CO-DESIGN OF NETWORKS AND CONTROL SYSTEMS WITH SYNTHESIZED CONTROLLER

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Abstract. This paper investigates a novel control scheme, jointly controlling networks and plant, to improve and increase the overall performance of networked control systems. There is a fundamental tradeoff between power consumption, data transmission rates, and congestion levels in a wireless network. Based on the joint rate and power control model available, we formulate the state-space model of networked control systems. Co-design is pursued by determining control signals that help meet certain performance criteria (such as desired levels of signal-to-interference ratio and stability). Finally, a numerical example is given to illustrate the effectiveness of the proposed method.

Key Words. Wireless networked control systems, rate adaptation, power control, stability, and congestion control.

1. Introduction

A feedback system in which several signals are transmitted over wireless communication links is called the wireless networked control systems (WNCS)[1,2]. By inserting wireless network, some issues associated communication including delay, packet loss, disordering and resource allocation have to be taken into account[3,4]. Obviously, the quality of service (QoS) of network communication influences the performance of control systems, and whose design in turn also interferes the QoS of network. For instance, control performance may be improved by sampling more frequently requiring more communication bandwidth. Sending sensor values more frequently over a bandwidth limited channel may lead to network congestion resulting in increased delays and packet dropouts that negatively affect the control performance[5]. However, the major problem appeared by using all current methods is that these methods either focus on the control system and ignore the effect of networking[6,7], or vice versa[8,9], thereby producing results which are sub-optimal. More effort needs to be put into co-design which acknowledges the interdependence between the control system and the communications network and which aims to optimisation the performance of both.

For simplicity, there are two methodologies to the joint design control systems and communication in existing literature. One is distributed control, that is communication system parameters (such as communication protocols, network bandwidth, transmission rate, and transmission power) and control system parameters (such as control algorithms, controller gains, sampling period) are selected separably, see

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The other is control system parameters are designed or turned by virtue of the quality of service (QoS) of network determined by communication system parameters, or vice versa, which means that there is a tradeoff between design of control systems and communication systems, see [11, 12]. It is obvious that the second approach due to high robustness to the practical application is enjoyed better than the first one. In this paper, we choose the second approach to jointly design control and communication systems for WNCS. Then, whether can we design an unified control law to realize joint optimism of them? [13] made a preliminary attempt by constructing state-space model of network only based on power and rate control, which inspired us.

Power control is an important means for resource allocation due to the nodes with limited power capabilities[14]. More important, nodes need to cater to certain data rates, which requires nodes to adjust their power in view of Shannon’s capacity theorem. Actually, there is a tradeoff exists between power levels, data rates, and congestion in a network, in the sense that the nodes need to be responsive to congestion conditions in the network, and therefore, they should be able to adjust their transmission rates and, hence, their power levels accordingly[13].

As summarized by [13], several power control strategies using different objective measures have appeared in recent years, such as balancing the signal-to-interference ratios in a distributed way[15,16], QoS-based approach, as well as joint control of power and rate[17,18]. To the best of the authors’ knowledge, co-design of network and control systems has not been fully investigated to date. Especially for formulating the unified state-space model for power, rate and application plant and for which counting for stability, no results have been available in the literature so far, which motivates the present study.

This research presented aims at designing a synthetical controller, which has capable to control synchronously the network, by joint adjusting the power and rate to achieve desired signal-to-interference ratio (SNR), and application plant to improve and increase the performance of networked control systems. The key technology lies in the modeling method, more precisely, the method of constructing network and plant state-space model.

2. Problem formulation

2.1. Construction communication systems model. Inspired by [13], we consider a wireless network operating under dynamic network conditions. The space is divided into virtual geographical cells with each cell having one master node. A frequency slot is allocated to each node that wishes to communicate to the master node in a cell. The nodes communicating in the same frequency slot in other cells cause interference with this cell. The interference is measured in terms of the SNR, which is defined as follows. The SNR for node $i$ at time $k$ is defined by

\[
\gamma_i(k) = \frac{G_{ii}(k)p_i(k)}{\sum_{j \in \mathcal{A}} G_{ij}(k)p_j(k) + \sigma_i^2}
\]

where, for each time instant $k$, denotes the channel gain $G_{ij}$ from the $j$th node to the intended master node of the $i$th node and is assumed to have a log-normal channel distribution, $p_i$ is the transmission power from the $i$th node, and $\sigma_i^2$ is the power of the white noise at the receiver of the master node to which node $i$ is connected. Moreover, $\mathcal{A}$ denotes the set of all nodes that are interfering with node $i$ from all cells (see [13]).
Choosing the following flow-rate control algorithm applied in the communication network (see, e.g., [19]):

\[
f_i(k+1) = f_i(k) + \mu [d(k) - c(k)f_i(k)]
\]

where \( \mu \) is a positive step-size, \( c(k) \) is a measure of the amount of congestion in the network at time \( k \), \( d(k) \) and \( f_i(k) \) controls the amount of rate increase per iteration. In our subsequent derivations, we shall assume that is independent of the flow rates at different nodes.

