Design and Kinematics of a Parallel Manipulator for Manufacturing

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Abstract

This paper presents a new model of parallel manipulator for manufacturing, whose mechanism comes from the general Stewart platform which has the advantages of high rigidity and high load capacity. A scheme of the link mechanism is also proposed to make the link structure very simple. The mechanism parameters are optimized by workspace analysis of the parallel mechanism. With the view of kinematics equivalence between parallel mechanism and serial mechanism, by way of hypothesis serial mechanism and branch hypothesis serial mechanism, an efficient kinematics algorithm has been developed. The problem, derived motion produced by the link structure, has also been solved conveniently.

Keywords: parallel manipulator, mechanism design, kinematics analysis

1. Introduction

Industrial robot manipulators have been widely used in the field of painting, welding, assembling and dangerous environments, but few of them can be used for manufacturing, such as milling and grinding. The main reason is the view of bionics and the goal of flexibility, robots take the form of man's arms and their mechanisms are serial linkages which are inherently not very rigid and have poor dynamic performance at high dynamic loading operating conditions[2]. Fortunately, parallel platforms are attracting more and more attention of robot researchers, because of the advantages as compared to the traditional serial manipulators: more rigidity and accuracy due to the lack of cantilever-like structure, high force and torque capacity for the number of actuators as the actuators are arranged in parallel rather than in series, and relatively parallel manipulators have potential applications where the dynamic loading is severe and high speed and precision motion are of primary concerns[9].

As we know, the traditional architectural model of the machine tool is facing more and more challenges. The concept of flexibility is considered to be the most important factor with respect to the classical system. Even O. Garro states that "machine tool concept is becoming vague and seems to be disappearing with the emergence of integration within the workshop or flexible concept"[3].

As result of combination of the above view of robot researchers and machine tool researchers, a prototype of a parallel manipulator for manufacturing, looking like a flexible cell, has been developed in Robotics laboratory shenyang Institute of Automation Chinese Academy of Sciences. In fact, some machine tool researchers have mentioned the advantages of the parallel mechanism regarded as the mechanism of high speed machine tool, and various types of such parallel manipulators have been developed in recent years[4]. Coma Company in Italy has made a three degree-of-freedom(DOF) parallel platform for manufacturing. Ingersoll, Gidings & Lewis in U. S. A., Lapik company in Russia, Toyota in Japan and so on have all developed their own parallel structural CNC machine tools[10].

This paper highlights the mechanism design and kinematics of our prototype which is based on a six DOF general Stewart platform. Section two describes the parallel mechanism, section three design of the prototype, section four the kinematics of parallel mechanism, section five the kinematics of the prototype. At last, section six is the conclusion.

2. Parallel Mechanism

For hundred years, a camera tripod, a typical parallel mechanism, has been used as photograph auxiliary apparatus. In this paper, another typical parallel mechanism, a six DOF Stewart platform, is under discussion. Shown in Fig. 1, the mechanism of the model parallel manipulator, basically consists of a platform called the mobile which houses the cutting tool, six extensible links and a fixed platform called the base. The mobile is connected to the links by means of ball joints. The other ends of the links are connected to the base through
universal joints. With the link lengths changing, the mobile can be manipulated and the tool changes its position and orientation with respect to the base.

Compared to that of a classical machine tool, the kinematics of the parallel manipulator is more complex. In general, the kinematics includes two aspects: forward kinematics and inverse kinematics. Of particular interest here is that, whereas in serial mechanism, the forward kinematics problem is easy and the inverse kinematics problem is challenging, the converse is true of parallel mechanism. And an efficient algorithm of kinematics is discussed in section four.

3. Manipulator Design

3.1 Mechanism Design Criteria

The purpose of this design is to construct a parallel manipulator for milling. The position workspace is $380 \times 200 \text{mm}$. Because of mechanical limits on the passive joints, the angle between the normal of the mobile and the Z-axis forward direction is $0 - 20^\circ$. In order to simplify the structure, if every two links is a pair, then the three pairs are symmetric by $120^\circ$.

3.2 Mechanism Workspace Analysis

In fact, the workspace of the parallel mechanism has been investigated and some effective algorithms have been developed[6][7]. But these algorithms are relatively complex. In this paper, a simple algorithm of workspace based on the inverse kinematics of parallel mechanism is proposed.

Assuming that the orientation matrix of the mobile $[A]=[I]$, only the position of mobile changes with respect to the fixed base frame OXYZ, according to the solution of the inverse kinematics, the lengths of the links $[L]$ are found, because $[L]$ are limited by the link extend range, so it is easy to get the corresponding position workspace of the model machine tool.

