disturbance attenuation.

I. INTRODUCTION

Observer design is of both theoretical importance and practical significance in the control community since the observer is widely used in controller design and fault diagnosis. Therefore, observer design has been intensively studied since 1970s [1–3]. It is known that the theory of observer design for nominal linear system is well established. However, parameter uncertainties, external disturbances and noise are inevitable in practice. The estimation of the states may not be sufficiently accurate in such a Luenberger observer because of these non-ideal factors.

To deal with this problem, robust observer design has received considerable attention in the last two decades [4–6]. The H_∞ technique has been widely used to design robust observer to attenuate the effect of exogenous disturbance [5]. Although the H_∞ observer is able to attenuate the effect of disturbance in some sense, its estimate may not accurately converge to the actual states [7], especially for systems with model uncertainties. In [6], a robust observer is proposed to deal with state estimation for linear systems with model uncertainty. However, [8] comments that the method presented in [6] cannot make the estimation error converge to zero unless equilibrium point of estimation error equation is zero.

Despite conventional estimator cannot converge to the real value of the state in the presence of model uncertainty, an interval estimation may still remain feasible. Interval estimations methods can be generally classified as interval observer, zonotopic technique and peak-to-peak analysis methods. The interval observer has become a popular estimation design method to deal with uncertainty during the last decade [9–11]. In continuous-time case, this method requires that the estimation error state matrix is not only a Hurwitz matrix but also a Metzler matrix (*i.e.* all its off-diagonal elements are nonnegative) [11]. However, searching for a qualified observer gain is not a trivial task [12]. The Zonotopic technique is an alternative approach to build state bounding of the observer [13–16]. The Zonotopic technique provides an approximate reachable set in sampled time series, which is used in discrete-time system generally. Peak-to-peak analysis method has been used to attenuate the bounded disturbance with concern of minimizing the maximum (peak) amplitude of the tracking error [17,18]. The peak-to-peak performance was first introduced in [17] and then the peak-to-peak filtering problems for linear time-variant systems were studied in [19,20]. [21] shows an application of the robust peak-to-peak filter in genetic regulatory networks. However, parameter uncertainty is not considered in the above-mentioned papers on peak-to-peak filtering. Peak-to-peak analysis methods are also referred to as state estimation using quadratic boundedness in [22,23]. Disturbance attenuation was also discussed for systems with parameter uncertainty using ellipsoids-based method in [24,25]. Peak-to-peak analysis requires the induced L_∞ norm performance prescribed bounded from the unknown inputs

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to the estimation error. To the best of our knowledge, peak-to-peak analysis for uncertain systems in the existing literature is complicated by bounding the reachable set with inescapable ellipsoids.

In this note, we develop a new peak-to-peak analysis and design method based on Lyapunov stability theory. Compared with the well-known ellipsoids-based design method, the proposed method is more simple and straightforward. An L_∞ observer design method is proposed for linear systems with parameter uncertainty. The proposed L_∞ observer design approach not only attenuates disturbance but also gives an upper bound of estimation error norm by bounding peak-to-peak gain of error system. Sufficient conditions of the L_∞ observer design are given as linear matrix inequalities (LMIs). Numeral simulation results illustrate the effectiveness of the proposed approach.

Notation. $\mathbb{R}^{n\times m}$ represents the set of all $n \times m$ real matrices. The superscript $(\cdot)^T$ denotes the transpose of a real matrix. $\mathbf{0}$ and $\mathbf{I}$ denote the zero and identity matrices with an appropriate dimension, respectively. The vector $\mathbf{1}_n$ denotes the n-dimensional column vector comprising of all ones. He$(\cdot)$ denotes the sum of matrix and its transpose, *i.e.*, He $(A) = A + A^T$. Asterisk $*$ represents a term that is induced by symmetry in symmetric block matrix. For a symmetric matrix P, $P > (<)0$ means that P is positive (negative) definite. For a signal $x(t) \in \mathbb{R}^n$, its L_∞ norm is defined as $\|x\|_\infty = \sup_{t\geqslant 0} \|x(t)\|$, where $\|x(t)\|$ denotes the Euclidean norm of $x(t)$, *i.e.* $\|x(t)\| = \sqrt{x^T(t)x(t)}$.

