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Iterative Learning Control of Discrete-time System with Variable Initial Conditions Based on 2-D System Theory

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Abstract: Iterative learning control (ILC) is a simple and effective method for the control of systems that perform same task repetitively. For real-time implementation, all ILC schemes have to be designed in the discrete-time domain. The initial state problem is very important in learning control. A closed-loop iterative learning controller is developed based on two-dimension (2-D) discrete system for the application of discrete-time system with variable initial states. The sufficient and necessary conditions guaranteeing the convergence of the proposed iterative learning control scheme is proved based on 2-D system theory. Some simulation results are given to demonstrate the control scheme.
Keywords: iterative learning control; discrete-time system; variable initial state; 2-D system theory

基于 2-D 系统理论的变初始条件离散系统的迭代学习控制

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摘 要：对于具有重复运动特性的系统，迭代学习控制是一种简单有效的控制方法。为便于实时应用，所有迭代学习控制方案的设计必须在离散时间域进行。初始状态问题是学习控制设计中遇到的一个重要问题。针对具有变初始状态的离散时间系统，利用 2-D 线性连续-离散型系统理论设计了闭环迭代学习控制器，并给出了保证控制器收敛的充分必要条件。仿真结果证明了该方案的有效性。
关键词：迭代学习控制；离散时间系统；变初始条件；2-D 系统理论
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Introduction[1]

Iterative Learning Control, or ILC, was originally proposed in the robotics community as intelligent teaching mechanism for robot manipulators[1]. ILC has received considerable attention in recent years, and various schemes have been developed. The classical formulation of the Iterative Learning Control problem is, given a reference trajectory and a system, find (using an iterative procedure) the input to the system such that the output follows the desired trajectory as well as possible over a fixed time interval. The controllers are updated iteratively after each operation using the error measurement in the previous trials (open-loop) and the current trial (closed-loop). Most ILC research has focused on continuous-time dynamic systems. But for real-time implementation, all ILC schemes have to be designed in the discrete-time domain[2]. The technical difficulty of ILC lies in the two-dimensionality (in the mathematical sense) of the overall system[3]. Two-dimensional (2-D) iterative learning models provide a clear description on the dynamics of the control system and the behavior of the learning process. In some recent papers, effective ILC schemes are proposed based on 2-D system theory. Tommy[4] proposed a D-type open-loop updated law based on 2-D continuous-discrete system theory. Ding[5] then extended the algorithm to closed-loop updated law.

The initial state problem is very important in learning control[6]. Recently, there have been many ILC schemes on linear multivariable systems and certain classes of nonlinear system. But most of these theoretical results are restricted to the requirement for convergence that the initial condition at each cycle should be reset to the initial condition corresponding to the desired trajectory. It is not practical to assume that each learning iteration starts at the same point, because it is impossible to repeat the same initial condition in practical engineering application. This study is inherent to improve the tracking performance and is, thus, meaningful in itself[7]. Several researchers proposed some schemes to tackle the ILC problem with variable initial conditions[8].

In this paper, we proposed a closed-loop ILC updated law based on 2-D system theory for a discrete-time system with variable initial conditions. Sufficient and necessary conditions for guaranteeing the convergence of the ILC scheme are given. Some simulation results are included to verify the results. The paper is organized as follows. In section 1, we state the closed-loop ILC for 2-D system. In section 2, sufficient conditions for the monotonic convergence of the iterative process are given. Section 3 contains a numerical example to illustrate the new results. And finally, the conclusions appear in section 4.