In view of Shannons capacity formula,

\[
f_i(k) = \frac{1}{2} \log_2 [1 + \gamma_i'(k)]
\]

Let \( \bar{x} \) denote the decibel value of a variable \( x \), namely, \( \bar{x} = 10 \log(x) \). According to (2), then the desired SIR level (in decibel scale) should vary according to the rule

\[
\bar{\gamma}_i'(k+1) = [1 - \alpha_i] \bar{\gamma}_i(k) + \alpha_i \bar{\gamma}_i'(k) + n_i(k)
\]

where \( \alpha_i \) is given as step-size parameter that is to vary from one node to another. Let

\[
\beta_i(k) = \frac{G_{ii}(k)}{\sum_{j \in \mathcal{A}} G_{ij}(k)p_j(k) + \sigma_i^2}
\]

Actually, \( \beta_i(k) \) can be the following random-walk model [13]

\[
\bar{\beta}_i(k+1) = \bar{\beta}_i(k) + n_i(k)
\]

By (5) and (6), we have [13]

\[
\bar{\gamma}_i(k+1) = [1 - \alpha_i] \bar{\gamma}_i(k) + \alpha_i \bar{\gamma}_i'(k) + n_i(k)
\]

This paper mainly focused on co-design power and rate sequence such that the actual SNR levels, as shown in (7), tend to desired SNR levels, defined by (4). For simplicity, node index \( i \) is dropped, which can indicated every node may implement this control mechanism. Denote \( \epsilon \) as the error between actual and desired SNR, which means \( \epsilon = \bar{\gamma} - \bar{\gamma}' \) we can obtain

\[
\epsilon(k) = (1 - \alpha - \mu c(k))\epsilon(k) + \mu c(k)\bar{\gamma} + n_i(k) - \mu' d(k)
\]

Then, let

\[
x_k = \begin{bmatrix} \bar{\gamma} \\ \epsilon \end{bmatrix}
\]

Combining (7) and (8), we have

\[
x_{k+1} = \tilde{A}_k x_k + w_k
\]

where

\[
\tilde{A}_k = \begin{bmatrix} 1 - \alpha & \alpha \\ 0 & 1 - \mu c(k) \end{bmatrix}, w_k = \begin{bmatrix} n_i(k) \\ \mu' d(k) \end{bmatrix}
\]
2.2. Co-model of Communication and Control Systems. Considering the following plant given by
\[ \xi_{k+1} = A_p \xi_k + v_k \]
where \( \xi_k = \xi(kT) \in \mathbb{R}^n \) and \( v_k = v(kT) \in \mathbb{R}^M \) are the state vector and the exogenous disturbance signal, respectively; \( A_p \) is some constant matrices of appropriate dimensions. Design state feedback controller to select the power control sequence and rate control sequence such that the actual SNR level \( \bar{\gamma}_i(k) \) will tend to the desired SNR level \( \bar{\gamma}_i'(k) \), and stabilize the system. Let \( z_k = [\xi_k^T x_k^T]^T \), we have
\[ z_{k+1} = \hat{A}_k z_k + B u_k + o_k \]
where \( \hat{A}_k = \begin{bmatrix} A_p & 0 \\ 0 & A_k \end{bmatrix} \), \( o_k = \begin{bmatrix} v(k) \\ w(k) \end{bmatrix} \)
\( B u_k \) denote the individual entries of \( B u_k \) to be designed. Then, the inclusion of the term \( B u_k \) in (10) amounts to adding the control signal \( u_p(k) \) to the power update (5), the control signal \( u_f(k) \) is added to the desired SIR update (4), likewise, the control signal \( u_\xi(k) \) is added to the application system.