II. PROBLEM FORMULATION

Consider the following linear systems including parameter uncertainty:

$$\begin{aligned} \dot{x}(t) &= (A + \Delta A(t))x(t) + Bu(t) + B_d d(t) \\ y(t) &= Cx(t) + D_d d(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$ and $y(t) \in \mathbb{R}^p$ are the state, the input and the measurement output, respectively, $d(t) \in \mathbb{R}^q$ is the unknown input including process disturbance and measurement noises, $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times r}, C \in \mathbb{R}^{p\times n}, B_d \in \mathbb{R}^{n\times q}$, and $D_d \in \mathbb{R}^{p\times q}$ are known constant matrices with appropriate dimensions, and $\Delta A(t) \in \mathbb{R}^{n\times n}$ is used to represent parameter uncertainty.

The following assumptions are considered in this paper.

Assumption 1. The pair (C, A) is observable.

Assumption 2. The parameter uncertainty matrix is equal to $\Delta A(t) = M\Sigma(t)N$, where M and N are known matrices that represent the structure of the uncertainties and $\Sigma(t)$ is an unknown time-varying matrix satisfying $\Sigma^T(t)\Sigma(t) \leqslant \mathbf{I}$ for all t.

Assumption 3. The unknown input d is bounded as $\|d(t)\| \leqslant \|d\|_\infty$, where $\|d\|_\infty$ is a known constant.

For system (1), we propose the following observer

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t) - C\hat{x}(t)) \tag{2}$$

where $L \in \mathbb{R}^{n\times p}$ is observer gain matrix to be designed.

To analyze and synthesize observer (2), we define the estimation error as

$$e(t) := x(t) - \hat{x}(t). \tag{3}$$

Subtracting (2) from (1) it yields to the following error equation

$$\begin{aligned} \dot{e}(t) &= (A - LC)e(t) + \Delta A(t)x(t) + (B_d - LD_d)d(t) \\ &= (A_c + \Delta A(t))e(t) + \Delta A(t)\hat{x}(t) + D_c d(t) \end{aligned} \tag{4}$$

where $A_c = A - LC$, $D_c = B_d - LD_d$.

By substituting $\Delta A(t) = M\Sigma(t)N$ into (4), we obtain

$$\dot{e}(t) = (A_c + \Delta A(t))e(t) + M\Sigma(t)N\hat{x}(t) + D_c d(t) \tag{5}$$

Note that $\Sigma(t)N\hat{x}(t)$ in (5) can be treated as disturbance. By using the L_∞ gain minimization method in [26] and [27], the effect of disturbances can be attenuated.

Based on (5), we aim to design observer (2) such that

(i) For each parameter uncertainty Σ such that $\Sigma(t)^T\Sigma(t) \leqslant \mathbf{I}$, the error system (5) is uniformly bounded-input bounded-output stable.

(ii) Let $e(t) = x(t) - \hat{x}(t)$ be the error corresponding to initial condition $x(0) = x_0$ and $\hat{x}(0) = \hat{x}_0$. Give $\gamma > 0$, then the estimation error $e(t)$ in (5) satisfies

$$\|e(t)\| \leqslant \gamma\sqrt{V(0)\mathrm{e}^{-\alpha t} + \|d\|_\infty^2 + v(t)} \tag{6}$$

where $V(0) = e(0)^T Pe(0)$, $P > 0$ is a positive definite matrix to be specified, $\alpha > 0$ is a given positive scalar, and $v(t) := \alpha\int_0^t \mathrm{e}^{-\alpha(t-\tau)}\|N\hat{x}(\tau)\|^2 \mathrm{d}\tau$.

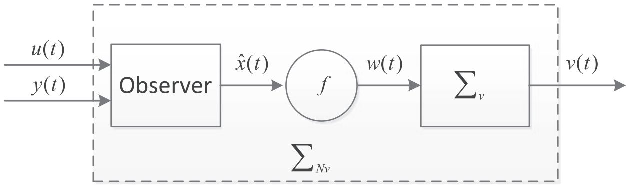

Fig. 1. Nonlinear system for calculating $v(t)$.

Remark 1. It is noted that $v(t)$ is the solution of following equation

$$\dot{v}(t) = -\alpha v(t) + \alpha \|N\hat{x}(t)\|^2 \tag{7}$$

with zero initial condition, *i.e.* $v(0) = 0$. Calculation of $v(t)$ is shown in Fig. 1, where Σ_v represents equation (7) and $w(t) = f(\hat{x}(t)) = \hat{x}^T N^T N\hat{x}$. The calculation can be viewed as a nonlinear system with input $u(t), y(t)$ and output $v(t)$.