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1 System description and preliminaries

Consider the linear time invariant multivariable system described by following state space model:

$$\begin{cases} x(t+1) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

Where $x(t) \in R^n$ is state vector, $u(t) \in R^m$ is input vector, $y(t) \in R^p$ is output vector, and $A \in R^{n\times n}$, $B \in R^{n\times m}$,$C \in R^{p\times n}$ are real matrices. Without loss of generality, it is assumed that matrixes B and C are of full rank. The system performs a given task repeatedly on the finite interval $[0, N]$ in the k-th repetitive operation of the system. Thus, the above system can be presented in the following 2-D form:

$$\begin{cases} x(t+1,k) = Ax(t,k) + Bu(t,k) \\ y(t,k) = Cx(t,k) \end{cases} \tag{2}$$

Where subscript k denotes the iteration index. We hope the system output *y(t,k)* to track a given trajectory $y_r(t)$ for all $t \in [0, N]$. To do this, we propose a closed-loop ILC law based on current cycle error,

$$u(t,k+1) = u(t,k) + \Delta u(t,k+1) \tag{3}$$

The equation (3) describes the 2-D model of the ILC updated law. The boundary conditions for the 2-D system (2-3) are presented as follows:

$$\begin{cases} x(0,k) = x_k^0, \quad k = 0,1,2\Lambda, \\ u(t,0) = 0, \quad t = 0,1,2\Lambda, \end{cases} \tag{4}$$

Where x_k^0 denotes the initial system state. Every control cycle starts at the different initial state.

Suppose the initial state input at k-th iteration is boundary, i.e.

$$\left\| x_k^0 - x_0 \right\| < \gamma \tag{5}$$

Where x_0 denotes the desired initial state and γ denotes a positive vector.

2 Analysis of Convergence

The closed-loop ILC updated law is given by:

$$\Delta u(t,k+1) = Ke(t+1,k+1) \tag{6}$$

Where there is a prediction for the expression, i.e. $e(t+1,k+1)$ [9].

Let us define the following error equations:

$$\begin{aligned} &\eta(t,k) = x(t-1,k+1) - x(t-1,k) \\ &e(t,k) = y_r(t) - y(t,k) \end{aligned} \tag{7}$$

Where $\eta(t,k)$ denotes the state vector error and $e(t,k)$ denotes the output error.

Therefore, we can write

$$\begin{aligned} \eta(t+1,k) &= x(t,k+1) - x(t,k) \\ &= A[x(t-1,k+1) - x(t-1,k)] + B[u(t-1,k+1) - u(t-1,k)] \\ &= A\eta(t,k) - B\Delta u(t-1,k+1) \\ &= A\eta(t,k) + BKe(t,k+1) \end{aligned} \tag{8}$$

$$\begin{aligned} &e(t,k+1) - e(t,k) \\ &= y(t,k) - y(t,k+1) = Cx(t,k) - Cx(t,k+1) \\ &= C[Ax(t-1,k) + Bu(t-1,k+1) - Ax(t-1,k+1) \\ &\qquad - Bu(t-1,k+1)] \\ &= -CA\eta(t,k) - CB\Delta u(t-1,k+1) \\ &= -CA\eta(t,k) - CBKe(t,k+1) \end{aligned} \tag{9}$$

Using (8-9), we can obtain the closed-loop Iterative Learning Control system that is presented in the form of 2-D Roesser's type model.

$$\begin{pmatrix} \eta(t+1,k) \\ e(t,k+1) \end{pmatrix} = \begin{pmatrix} A - BK(I+CBK)^{-1}CA & BK(I+CBK)^{-1} \\ -(I+CBK)^{-1}CA & (I+CBK)^{-1} \end{pmatrix} \begin{pmatrix} \eta(t,k) \\ e(t,k) \end{pmatrix} \tag{10}$$

Where I denotes the identity matrix with an appropriate dimension. The boundary conditions are given by

$$\begin{cases} \eta(1,k) = x(0,k+1) - x(0,k), \quad for \quad k = 0,1,2\Lambda, \\ e(t,0) = y_r(t) - CA^t x_0^0, \quad for \quad t = 1,2\Lambda. \end{cases} \tag{11}$$

Clearly, the initial error $e(t,0)$ is bounded and $|\eta(1,k)| = |x(0,k+1) - x(0,k)| < 2\gamma$ according to inequality (5).