In the case of a varying orientation $[A]$, at first we define the angle $\theta$, which is between the normal vector $\vec{z}$ of the mobile and the Z-axis forward direction $\vec{Z}$. Shown in Fig.2, to a constant position point $P(x,y,z)$, when the normal vector $\vec{z}$ rotates around vector $\vec{Z}$, whose origin point is $P$ and parallel to vector $\vec{Z}$, in fact what $\vec{z}$ travels forms a 3D cone, with the same link length constraints, the point $P$ can be deduced to be within the orientation workspace or not. So the region of the three-dimensional Cartesian space that can be attained by the parallel manipulator with a given $\theta$ forms the orientation workspace at angle $0$.

According to above workspace analysis, the compute simulation reveals that: the available workspace is seriously limited by the extend range of the links. As an example, table 1 lists two sets of parameters and Fig.3 is the comparison of the position workspace in Z-X plane. And Fig.4 is the comparison of the orientation workspace of the second set parameters at $\theta=0^\circ$, $10^\circ$, $20^\circ$.

<table>
<thead>
<tr>
<th>Table 1 Parallel mechanism parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>radium of mobile(Rt)</td>
</tr>
<tr>
<td>-----------------------</td>
</tr>
<tr>
<td>200mm</td>
</tr>
<tr>
<td>200mm</td>
</tr>
</tbody>
</table>

3.3 Mechanical Design

To design a parallel manipulator, the key component parts are links. In our experiment link structure was designed as shown in Fig.5. The three gears represent a universal joint, on the other hand have the function of transmitting the motion from motor to the ball screw. The ball screw meets the requirement of accuracy and is regarded as a cylinder joint. Connecting the mobile and the link is a universal joint. In fact, the cylinder joint and the latter universal joint can be regarded as the ball joint and the prismatic joint as the Fig.1 shows. So each link has 6 DOF, and the mobile has...
4. Kinematics of A General Stewart Platform

In the past decades, the kinematics of a general Stewart platform has been extensively studied. The approach of the kinematics can be divided into two aspects. One suggested method is using extra sensors to make it easy and another method is trying to find all the roots of the nonlinear equation[5][7][9][11]. But to the authors' knowledge, M.Y. Zhao proposed a relatively efficient way to solve the problem[12]. The main idea is that: with the view of kinematics equivalence between parallel mechanism and serial mechanism, a hypothesis serial mechanism which is kinematically equivalent to parallel mechanism is used to build the iteration algorithm which is based on the hypothesis serial mechanism's coordinates \([Q]\). It can be shown in Fig. 6.

![Fig. 6 Hypothesis serial mechanism](image)

In order to recount the algorithm clearly, we define two kinds of coordinates. One is the input coordinates \([L]=\{L_1 \ L_2 \ \ldots \ \ L_6\}^T\): the lengths of the links. Another is the generalized coordinates \([Q]=\{Q_1 \ Q_2 \ \ldots \ Q_6\}^T\): the coordinates of hypothesis serial mechanism. Referring to Fig. 6, the links parameters are given in table 2.

<table>
<thead>
<tr>
<th>link</th>
<th>([\theta])</th>
<th>([\alpha])</th>
<th>([a])</th>
<th>([d])</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>90°</td>
<td>90°</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>90°</td>
<td>90°</td>
<td>0</td>
<td>Q_1</td>
</tr>
<tr>
<td>2</td>
<td>90°</td>
<td>90°</td>
<td>0</td>
<td>Q_2</td>
</tr>
<tr>
<td>3</td>
<td>90°</td>
<td>0°</td>
<td>0</td>
<td>Q_3</td>
</tr>
<tr>
<td>4</td>
<td>90° + Q_4</td>
<td>90°</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>Q_5</td>
<td>-90°</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>-90° + Q_6</td>
<td>0°</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Based on the generalized D-H transformations the following homogeneous transformation matrix derived for the \(i\)th joint is given as[1]:

\[
T_{i,i-1} = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & a_i & 0 \\
\sin \theta_i & \cos \theta_i & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

Transformation matrix for the end-effector with respect to frame OXYZ is:

\[
\begin{align*}
T_e^6 &= T_6^5 T_5^4 T_4^3 T_3^2 T_2^1 T_1^0 \\
&= \begin{bmatrix}
-Sx+CzSz -SyCx -CzSt -SySt -CzSs StCt -SyStCzSs \\
Sx+CzSz SyCx -CzSt -SySt -CzSs StCt -SyStCzSs \\
CzSzSx SyCz -CzSt -SySt -CzSs StCt -SyStCzSs \\
Sx+CzSz SyCx CzSt -SySt -CzSs StCt -SyStCzSs \\
StSs +StSs -StSs CzSs +StSs CzSs +StSs \\
0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]  

where \(C_i=\cos Q_i \ , \ S_i=\sin Q_i\). And the position and orientation matrix of the mobile is:

\[
T_m = \begin{bmatrix}
nx & ox & ax & P_x \\
xv & oy & ay & P_y \\
xw & oz & az & P_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

According to the kinematics equivalence, \([T_e^6]=T_m\), from the equation (2) and (3), the general coordinates \([Q]\) can be found:

\[
\begin{align*}
Q_1 &= P_1 \\
Q_2 &= P_2 \\
Q_3 &= \frac{1}{a_3} a_3 \\
Q_4 &= \frac{1}{a_4} a_4 \\
Q_5 &= \frac{1}{a_5} a_5 \\
Q_6 &= \frac{1}{a_6} a_6
\end{align*}
\]

From equation (4), we find that: \(Q_1, Q_2\) and \(Q_3\) represent the arm coordinates of the hypothesis serial mechanism, \(Q_4, Q_5\) and \(Q_6\) represent the spherical wrist of it.