Remark 2. If there is no uncertainty, *i.e.* $\Delta A(t) \equiv 0$, we can choose $N = \mathbf{0}$ and $v(t)$ becomes zero. The problem becomes a conventional L_∞ disturbance attenuation problem. Condition (i) guarantees error bound existing and condition (ii) gives a bound of the error norm.

To be more precise, the following definitions about time-varying system are introduced.

Definition 1. [28]. The linear time-varying system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t) \\ y(t) &= C(t)x(t) \end{aligned} \tag{8}$$

is called uniformly bounded-input bounded-output stable if there exists a finite constant η such that for all t_0 and input signal $u(t)$ the corresponding zero-state response satisfies

$$\sup_{t\geqslant t_0} \|y(t)\| \leqslant \eta \sup_{t\geqslant t_0} \|u(t)\| \tag{9}$$

Definition 2. [28]. The linear time-varying system (8) with initial states $x(t_0) = x_0$ is called uniformly exponentially stable if there exist finite positive constants γ, λ such that for any t_0 and x_0 the corresponding solution satisfies

$$\|x(t)\| \leqslant \gamma \mathrm{e}^{-\lambda(t-t_0)} \|x_0\|, \quad t \geqslant t_0 \tag{10}$$

We recall the following lemma about uniformly bounded-input bounded-output stability.

Lemma 1. [28]. Suppose the linear time-varying system (8) is uniformly exponentially stable, and there exist finite constants β and μ such that for all t

$$\|B(t)\| \leqslant \beta, \quad \|C(t)\| \leqslant \mu. \tag{11}$$

Then the state equation also is uniformly bounded-input bounded-output stable.

Before giving the main result, we recall the following lemma.

Lemma 2. [29]. Assume $\Sigma^T(t)\Sigma(t) \leqslant \mathbf{I}$ for all t. For all positive scalar $\epsilon > 0$ and positive definite matrix $P > 0$, we have $P(M\Sigma(t)N) + (M\Sigma(t)N)^T P \leqslant \epsilon^{-1} PMM^T P + \epsilon N^T N$.

III. MAIN RESULTS

Now we are ready to state the main result.

Theorem 1. Given scalars $\alpha > 0, \gamma > 0$, the estimation error system (5) is uniformly bounded-input bounded-output stable for all $\Delta A(t)$ and the error $e(t)$ satisfies (ii) if there exist scalar, $\epsilon > 0$, a symmetric positive definite matrix $P \in \mathbb{R}^{n\times n}, P > 0$ and a matrix $W \in \mathbb{R}^{n\times p}$ such that

$$\begin{bmatrix} \Phi & PM & PB_d - WD_d & PM \\ * & -\alpha\mathbf{I} & \mathbf{0} & \mathbf{0} \\ * & * & -\alpha\mathbf{I} & \mathbf{0} \\ * & * & * & -\epsilon\mathbf{I} \end{bmatrix} < 0 \tag{12}$$

$$\mathbf{I} - \gamma^2 P < 0 \tag{13}$$

where $\Phi = \mathrm{He}\,(PA - WC) + \alpha P + \epsilon N^T N$. Furthermore, L can be determined as $L = P^{-1}W$.

Proof. First, we choose a candidate Lyapunov function as

$$V(t) = e^T(t)Pe(t), \quad P > 0. \tag{14}$$

By taking the time derivative of V, we obtain

$$\begin{aligned} \dot{V}(t) &= e^T(t)P\dot{e}(t) + \dot{e}^T(t)Pe(t) \\ &= e^T(t)(P(A_c + \Delta A(t)) + (A_c + \Delta A(t))^T P)e(t) \\ &\quad + 2e^T(t)P(M\Sigma(t)N\hat{x}(t) + D_c d(t)) \\ &= e^T(t)(PA_c + A_c^T P)e(t) + 2e^T(t)P(\Delta A(t)e(t) \\ &\quad + M\Sigma(t)N\hat{x}(t) + D_c d(t)) \\ &= e^T(t)(PA_c + A_c^T P)e(t) + 2e^T(t)PM\Sigma(t)NAe(t) \\ &\quad + 2e^T(t)PM\Sigma(t)N\hat{x}(t) + 2e^T PD_c d(t) \end{aligned} \tag{15}$$