Lemma[8] : Suppose that a discrete 2-D system given by as follows:

$$\begin{pmatrix} x^h(t+1,k) \\ x^v(t,k+1) \end{pmatrix} = \begin{pmatrix} A_1 & A_2 \\ A_3 & A_4 \end{pmatrix} \begin{pmatrix} x^h(t,k) \\ x^v(t,k) \end{pmatrix} \tag{12}$$

Where $x^h(t,k) \in R^m$ denotes horizontal state vector, $x^v(t,k) \in R^n$ denotes vertical state vector, and $A_1 \in R^{m\times n}, A_2 \in R^{m\times n}, A_3 \in R^{n\times m}, A_4 \in R^{n\times m}$ denote system matrixes.

The boundary conditions of the system is in the following forms,

$$\left\| x^h(1,k) \right\| < \alpha, \left\| x^v(t,0) \right\| < \beta \qquad \alpha > 0, \beta > 0. \tag{13}$$

If matrix A_1, matrix A_4 and matrix | $A_3(I-A_1)^{-1}A_2+A_4$| are asymptotic stable, i.e., their spectrum radius is represented in the following forms:

$$\rho(A_1) < 1, \rho(A_4) < 1 \ and \ \rho[\left| A_3(I - A_1)^{-1} A_2 + A_4 \right|] < 1 \tag{14}$$

then for all $t \geq 0$, when $k \to \infty$, $\begin{pmatrix} x^h(t,k) \\ x^v(t,k) \end{pmatrix} \to 0$.

According to above lemma, we can obtain the theorem as follows:

Theorem: Consider the system (2) and the iterative updated law (6). The tracking error of the system asymptotically convergence to zero if

$$\rho(A - BK(I+CBK)^{-1}CA) < 1 \tag{15}$$

$$\rho[(I+CBK)^{-1}] < 1 \tag{16}$$

$$\begin{aligned} \rho[\,|-(I+CBK)^{-1}CA(I - A + BK(I+CBK)^{-1}CA)^{-1}BK(I+CBK)^{-1} \\ +(I+CBK)^{-1}\,|\,] < 1 \end{aligned} \tag{17}$$

Proof : Straightforward from (10) , (11) and lemma.

The above convergent conditions are sufficient and necessary. The theorem shows the convergent conditions of the updated law (6) for the learning matrix K when the initial iterative condition is varying. If the initial condition is fixed, only inequality (16) is needed[5]. In equation (15), (16) and (17), $CB \in R^{p\times m}$. If the inverse, $(I + CBK)^{-1}$ and $(I - A + BK(I+CBK)^{-1}CA)^{-1}$, exists, the condition, p=m, should be satisfied.

3 Simulation Results

In this section, a simple simulation example is presented. Let us introduce a robot joint model expressed by the Laplace transfer function:

$$G_c(s) = \frac{1}{J \cdot s^2} \tag{18}$$

Where J is the joint moment of inertia.

From G_c, we obtain $G(z)$, which is the z-transform of $G_c(s)$ including the Zero Order Hold(ZOH) :

$$G(z) = \frac{T_s^2}{2J} \frac{z+1}{(z-1)^2} \tag{19}$$

Where T_s is the sampling time. $T_s = 0.001s$, $J = 0.0094 \ N \cdot s^2$.

Transfer function $G(z)$ is transferred into state equation as follows:

$$\begin{cases} x(t+1) = \begin{pmatrix} 2 & -1 \\ 1 & 0 \end{pmatrix} x(t) + \begin{pmatrix} 1 \\ 0 \end{pmatrix} u(t) \\ y(t) = \begin{pmatrix} 1 & 1 \end{pmatrix} x(t) \end{cases} \tag{20}$$

The variable initial conditions are given as $x_k(0) = x_0^k = (\mu_1 \quad \mu_2)^T$, where μ_1, μ_2 are two random numbers assumed to be uniformly distributed in [0 1]. For simplicity, the zero initial inputs are used in this simulation, i.e. $u(t,0) = 0$, $t = 0, 1, 2, \Lambda, N$.