We shall now assume that the congestion control function \( c(k) \) is not known exactly due to modeling errors in the network. Specifically, we shall assume that \( c_1 \leq c(k) \leq c_2 \). where \( c_1 \) and \( c_2 \) are some known positive scalars. In this way, the matrices \( \hat{A}_k \) themselves are not known exactly, but they can be modeled as \( \hat{A}_k = A_1 + \mu(c_2 - c_1)\theta A_2 \), where
\[ A_1 = \begin{bmatrix} 1 \\ \mu c_1 \end{bmatrix}, A_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}, \]
Then, we have \( \hat{A}_k = \hat{A} + D \theta E \), where
\[ \hat{A} = \begin{bmatrix} A_p & 0 \\ 0 & A_1 \end{bmatrix}, D = \begin{bmatrix} 0 \\ \mu(c_2 - c_1)I \end{bmatrix}, E = \begin{bmatrix} 0 & 0 \end{bmatrix} \]
Choose \( 0 \leq \theta \leq 1 \), the (10) can be rewritten as
\[ z_{k+1} = (\hat{A} + D \theta E) z_k + B K z_{k-T_k} + o_k \]

Remark 1. Different [13-18], where only communication parameters are adjusted. Compared with [20-22], we are capable to control network communication rather than under the given condition of communication. At some extent, modeling method in this paper is a new design direction of networked control systems in the sense that the characteristics of communication and control are characterized as a state-space model, and design a total control law to co-design communication and control systems. By contrast, the performance of networked control systems can be improved such that less conservative results can be derived.

Remark 2. In addition, networked communication delay is also taken into account when co-design for networked control systems, so the model obtained in this paper is general.
3. Robust Controller Design

In this section, we will present the sufficient condition on asymptotic stability of networked control systems based on co-design control scheme and the method of controller design.

Lemma 1: For any matrix $W$, $M$, $N$, $F(k)$ with $F^T(k)F(k) < I$, and any scalar $\varepsilon > 0$, the following inequality holds

$$W + MF(k)N + N^TF^T(k)M^T \leq W + \varepsilon MM^T + \varepsilon^{-1}N^TN$$

Lemma 2: Symmetric matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix}$$

where $S_{11}$, $S_{12}$ and $S_{22}$ are block matrices, and $S_{11}$, $S_{22}$ are square matrices. The following three conditions are equal in value.

1. $S < 0$;
2. $S_{22} < 0$, $S_{11} - S_{12}S_{22}^{-1}S_{12}^T < 0$;
3. $S_{11} < 0$, $S_{22} - S_{12}S_{11}^{-1}S_{12}^T < 0$.

Lemma 3: Assume that $a(\cdot) \in \mathbb{R}^{n_a}$, $b(\cdot) \in \mathbb{R}^{n_b}$, and $N \in \mathbb{R}^{n_a \times n_b}$. Then, for any matrices $X \in \mathbb{R}^{n_a \times n_a}$, $Y \in \mathbb{R}^{n_a \times n_b}$, $Z \in \mathbb{R}^{n_b \times n_b}$, the following holds:

$$-2a^TNb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} X & Y \end{bmatrix} \begin{bmatrix} a \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}$$

where \[
\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0.
\]

Theorem 1. For given scalars $h$ and a matrix $K$, the closed-loop system (13) is asymptotically stable, if there exist matrices $P_1 > 0$, $Q_i > 0$, $M_i$ ($i = 1, 2$) and $S > 0$, such that

$$\Theta = \begin{bmatrix} \Phi & N - P^T & 0 \\ * & -Q_1 & 0 \\ * & * & -Q_2 \end{bmatrix} < 0$$

and

$$\begin{bmatrix} W & N \\ N^T & S \end{bmatrix} \geq 0$$

where

$$\Phi = \begin{bmatrix} (h + 2)Q_1 + Q_2 & 0 \\ 0 & P_1 + hS \end{bmatrix} + hW + \text{sym} \begin{bmatrix} 0 & I \\ A - I + BK & -I \end{bmatrix}$$

$$+ \text{sym} \begin{bmatrix} N - P^T & 0 \\ 0 & BK \end{bmatrix}$$

and

$$P = \begin{bmatrix} P_1 & 0 \\ M_1 & M_2 \end{bmatrix}$$

Proof: We assume $a_k = 0$. If we augment $y_l = z_{k+1} - z_k$, it can be noted that $z_{k-1} = z_k - \sum_{l=k-1}^{k-1} y_l$. Similar to [26], the following equality holds:

$$\Pi = (\bar{A} + BK - I)z_k - y_k - BK \sum_{l=k-\tau_h}^{k-1} y_l = 0$$
Choosing Lyapunov function which is given by
\begin{equation}
V_k = V_{1,k} + V_{2,k} + V_{3,k} + V_{4,k} + V_{5,k}
\end{equation}
where
\begin{align*}
V_{1,k} &= z_k^T P_1 z_k, \\
V_{2,k} &= \sum_{i=k-1}^{k-1} z_i^T Q_1 z_i, \\
V_{3,k} &= \sum_{i=k-1}^{k-1} z_i^T Q_2 z_i, \\
V_{4,k} &= \sum_{i=1-h}^{k-1} \sum_{j=k+1}^{k+1} y_j^T S y_j.
\end{align*}
For simplicity, we will use the following notation \( e_k = [z_k^T \ y_k^T]^T \). By (15), it follows that
\begin{equation}
\Delta V_{1,k} = y_k^T P_1 y_k + 2z_k^T P_1 y_k + 2(z_k^T M_1^T + y_k^T M_2^T) \Pi
\end{equation}
where any weighting matrices \( M_1 \) and \( M_2 \) have compatible dimensions. By Lemma 3, we have
\begin{equation}
-2e_k^T P^T \begin{bmatrix} 0 \\ BK \sum_{l=k-\tau_k}^{k-1} y_l \end{bmatrix} \leq h e_k^T W e_k + \sum_{l=k-\tau_k}^{k-1} y_l^T S y_l
\end{equation}
and
\begin{equation}
\Delta V_{2,k} = z_k^T Q_1 z_k - z_{k-\tau_k}^T Q_1 z_{k-\tau_k} + \left( \sum_{l=k+1-\tau_k}^{k-1} - \sum_{l=k+1-\tau_k}^{k-1} \right) z_l^T Q_1 z_l
\end{equation}
where
\begin{equation}
\sum_{l=k+1-\tau_k}^{k-1} - \sum_{l=k+1-\tau_k}^{k-1} \leq \sum_{l=k+1-\tau_k}^{k-1} \leq \sum_{l=k+1-\tau_k}^{k-1} z_l^T Q_1 z_l
\end{equation}
and
\begin{equation}
\Delta V_{3,k} = z_k^T Q_2 z_k - z_{k-\tau_k}^T Q_2 z_{k-\tau_k}
\end{equation}
Similarly, we have
\begin{equation}
\Delta V_{4,k} = h z_k^T Q_1 z_k - \sum_{l=k-\tau_k}^{k-1} z_l^T T Q_1 z_l \leq h z_k^T Q_1 z_k - \sum_{l=k-\tau_k}^{k-1} z_l^T Q_1 z_l
\end{equation}
and
\begin{equation}
\Delta V_{5,k} = h z_k^T S z_k - \sum_{l=k-\tau_k}^{k-1} z_l^T S z_l
\end{equation}
Therefore, by (18)-(24), we can derive
\begin{equation}
\Delta V_k \leq y_k^T P_1 y_k + h e_k^T W e_k + (h + 2) z_k^T Q_1 z_k - z_{k-\tau_k}^T Q_1 z_{k-\tau_k} + z_k^T Q_2 z_k - z_{k-\tau_k}^T Q_2 z_{k-\tau_k} + h y_k^T S y_k
\end{equation}
\begin{equation}
+ 2e_k^T (N - P^T \begin{bmatrix} 0 \\ BK \end{bmatrix}) (z_k - z_{k-\tau_k}) + 2e_k^T P^T \begin{bmatrix} y_k \\ (A + BK - I) z_k - y_k \end{bmatrix}
\end{equation}
Let \( \eta_k = [e_k^T \ z_k^T \ y_k^T]^T \), we can obtain \( \Delta V_k \leq \eta_k^T \Theta \eta_k \). From (14), (15), we have \( \Delta V_k < 0 \). The closed-loop system (13) is asymptotically stable.

It is obvious that Theorem 1 cannot be directly used to obtain the control law of the system (13). Moreover, it should be noted that uncertainties still exist in (14),
Proof: Let $X$ as well as $\text{diag}(26)$ such that the following matrix inequalities

$$
\begin{bmatrix}
\Psi_1 & \Psi_2 & -\tilde{N}_1 & 0 & Z^T & Z^T & XE^T \\
* & \Psi_3 + \varepsilon^{-1}DD^T & BF - \tilde{N}_2 & 0 & Y^T & Y^T & 0 \\
* & * & -\tilde{Q}_1 & 0 & 0 & 0 & 0 \\
* & * & * & -\tilde{Q}_2 & 0 & 0 & 0 \\
* & * & * & * & -X & 0 & 0 \\
* & * & * & * & * & -\frac{1}{\varepsilon}S & 0 \\
* & * & * & * & * & * & -I
\end{bmatrix} < 0
$$

(26)

$$
\begin{bmatrix}
W_1 & W_2 & \tilde{N}_1 \\
* & W_3 & \tilde{N}_2 \\
* & * & 2X - \tilde{S}
\end{bmatrix} \geq 0
$$