By applying Lemma 2, we have

$$\begin{aligned}\dot V(t) \leqslant & 2e^T(t)PA_ce(t) + \epsilon^{-1}e^T(t)PMM^TPe(t)\\ &+ \epsilon e^T(t)N^TNe(t) + 2e^T(t)PD_cd(t)\\ &+ 2e^T(t)PM\Sigma(t)N\hat{x}(t)\end{aligned} \quad (16)$$

On the other hand, by using the Schur complement lemma [30], inequality (12) can be rewritten as

$$\begin{bmatrix} \Phi + \epsilon^{-1}PMM^TP & PM & PB_d - WD_d \\ * & -\alpha\mathbf{I} & \mathbf{0} \\ * & * & -\alpha\mathbf{I}\end{bmatrix} < 0 \quad (17)$$

which follows that

$$\begin{aligned}&e^T(t)(\Phi + \epsilon^{-1}PMM^TP)e(t) - \alpha d^T(t)d(t)\\ &+ 2e^T(t)PM\Sigma(t)N\hat{x}(t) + 2e^T(t)PD_cd(t)\\ &- \alpha(\Sigma(t)N\hat{x}(t))^T\Sigma(t)N\hat{x}(t) \leqslant 0\end{aligned} \quad (18)$$

Considering (16) and substituting $W = PL$ into (18) yield

$$\begin{aligned}\dot V(t) &\leqslant -\alpha V(t) + \alpha(\Sigma(t)N\hat{x}(t))^T\Sigma(t)N\hat{x}(t) + \alpha d^T(t)d(t)\\ &\leqslant -\alpha V(t) + \alpha(\Sigma(t)N\hat{x}(t))^T\Sigma(t)N\hat{x}(t) + \alpha\|d\|_\infty^2\end{aligned} \quad (19)$$

Inequality (19) implies

$$\begin{aligned}V(t) \leqslant\ & V(0)\mathrm{e}^{-\alpha t} + \alpha\|d\|_\infty^2\int_0^t \mathrm{e}^{-\alpha(t-\tau)}\mathrm{d}\tau\\ &+ \alpha\int_0^t \mathrm{e}^{-\alpha(t-\tau)}(\Sigma(\tau)N\hat{x}(\tau))^T(\Sigma(\tau)N\hat{x}(\tau))\mathrm{d}\tau\\ \leqslant\ & V(0)\mathrm{e}^{-\alpha t} + (1 - \mathrm{e}^{-\alpha t})\|d\|_\infty^2\\ &+ \alpha\int_0^t \mathrm{e}^{-\alpha(t-\tau)}\|N\hat{x}(\tau)\|^2\mathrm{d}\tau\\ \leqslant\ & V(0)\mathrm{e}^{-\alpha t} + \|d\|_\infty^2 + \alpha\int_0^t \mathrm{e}^{-\alpha(t-\tau)}\|N\hat{x}(\tau)\|^2\mathrm{d}\tau\end{aligned} \quad (20)$$

where $V(0) = e^T(0)Pe(0)$.

Then, we have $V(t) \leqslant V(0)\mathrm{e}^{-\alpha t}$ for all $\Delta A(t)$ when $\hat{x}(t)$ and $d(t)$ are zero, which implies

$$\begin{aligned}\|e(t)\|^2 &\leqslant \frac{1}{\lambda_{\min}}e^T(t)Pe(t)\\ &\leqslant \frac{1}{\lambda_{\min}}e^T(0)Pe(0)\mathrm{e}^{-\alpha t}\\ &\leqslant \frac{\lambda_{\max}}{\lambda_{\min}}\mathrm{e}^{-\alpha t}\|e(0)\|^2\end{aligned} \quad (21)$$

where $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum eigenvalue and the maximum eigenvalue of positive definite matrix P, respectively. Hence, error system (5) is uniformly exponentially stable. Further, from Lemma 1, error system (5) is uniformly bounded-input bounded-output stable.