The reference trajectory is given as follows:

$$y_r = 10\sin(3.14/20t) \ , \quad t = 0, 1, 2, \Lambda, N.$$

It is important to choose an appropriate K. In this simulation a direct method is used to choose K. The essential idea of the method is as follows:

(1) presetting a initial experiment value K^0;

(2) searching an appropriate direction P^d, d=0,1,2,⋯,P^d is $(d+1)$th searching direction.

(3) Choosing appropriate step size λ^d.

(4) Getting a new value K^{d+1}, $K^{d+1} = K^d + \lambda^d P^d$.

(5) Judging whether K^{d+1} is met with the convergence conditions. If the conditions are satisfied, searching is stopped. If not, return (2) and continue.

We preset an initial value with K=0.1 and $\lambda = 0.1$. In the searching process we find Criterion (16) is decreased monotonically, criterion (17) approximates 0 and meets the condition, and criterion (15) is not variable monotonically. The optimal K is difficult to find. We search for K from 0.1 to 100. When K=8, the smallest value of criterion (15) is obtained, the value of criterion (16) is not the smallest. In this simulation we use the closed-loop updated law (6) with K=8. Thus, according to above theorem, we can obtain the following forms:

$$\rho(A - BK(I + CBK)^{-1}CA) = 0.3333$$

$$\rho[(I + CBK)^{-1}] = 0.1111$$

$$\rho[|-(I+CBK)^{-1}CA(I-A+BK(I+CBK)^{-1}CA)^{-1}BK(I+CBK)^{-1} + (I+CBK)^{-1}|] = 0.0001$$

Clearly, the convergence conditions are met. This conclusion is verified by the simulation results shown in Fig.1 and Fig.2. After only several iterations, the output can track the reference trajectory perfectly. Because the closed-loop updated law is used, the convergence rate is faster than the learning rule described in [8], where an open-loop updated law is presented.

4 Conclusions

We have presented a closed-loop iterative learning controller based on 2-D discrete system model. The convergence conditions are developed based on 2-D system theory. The inclusion of closed-loop algorithm in the ILC scheme makes a faster convergence rate. We show that the convergence conditions are given as the forms of spectrum radius, which has less restriction than the form of norm. Finally, simulation result is presented to demonstrate the feasibility of the ILC scheme.

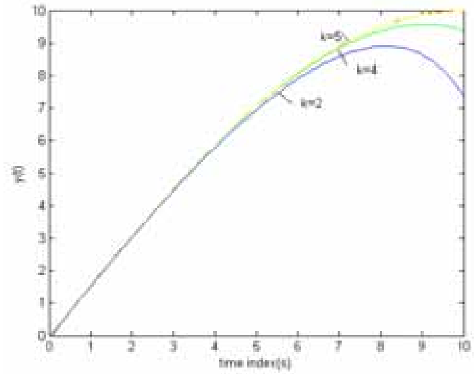

Fig.1 ILC output with variable initial states

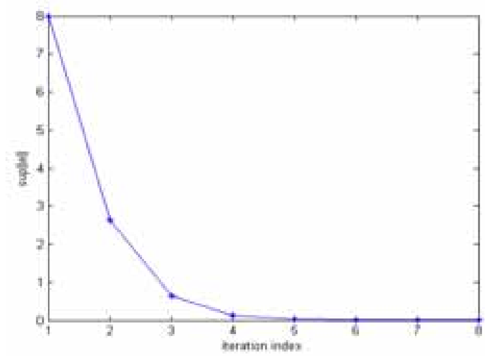

Fig.2 Tracking error versus the number of iteration

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