Suppose we know the position and orientation matrix \([T_m]\), then the inverse kinematics can be expressed by (5), and the equation (5) also describes the relation between \([L]\) and \([Q]\).

\[
F(Q) = \|\ddot{O}_{OZ} - \dot{A}\ddot{R}_{OZ} - \hat{R}_{OZ}\| = \|F(Q)\|
\]

In forward kinematics analysis of the parallel mechanism, let \(X = [P_1 \ P_x \ P_z \ \omega_x \ \omega_y \ \omega_z]^T\) a velocity vector of the center of the mobile and \(Q = [Q_1 \ Q_2 \ \ldots \ Q_6]^T\), then their relation can be derived:

\[
X = J_Q Q
\]

where \(J_Q\) is the Jacobian matrix.

Let \(L = [L_1 \ L_2 \ \ldots \ L_6]^T\) a velocity vector of input coordinates, then:

\[
\dot{L} = J_L X
\]

where

\[
J_L = \begin{bmatrix}
L_{10}^T \ (R_{1x} \times L_{60})^T \\
\vdots \ \vdots \\
L_{60}^T \ (R_{16} \times L_{60})^T
\end{bmatrix}
\]

In equation (9), \(L_{60}\) is an unit vector of \(L_6\). Then:

\[
\dot{Q} = J_Q^{-1} J_L \dot{L} = (J_Q J_L)^{-1} \dot{L} = J_Q^{-1} \dot{L}
\]
From (5) and (9), based on Newton iteration equation, (10) can be deduced:

\[
\begin{align*}
L &= F(Q) \\
\eta &= Q(k - 1) + J^{-1}(k) \Delta L(k) \\
Q(k) &= h + J^{-1}(k) (L(k) - F(\eta)) \\
& \quad (k = 1, 2, \ldots, n)
\end{align*}
\]

Given the initial iteration value \([Q_0] = [0,0,0,0.1,0.1,0.1]\), according to (10), we can find the general coordinates \([Q]\) corresponding to the known input coordinates \([L]\). Then the position and orientation matrix \([T_F]\) can be found by using equation (2), and forward kinematics of the general Stewart platform is found.

5. Kinematics of the Model Parallel Manipulator

Different structural designs of joints and links may have different influence on kinematics of parallel manipulator. With the structure mentioned above, the changes of position and orientation of mobile follow some extra changes of link lengths, which are called the derived rotations. Apparently, the three derived rotations are different influence on kinematics of parallel manipulator. So the six links are also regarded as six derived rotation of \(R_h\) link is

\[\Delta \Phi_i = \Phi_i - \Phi_{i0} \quad (i=1,2,\ldots,6, j=1,2,4)\]  

where \(\Phi_{i0} = 1,2,\ldots,6, j=1,2,4\) is the coordinates of \(i^{th}\) link in the branch serial mechanism when the mobile is in a certain position and orientation \([T_F]\), whose derived rotations are regarded as zero. \(\Phi_i = 1,2,\ldots,6, j=1,2,4\) are the coordinates of \(i^{th}\) link in the branch serial mechanism at \([T_F]\).

The total derived rotation of \(i^{th}\) link is

\[\Delta \Phi_i = \sum \Delta \Phi_j\]  

and

\[\Delta L_i = \Delta \Phi_i \cdot S\]  

\[L_i = L_i' - \Delta L_i\]

where \(i=1,2,\ldots,6, j=1,2,4, S\) is the screw-pitch of the ball screw. \([L_i']\) is the real length of the motor rotation. Equation (13) means that: the motor must turn \(-\Delta \Phi_i\) to compensate the derived rotations to eliminate the effect of them and keep link length unchanged. Equation (14) means the length should be compensated and the theoretical length is \([L_i]\). Now we can solve the kinematics problem of our prototype with theoretical length \([L_i]\) and equations (2) (5) (9) and (10).

According to above analyses, the simulation and experiment results show that this algorithm can avoid multi-solution, spend less time and be easily carried out.

6. Conclusion

From the view of simple structure and large workspace, a parallel manipulator for milling has been designed. Although the link structure leads to the problem of derived rotations, we introduce the hypothesis serial mechanism and hypothesis branch serial mechanism, which make it possible to solve the kinematics problem by iteration algorithm. In addition, we are doing our best to realize the function of milling.

7. Acknowledgments

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8. References