With the inequality in (13) and the definition of $v(t) := \alpha\int_0^t \mathrm{e}^{-\alpha(t-\tau)}\|N\hat{x}(\tau)\|^2\mathrm{d}\tau$, we have

$$\begin{aligned}\|e(t)\|^2 &\leqslant \gamma^2e^T(t)Pe(t)\\ &\leqslant \gamma^2(V(0)\mathrm{e}^{-\alpha t} + \|d\|_\infty^2 + v(t))\end{aligned} \quad (22)$$

That is, L_∞ performance index (6) is satisfied. Therefore, conditions (i) and (ii) are both satisfied.

Remark 3. Error system (5) is uniformly bounded-input bounded-output stable. Bound of $\|e(t)\|$ exists with bounded disturbances. We propose an approach to calculate the bound of $\|e(t)\|$ in Theorem 1. From peak-to-peak performance index (6), we can minimize γ to make threshold small. In fact, α is the smallest convergence rate of error system (5) [31].

Remark 4. Note that the proposed method can also be used for interval estimation. For the ith component of $x(t)$, its interval is $\hat{x}_i(t) - \|e_i(t)\| \leqslant x_i(t) \leqslant \hat{x}_i(t) + \|e_i(t)\|$, where $e_i(t)$ satisfies $\|e_i(t)\| \leqslant \|e(t)\|$. It follows that $x_i(t)$ satisfies $\hat{x}_i(t) - \|e(t)\| \leqslant x_i(t) \leqslant \hat{x}_i(t) + \|e(t)\|$. Then we have the following interval estimation result.

$$\hat{x}(t) - \mathbf{1}_n\bar{e}(t) \leqslant x(t) \leqslant \hat{x}(t) + \mathbf{1}_n\bar{e}(t). \quad (23)$$

where $\bar{e}(t) = \gamma\sqrt{V(0)\mathrm{e}^{-\alpha t} + \|d\|_\infty^2 + v(t)}$.

IV. SIMULATIONS

In this section, a numerical example borrowed from [32] is simulated to illustrate L_∞ observer design.

Considering an uncertain system represented as (1), parameter matrices of the system are given as

$$A = \begin{bmatrix} 1 & 0 & 1\\ 0 & -3 & 1\\ 1 & -5 & -16\end{bmatrix}, B = \begin{bmatrix} 1 & 0\\ 0 & 1\\ 0 & 0\end{bmatrix}, B_d = \begin{bmatrix} 1\\ 0\\ 1\end{bmatrix},$$
$$C = \begin{bmatrix} 0 & 1 & 1\\ 1 & 0 & 0\end{bmatrix}, D_d = \begin{bmatrix} 0\\ 0.1\end{bmatrix}, \Delta A(t) = \begin{bmatrix} 0 & 0 & 0\\ 0 & \delta_1(t) & 0\\ \delta_2(t) & 0 & 0\end{bmatrix}.$$

where $\delta_1(t) = 0.1\sin(2t), \delta_2(t) = -0.1\sin(4t)$.

Parameter uncertainty $\Delta A(t)$ can be constructed as

$$M = \begin{bmatrix} 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}, \Sigma(t) = \begin{bmatrix} \sin(2t) & 0\\ 0 & -\sin(4t)\end{bmatrix}, N = \begin{bmatrix} 0 & 0.1 & 0\\ 0.1 & 0 & 0\end{bmatrix}.$$

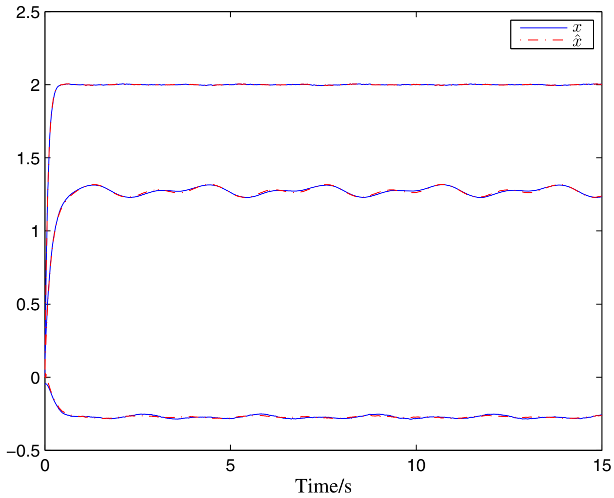

Fig. 2. States x and their estimation $\hat{x}$. [Color figure can be viewed at wileyonlinelibrary.com]