(27)

hold for all admissible uncertainties. Where

$$
\Psi_1 = (h + 2)\tilde{Q}_1 + \tilde{Q}_2 + Z + Z^T + hW_1 + \tilde{N}_1 + \tilde{N}_1^T
$$

$$
\Psi_2 = hW_2 + Y + \tilde{N}_2^T + (\hat{A}X + BF - X - Z)^T
$$

$$
\Psi_3 = hW_3 - Y - Y^T
$$

Proof: Let $X = P^{-1}_1$, $Y = M^{-1}_2$, $Z = -M^{-1}_2M_1P^{-1}_1$, we have

$$
P^{-1} = \begin{bmatrix} X & 0 \\ Z & Y \end{bmatrix}
$$

Choosing

$$
N = P^T \begin{bmatrix} N_1 \\ N_2 \end{bmatrix}
$$

Pre- and post-multiplying $\text{diag}((P^{-1})^T, X, X)$ and its transpose to (14) respectively as well as $\text{diag}((P^{-1})^T, X)$ and its transpose to (15) respectively. Let $\tilde{S} = S^{-1}$, $\tilde{Q}_i = XQ_iX$, $\tilde{N}_i = N_iX$ $(i = 1, 2)$, $F = KX$, and by Lemma 2, (14) is equivalent to

$$
\begin{bmatrix}
\psi_1 & \varphi_2 & -\tilde{N}_1 & 0 & Z^T & Z^T \\
* & \psi_3 & BF - \tilde{N}_2 & 0 & Y^T & Y^T \\
* & * & -\tilde{Q}_1 & 0 & 0 & 0 \\
* & * & * & -\tilde{Q}_2 & 0 & 0 \\
* & * & * & * & -X & 0 \\
* & * & * & * & * & -\frac{1}{\varepsilon}S
\end{bmatrix} < 0
$$

(28)

where

$$
\varphi_2 = hW_2 + Y + \tilde{N}_2^T + ((\hat{A} + D\theta E)X + BF - X - Z)^T
$$

and (15) is equivalent to (27). By Lemma 1, Theorem 2 can be driven.

Remark 3. Different [13], in which only power and rate are designed so as to improve the performance of networks. It is worthwhile mentioned that the control sequence obtained in this paper is capable to not only minimize the distance between actual and desired SNR with increasing time, but also control plant such that the stability of the overall networked control systems can be guaranteed.
Remark 4. By constructing the state-space model of communication and control, time-varying congestion parameter $c(k)$ is inverted to the structural uncertainty of system, obviously, which makes it easy to analyze and control networked control systems by virtue of robust control theory.

4. Numerical Examples

Considering the following the application system

$$x_{k+1} = \begin{bmatrix} -0.2163 & 0.0627 \\ -0.8328 & 0.1438 \end{bmatrix} x_k$$

We choose step-size parameter $\alpha = 0.3$ and $\mu = 0.5$ in power control and rate control, respectively. Congestion parameter $c(k)$ belongs to the interval $[0.1, 0.6]$, that is $c_1 = 0.1$ and $c_2 = 0.6$. Without loss of generality, we assume that maximum bound of communication delay is $h = 2$. Choose

$$B = \begin{bmatrix} -0.5732 \\ 0.5955 \\ 0.5946 \\ -0.0188 \end{bmatrix}$$

In this case, we obtain augment system as follows

$$z_{k+1} = \begin{bmatrix} -0.2163 & 0.0627 & 0 & 0 \\ -0.8328 & 0.1438 & 0 & 0 \\ 0 & 0 & 1 & -0.3 \\ 0 & 0 & 0.05 + 0.25\theta & 0.65 - 0.25\theta \end{bmatrix} z_k$$

where $0 \leq \theta \leq 1$. By Theorem 2, the controller gain matrix $K = [0.0589 \ -0.0380 \ -1.3548 \ 1.3160]$ is obtained, under which the state response curves of the networked control system is shown in Fig.1 at the initial state value $x_0 = [1 \ 0.8 \ 0 \ -0.5]^T$, where $p_1$, $p_2$, $p_3$ and $p_4$ denote the state 1 and state 2 of plant, the actual SNR and error between the actual and the desired SNR, respectively. Fig. 2 shows the difference between the actual and the desired SNR. Obviously, the simulations show the effectiveness of the proposed method.
5. Conclusion

This paper focuses on co-design for networked control systems. The main contributions include two aspects. One is formulating communication and control systems as a state-space model based on the joint rate and power control model available and application systems. The other is co-design method is derived, which jointly controlling networks and plant so as to actual SNR trace to the desired SNR and guarantee stability of the overall networked control systems. Finally, A numerical example is given to illustrate the effectiveness of the proposed method.

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