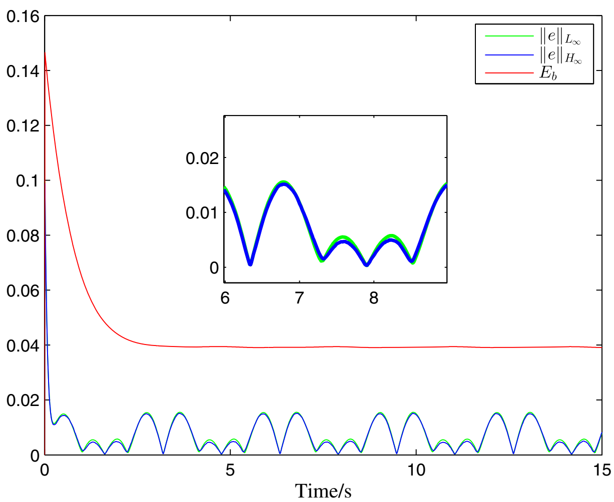

Fig. 3. The norm of error signal e and its interval. [Color figure can be viewed at wileyonlinelibrary.com]

To obtain L_∞ observer, we solve inequality (12) and (13). By setting $\alpha = 2$, we get $\gamma = 0.1525, \epsilon = 12775$. The observer matrices are found out as follows

$$P = \begin{bmatrix} 6121.2 & 160.8 & -169.4 \\ 160.8 & 172.1 & 120.2 \\ -169.4 & 120.2 & 172.3 \end{bmatrix}, L = \begin{bmatrix} 0.4339 & 9.9774 \\ 81.4407 & 0.4830 \\ 82.3140 & 10.6087 \end{bmatrix}.$$

With these parameters, L_∞ observer is designed. In our simulation, the initial conditions are set as $x(0) = [0 \quad 0.1 \quad -0.05]^T$ and $\hat{x}(0) = [0 \quad 0 \quad 0]^T$. The disturbance $d(t)$ is chosen as a random vector with $\|d\|_\infty = 0.1$. The system states and their estimation are presented in

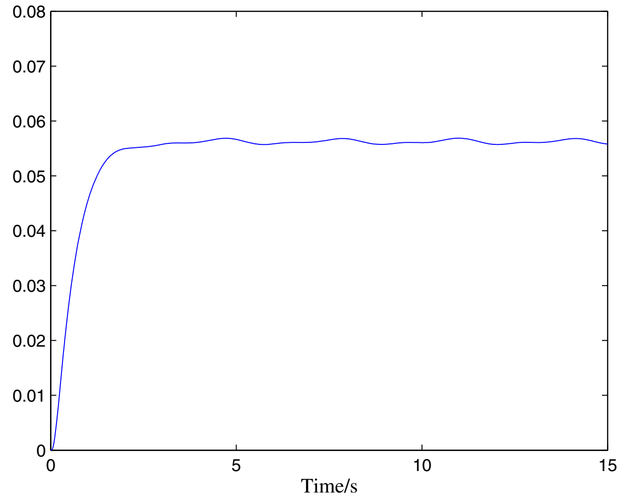

Fig. 4. The trajectory of the variable $v(t)$. [Color figure can be viewed at wileyonlinelibrary.com]

Fig. 2. The result shows that L_∞ observer designed in this paper can estimate system states. In Fig. 3, we compare performances of L_∞ observer and H_∞ observer in the sense of estimation error norm $\|e(t)\|$. We zoom in the local details of the comparison in Fig. 3. In attenuation of uncertainty and disturbances, two approaches can achieve almost the same performance. Besides, L_∞ method can give a time-varying upper bound of estimation error norm, which is shown in Fig. 3. The trajectory of the variable $v(t)$ is depicted in Fig. 4. A time-varying bound of estimation error norm has a significance of process monitoring and fault diagnosis.

V. CONCLUSION

This technical paper proposes an L_∞ observer design approach for uncertain linear systems. The main advantage of the observer is that it can not only attenuate bounded disturbance but also give upper bound of estimation error norm. Finally, the effectiveness of the proposed L_∞ observer is illustrated by simulation results. A potential application of the proposed L_∞ observer is fault detection. In the future, the proposed method can be used to deal with fault detection for more complicated systems such as linear time-varying systems [33] and Markovian jump linear systems [34].

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